

# SICOPOLIS-AD v2: tangent linear and adjoint modeling framework for ice sheet modeling enabled by automatic differentiation tool Tapenade

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## Summary

Simulation COde for POLythermal Ice Sheets ([SICOPOLIS](#)) is an open-source, 3D dynamic/thermodynamic model that simulates the evolution of large ice sheets and ice caps. SICOPOLIS has been developed continuously and applied to problems of past, present, and future glaciation of Greenland, Antarctica, and others. It uses the finite differences discretization on a staggered Arakawa C grid and employs the shallow ice and shallow shelf approximations, making it suitable for paleoclimatic simulations. We present a new framework for generating derivative code, i.e., tangent linear, adjoint, or Hessian models, of SICOPOLIS. These derivative operators are powerful computational engines to efficiently compute comprehensive gradients or sensitivities of scalar-valued model output, including least-squares model-data misfits or important quantities of interest, to high-dimensional model inputs (such as model initial conditions, parameter fields, or boundary conditions). The new version 2 ([SICOPOLIS-AD v2](#)) framework is based on the source-to-source automatic differentiation (AD) tool [Tapenade](#) which has recently been open-sourced. The switch from a previous AD tool ([OpenAD](#)) used in SICOPOLIS-AD version 1 to Tapenade overcomes several limitations outlined here. The framework is integrated with the SICOPOLIS model's main trunk and is freely available.

## Statement of need

The two contemporary ice sheets, Greenland and Antarctica, are dynamic entities whose evolution is governed by a set of nonlinear partial differential equations (PDEs) that describe the conservation of mass, momentum, and energy, as well as constitutive laws for the material properties of ice. In general, these equations cannot be solved analytically but must be solved numerically. Ice sheet models are a computer representation of these PDEs. They require as input parameters (i) initial conditions of the state of the ice sheet, (ii) surface boundary conditions, such as precipitation, (iii) basal boundary conditions, such as geothermal flux, and (iv) model parameters, such as flow law parameters. Despite advances in numerical modeling of ice sheets, the effects of ad-hoc initialization and the uncertainties in these independent input parameters propagate to quantities of interest (QoI), such as future projections of sea-level rise, which is of economic and societal importance ([Schinko et al., 2020](#)). It is thus desirable to evaluate the sensitivities of our QoI to these independent input variables.

In the context of ice sheet modeling, sensitivities of model-data misfits or other QoI are a key

42 ingredient for performing model calibration, state estimation, or uncertainty quantification (UQ),  
43 which guide the improvement of model simulations through PDE-constrained gradient-based  
44 optimization.

45 SICOPOLIS-AD v2 leverages the recently open-sourced AD tool Tapenade (Hascoët & Pascual,  
46 2013) to generate code for the adjoint model of the open-source ice sheet model, SICOPOLIS  
47 (Greve, 1997; Greve et al., 2011; Greve & Blatter, 2009). Sensitivities can be calculated using  
48 a single forward and adjoint model evaluation, instead of the  $\mathcal{O}(N)$  forward model evaluations.  
49 Empirically, one adjoint model evaluation is about 5-10 times as expensive as a forward model  
50 run. The adjoint computation is highly efficient for calculating sensitivities when  $N$  is large  
51 (typically,  $N \sim 10^4 - 10^6$ ).

52 The functionality to generate a tangent linear version of the forward model is also included,  
53 which was not available in SICOPOLIS-AD v1. This is valuable for UQ of the inferred parameters,  
54 as well as uncertainty propagation to Qols. It can also be used to verify the results of the  
55 adjoint model.

## 56 Target Audience

57 This package is intended as a resource that enables sensitivity analysis, model calibration, and  
58 uncertainty quantification of a continent-scale ice sheet model. Our package is also intended  
59 to serve as a guide for future work in the application of open-source AD tools for physics-based  
60 simulation codes written in Fortran.

## 61 State of the field

62 SICOPOLIS is among the early thermo-mechanical models to simulate contemporary and  
63 paleo continental-scale ice sheets (Greve, 1997). Like similar models developed at the time,  
64 including Glimmer and its successor, the Community Ice Sheet Model (CISM) (Rutt et al.,  
65 2009), GRISLI (Ritz et al., 1996), the model by Huybrechts (1990), or by Pollard & DeConto  
66 (2009), SICOPOLIS has been based (until recently) on the so-called shallow ice approximation  
67 to simplify the Cauchy stress tensor in the momentum conservation equation, implemented on  
68 a regular, finite-difference mesh. See Hindmarsh (2004) for other approximations commonly  
69 employed in ice sheet models. This approximation enabled the efficient computation of ice  
70 sheet evolution over long, glacial/deglacial cycles.

71 The last decade has seen substantial advances in continental-scale ice sheet modeling, with the  
72 development of several new ice sheet models (some of which are on unstructured grids using  
73 finite element methods), notably the Ice Sheet System Model ISSM (Larour et al., 2012), the  
74 Parallel Ice Sheet Model PISM (Bueler et al., 2007), Elmer/Ice (Gagliardini et al., 2013), or the  
75 MPAS-Albany Land Ice MALI (Hoffman et al., 2018). While designed to capture the evolution  
76 of short-term, fast-flowing, or fast-changing outlet glaciers via horizontal stress contributions,  
77 these models have so far found little application in paleo-ice sheet simulations due to their  
78 extensive computational costs. A compilation of the suite of ice sheet models used for the  
79 latest Ice Sheet Model Intercomparison Project, Phase 6 (ISMIP6) in support of the IPCC's  
80 Sixth Assessment Report is available in Payne et al. (2021) and Nowicki et al. (2016).

81 Relevant to this paper, of all the time-evolving models listed, apart from SICOPOLIS-AD  
82 (Heimbach & Bugnion, 2009; Logan et al., 2020), only the ISSM model and variants thereof  
83 possess adjoint model codes which have been generated, in part, using automatic differentiation  
84 (Hascoët & Morlighem, 2018; Larour et al., 2014). Multi-centennial and longer integrations  
85 with the adjoint model have so far been conducted only with SICOPOLIS-AD.

## 86 Features

87 AD tools such as the commercial TAF ([Giering & Kaminski, 1999](#)) and the open-source  
88 OpenAD ([Utke et al., 2008](#)) have been used previously with SICOPOLIS ([Heimbach & Bugnion,  
89 2009](#); [Logan et al., 2020](#)). OpenAD is no longer actively developed because it is based on  
90 the Open64 compiler which ceased development in 2011. The differentiation of SICOPOLIS,  
91 therefore, must be performed using a different tool. Compared to OpenAD, the Tapenade  
92 enabled implementation has the following advantages:

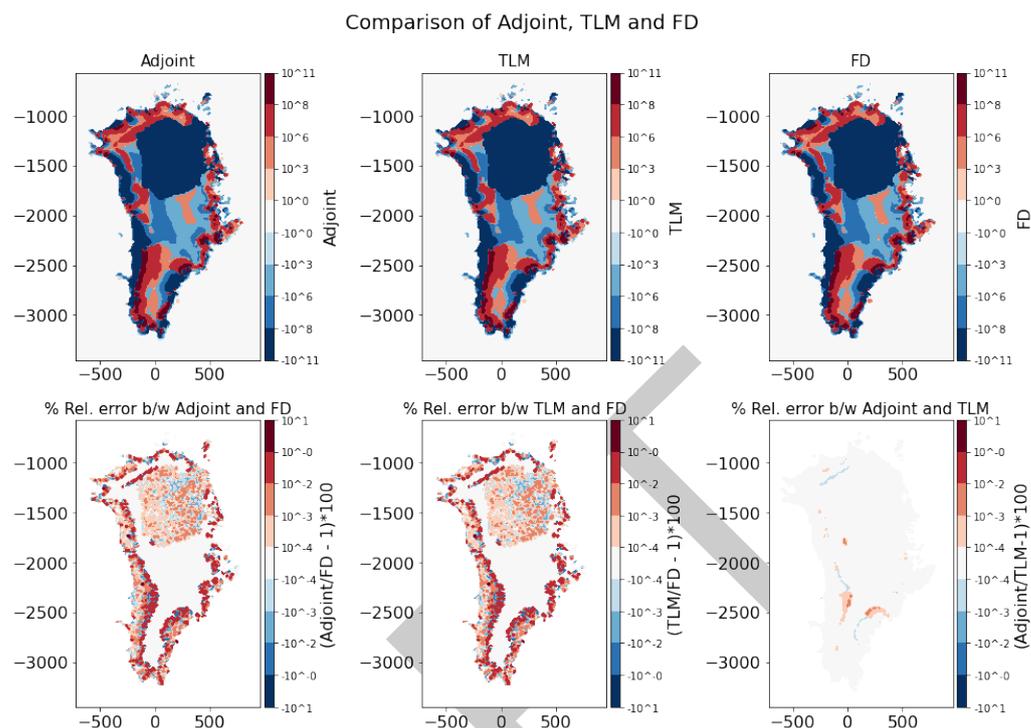
- 93     ▪ It is up-to-date with the latest SICOPOLIS code.
- 94     ▪ The AD tool Tapenade is open-source and actively maintained.
- 95     ▪ A new tangent linear code generation capability is introduced.
- 96     ▪ The AD-generated codes can accept [NetCDF](#) inputs.
- 97     ▪ The external library [LIS](#), its tangent linear code, and adjoint code are correctly incorpo-  
98     rated which can improve the simulation of Antarctic ice shelves and Greenland outlet  
99     glaciers.
- 100    ▪ [Gitlab-CI](#), a [Docker](#), and the [pytest](#) framework are leveraged for Continuous Integration  
101    (CI) to track changes in the trunk that “break” the AD-based code generation.
- 102    ▪ The entire code is parsed by Tapenade, preventing cumbersome manual maintenance of  
103    subroutines to initialize the adjoint runs.
- 104    ▪ Python scripts are provided for quick setup of the compilation, I/O, and execution  
105    processes based on user-provided metadata.
- 106    ▪ The setup is [well-documented](#), along with tutorials.

## 107 Software requirements and external usage

108 SICOPOLIS-AD v2 is built on top of the ice sheet model SICOPOLIS and uses Tapenade to  
109 differentiate this model. All the prerequisites of using SICOPOLIS and Tapenade need to be  
110 satisfied. A Python installation is needed to use the automation tools.

## 111 Example

112 We illustrate the use of our tool with the example of a steady-state simulation of the Greenland  
113 ice sheet under modern climate conditions. The corresponding SICOPOLIS configuration header  
114 file, `v5_grl16_bm5_ss25ka`, is provided as a reference template in the standard SICOPOLIS  
115 distribution. We shorten the total integration time to 100 simulated years to keep the  
116 computational cost of the tangent linear and finite differences reasonable. Our Qol (i.e.,  
117 dependent variable) is the total volume of the ice sheet at the end of the run (`fc`). The  
118 sensitivity is evaluated with respect to the geothermal heat flux, `q_geo` (independent variable),  
119 a 19,186-dimensional field. The results are shown in Figure 1.



**Figure 1:** Validation exercise for adjoint (ADM) and tangent linear (TLM) models using the finite differences (FD) results for the sensitivity of  $fc$  with respect to  $q_{geo}$ . The upper row shows the sensitivities computed using the adjoint model (reverse-mode AD), tangent linear (forward-mode AD), and finite differences, respectively. The bottom row illustrates the relative error between (ADM, FD), (TLM, FD), and (ADM, TLM) respectively. For the bottom row, note that the values of relative error are only shown for points where the value of the gradient is “significant”, i.e. within 4 orders of magnitude of the maximum absolute value of the gradient.

120 The results show good agreement between all three modes used to evaluate this sensitivity.  
 121 The error is less than 6% between AD-generated (adjoint/tangent linear codes) and finite  
 122 differences at all but one point with “significant” gradient values, i.e. within 4 orders  
 123 of magnitude of the maximum absolute value of the finite differences gradient. The relative  
 124 error between the AD-generated adjoint and tangent linear models is less than 0.002% at  
 125 all points with values within 4 orders of magnitude of the maximum absolute value of the  
 126 finite differences gradient. However, the adjoint model is much faster than the other two, as  
 127 shown in Table 1, because the number of evaluations of the latter two scales linearly with the  
 128 parameter dimension ( $\sim \mathcal{O}(N)$ ). The discrepancy will be even larger if a finer mesh is used.

**Table 1:** Comparison of the time taken by various methods to evaluate the gradient for a scalar objective function with respect to a 19,186-dimensional 2D field (16 km mesh) in a typical SICOPOLIS run. The runs are performed on Intel Xeon CPU E5-2695 v3 nodes (2.30 GHz clock rate, 35.84 MB L3 cache, 63.3 GB memory).

Gradient calculation method	Time (in seconds) for 16 km mesh
Finite Differences	$1.640 \times 10^5$
Tangent Linear Model	$9.793 \times 10^4$
Adjoint Model	$2.214 \times 10^1$

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