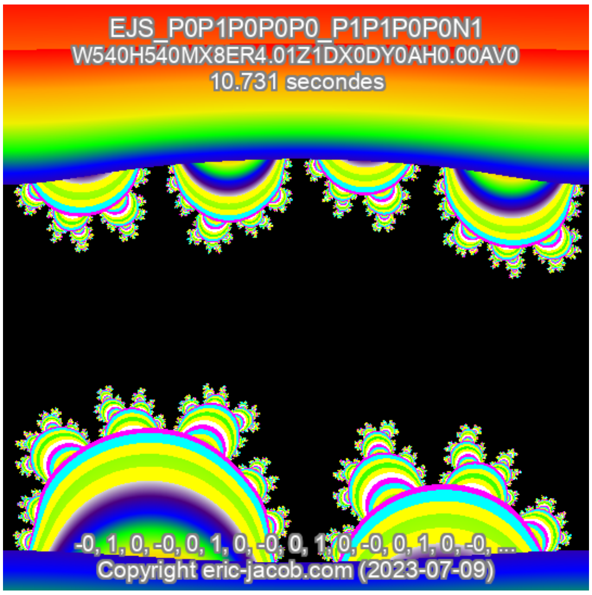
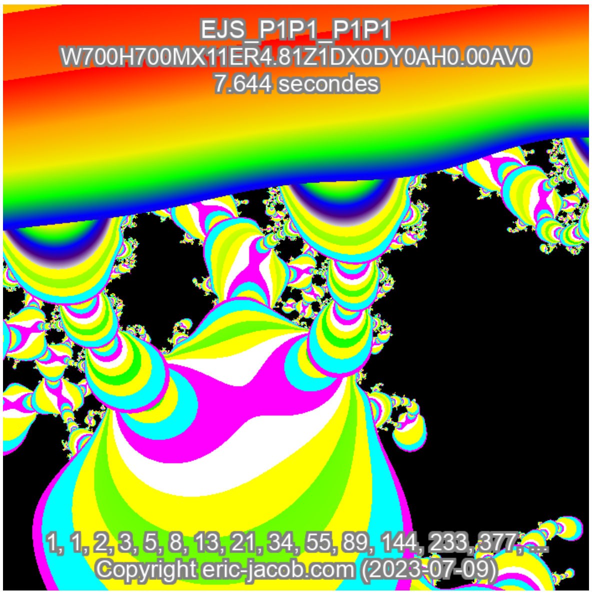
An innovative approach for the calculation of Linear Recurrences

General global recurrence formula/equation

Global generating function for recurrent sequences





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04/02/2023

MATHEMATICS AND SCIENCES

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ABSTRACT

This article presents an innovative and efficient approach for calculating linear recurrences, offering a powerful and versatile solution to solve these complex mathematical problems. The proposed approach is based on a specific global formula and a perception of physical space that efficiently generates the successive terms of any recurrent sequence. Although the complete details of these different formulas are not disclosed in this initial article, readers can visit the website https://eric-jacob.com for more information and access the online equation calculator tool for all kinds of recurrent sequences.

INTRODUCTION

Linear recurrences constitute an important domain of applied mathematics, with applications in various fields such as computer science, macroscopic and quantum physics, chemistry, genetics and botany, and finance. The ability to solve these recurrences quickly and efficiently can have a significant impact on solving complex problems in these domains. In this article, an innovative approach is presented to calculate the terms of a linear recurrence without using traditional iterative solving methods. Instead of revealing the complete formulas in this article (since mathematical works cannot be protected even if their value is immense), readers are encouraged to visit my website to discover the existence of such a theory (since 2019). I also invite them to explore the potential of these global formulas obtained with a small web server using PHP and/or Python, and to use the online calculation tools to verify the authenticity of an innovative theory.

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Linear recurrences are a mathematical concept that finds applications in various scientific and technical fields. Here are some examples of domains where linear recurrences are used:

- Mathematics: Linear recurrences are part of the theory of sequences and series, which is an important branch of mathematics. They are used to study the properties of numerical sequences and to solve mathematical problems related to recurrent sequences.
- Computer Science: Linear recurrences are used in computer science for modeling and analyzing recurrent data structures. They are commonly used in analyzing the efficiency of recursive algorithms and in designing dynamic programming algorithms.
- Physics: Linear recurrences are used in physics to model recurrent dynamic systems, such as oscillations, feedback systems, chains of coupled oscillators, etc. They play a key role in the study of periodic phenomena and in solving recurrent differential equations.
- Botany: Linear recurrences can be used in botany to model plant growth and development processes. For example, linear recurrences can be used to study the growth of stems or branches of a tree.
- Chemistry: Linear recurrences can be used in chemistry to model recurrent chemical reactions or transformation processes. They can help predict the temporal evolution of concentrations of different chemical species involved in a reaction.

PROCEDURE

The website allows for two different approaches: partial knowledge of a sequence of integers, where the site will propose similar or neighboring linear recurrences if possible, or perfect knowledge of the sequence. If the linear recurrence is perfectly identified, the mathematical formula for integers, real numbers, and complex numbers will be immediately provided, even for high degrees of recurrence.

The calculations rely on a global formula primarily intended for specialized mathematical software. However, I have endeavored to adapt it for PHP and then for Python. This has led me to further explore the real and complex space to which this function applies. The sequences currently described on the web (outside of my website) are limited in number, and their equations often appear in an outdated and approximate form, often representing only a projection of physical reality onto the space of integers. Many pieces of information are sacrificed to display an appearance of simplicity, clarity, or insight.

As for myself, based on a theoretical understanding of the entire universe, both quantum and macroscopic (including all existing entities and phenomena, from subatomic particles to macroscopic objects such as stars, galaxies, and cosmic structures), I preferred to develop a mathematical theory suited to my conception of the cosmos and my demonstration needs on a limited platform (PHP, web server). A global theory that unifies the universe from the smallest to the largest inevitably requires a global theory of linear recurrences. It has been almost 30 years since I last published anything, but the recent advent of AI has prompted me to come out of my reserve, fearing that these machines will unravel all secrets and leave no merit to human beings who do not possess the first machines.

I have adapted the general formula so that it can be used with the PHP language in the real world. The real world operates with symmetries and limited quantities of matter… and I have found that despite the significant simplifications required for PHP calculations, the final results were similar in all aspects between a basic theory requiring powerful calculations and a simplified theory requiring much fewer resources. By disregarding symmetry phenomena, the calculations quickly exceed 10^40 and reach 10^300 and beyond; in PHP, using such numbers compromises the precision and relevance of results, and it is challenging to provide accurate results within acceptable timeframes for Google with a small web server, especially for numbers with precision exceeding 6 digits. In reality, why should we separate mathematics from physical reality when they are supposed to be a tool for describing it? In other words, it is not necessary to subtract two infinities if their difference yields a finite result. Therefore, calculations can be simplified to fit the real world. This is essential because PHP is very limited in terms of computing power, just as nature is. Furthermore, PHP does not provide calculation tools for complex numbers or mathematical tools for calculating roots, fractions, simplifications, and others. I had to build all of that to accomplish this demonstration.

My website is optimized enough to demonstrate the potential of the theory. However, it is not perfect, as the precision is on the order of 10^-6 for "hard cases" (1.6% of linear recurrences) and 10^-14 for "easy cases". When numbers become larger, exceeding 6 to 14 digits, the calculations on the website become imprecise, and the results can even be completely incorrect for degrees > 10. However, the theory remains valid, and thus the underlying (hidden) basic formula remains viable, even in PHP. It is the demonstration that fails, not the theory. PHP is not suited for advanced mathematics unless you use mathematical libraries, which have the drawback of making the website very slow. As search engines like Google penalize slow pages and overall slow sites, you quickly end up relegated to the depths of Google research for wanting to perform precise calculations.

I attempted to create a graphical representation of linear recurrences. According to my overall understanding of the universe, everything can be connected to the construction of a vast and infinite structure made up of infinitesimal bricks that are not really bricks, but already constructions in themselves. However, these quantum elements of the world are not perceptible and can only be apprehended through statistics, wave phenomena, and holographic effects, whose projections become solid thanks to the existence of symmetries, oppositions, cancellations, and accumulations. It is strange to observe that an hologram can become alive and autonomous, while its constituent elements are themselves alive and invisible. Each sequence can project itself into physical space according to a crystallographic pattern. A two-dimensional simulation actually translates into a three-dimensional representation through complex space, while the structure of this construction mainly depends on the amount of matter used. The evolution of matter/energy is governed by sequences (linear recurrences) and thresholds. The website used in Python mode allows for the configuration of virtual, and even real, mini-quantum universes.

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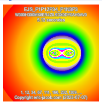
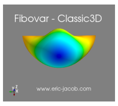
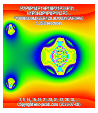
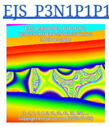
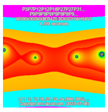
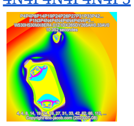
EXEMPLES

The general theory allows, in principle, to evaluate and determine the formulas for any particular case. Here, I will take random examples.

Nomenclature	Linear recurrence	Generating function and Equation
EJS_P1P12P34_P1N3P3  	a(n) = 1*a(n-3)-3*a(n-2)+3*a(n-1) a(0)=1, a(1)=12, a(2)=34	$EJSGF(x) = -\frac{1\cdot x^2+9\cdot x^1+1\cdot x^0}{1\cdot x^3-3\cdot x^2+3\cdot x^1-1\cdot x^0}$ $a(n) = \frac{11n^2}{2} - \frac{11n}{2} + 1$ 1, 12, 34, 67, 111, 166, 232, 309, 397, 496, 606, 727, 859, 1002, 1156, 1321, 1497, 1684, 1882, 2091, 2311, 2542, 2784, 3037, 3301, 3576, 3862, 4159, 4467, 4786
EJS_P2P9P14P19P19P21P28P31_N1P3N3P1P0P1N3P3 	a(n)=-1.a(n-8) + 3.a(n-7) -3.a(n-6) + 1.a(n-5) + 0.a(n-4) + 1.a(n-3) -3.a(n-2) + 3.a(n-1) a(0)=2 a(1)=9 a(2)=14 a(3)=19 a(4)=19 a(5)=21 a(6)=28 a(7)=31	$EJSGF(x) = -\frac{-2\cdot x^7+5\cdot x^5-5\cdot x^4+2\cdot x^3-7\cdot x^2+3\cdot x^1+2\cdot x^0}{-1\cdot x^8+3\cdot x^7-3\cdot x^6+1\cdot x^5+1\cdot x^3-3\cdot x^2+3\cdot x^1-1\cdot x^0}$ $a(n) = -\frac{n^3}{15} + \frac{4n^2}{5} + \frac{16n}{15} - \frac{164306721\sin\left(\frac{2\pi n}{5}\right)}{64065380} - \frac{76891811\sin\left(\frac{4\pi n}{5}\right)}{80584391} - \frac{92100937\cos\left(\frac{2\pi n}{5}\right)}{60770966} - \frac{7982578\cos\left(\frac{4\pi n}{5}\right)}{94515079} + 3.6$ 2, 9, 14, 19, 19, 21, 28, 31, 32, 26, 20, 17, 8, -5, -27, -51, -74, -105, -142, -190, -242, -295, -358, -429, -513, -603, -696, -801, -916, -1046
EJS_P3N1P1P1_P1P1P1P1 	a(n)=1.a(n-4) + 1.a(n-3) + 1.a(n-2) + 1.a(n-1) a(0)=3 a(1)=-1 a(2)=1 a(3)=1	$EJSGF(x) = -\frac{-2\cdot x^3-1\cdot x^2-4\cdot x^1+3\cdot x^0}{1\cdot x^4+1\cdot x^3+1\cdot x^2+1\cdot x^1-1\cdot x^0}$ $a(n) = -\frac{598999(-1)^n 0.774803997405886^n}{287591} + 2\cdot 0.81827598250762^n\left(\frac{63863267\sin\left(\frac{76729703n}{46104017}\right)}{88870514} - \frac{1118463\cos\left(\frac{76729703n}{46104017}\right)}{80503969}\right) + \frac{137411\cdot 1.92756088382292^n}{1242391}$ 3, -1, 1, 1, 4, 5, 11, 21, 41, 78, 151, 291, 561, 1081, 2084, 4017, 7743, 14925, 28769
EJS_P3P7P12P12P18P27P27P31_P9P9P9P9P9P9P9P9 	a(n)=9.a(n-8) + 9.a(n-7) + 9.a(n-6) + 9.a(n-5) + 9.a(n-4) + 9.a(n-3) + 9.a(n-2) + 9.a(n-1) a(0)=3 a(1)=7 a(2)=12 a(3)=12 a(4)=18 a(5)=27 a(6)=27 a(7)=31	$EJSGF(x) = -\frac{-923\cdot x^7-684\cdot x^6-441\cdot x^5-288\cdot x^4-186\cdot x^3-78\cdot x^2-20\cdot x^1+3\cdot x^0}{9\cdot x^8+9\cdot x^7+9\cdot x^6+9\cdot x^5+9\cdot x^4+9\cdot x^3+9\cdot x^2+9\cdot x^1-1\cdot x^0}$ $a(n) = -\frac{10715787(-1)^n 0.975500001793641^n}{748819} + 0.978384038989203^n\left(\frac{1060801\sin\left(\frac{4917677\pi n}{6534693}\right)}{14915707} - \frac{23549633\cos\left(\frac{4917677\pi n}{6534693}\right)}{1664564}\right) + 0.978384038989203^n\left(\frac{3213589\sin\left(\frac{4917677\pi n}{6534693}\right)}{45185621} - \frac{1397189835\cos\left(\frac{4917677\pi n}{6534693}\right)}{98757883}\right) + 0.986171901220608^n\left(-\frac{142151310\sin\left(\frac{1803951\pi n}{3579934}\right)}{68374337} - \frac{500328371\cos\left(\frac{1803951\pi n}{3579934}\right)}{32136369}\right) + 0.986171901220608^n\left(-\frac{96401500\sin\left(\frac{1803951\pi n}{3579934}\right)}{46368821} - \frac{900061924\cos\left(\frac{1803951\pi n}{3579934}\right)}{57811477}\right) + 0.99551002168252^n\left(-\frac{244174900\sin\left(\frac{804934\pi n}{3181137}\right)}{49733901} - \frac{1007033545\cos\left(\frac{804934\pi n}{3181137}\right)}{69903221}\right) + 0.99551002168252^n\left(-\frac{384794083\sin\left(\frac{804934\pi n}{3181137}\right)}{78375422} - \frac{1233169147\cos\left(\frac{804934\pi n}{3181137}\right)}{85600421}\right) + \frac{77\cdot 9.99999991^n}{62624563}$ 3, 7, 12, 12, 18, 27, 27, 31, 1233, 12303, 122967, 1229562, 12295512, 122954958, 1229549337, 12295493127, 122954930991, 1229549298813, 12295492877403, 122954927667327
EJSGF_P4P4P8P14P19P24P26P27P31P33P42_P1N3P4N4P4N4P4N4P4N4P3 	a(n)=1.a(n-11) -3.a(n-10) + 4.a(n-9) -4.a(n-8) + 4.a(n-7) -4.a(n-6) + 4.a(n-5) -4.a(n-4) + 4.a(n-3) -4.a(n-2) + 3.a(n-1) a(0)=4 a(1)=4 a(2)=8 a(3)=14 a(4)=19 a(5)=24 a(6)=26 a(7)=27 a(8)=31 a(9)=33 a(10)=42	$EJSGF(x) = -\frac{15\cdot x^{10}-12\cdot x^9+10\cdot x^8-7\cdot x^7+6\cdot x^6-9\cdot x^5+9\cdot x^4-10\cdot x^3+12\cdot x^2-8\cdot x^1+4\cdot x^0}{1\cdot x^{11}-3\cdot x^{10}+4\cdot x^9-4\cdot x^8+4\cdot x^7-4\cdot x^6+4\cdot x^5-4\cdot x^4+4\cdot x^3-4\cdot x^2+3\cdot x^1-1\cdot x^0}$ $a(n) = n^2 - \frac{41n}{5} - \frac{369198938\sin\left(\frac{\pi n}{5}\right)}{55867751} - \frac{31409891\sin\left(\frac{2\pi n}{5}\right)}{23372368} - \frac{25179701\sin\left(\frac{3\pi n}{5}\right)}{27663842} - \frac{44043880\sin\left(\frac{4\pi n}{5}\right)}{95218093} - \frac{833974081\cos\left(\frac{\pi n}{5}\right)}{82221992} - \frac{18374665\cos\left(\frac{2\pi n}{5}\right)}{61248886} + \frac{895221\cos\left(\frac{3\pi n}{5}\right)}{6262198} - \frac{20289683\cos\left(\frac{4\pi n}{5}\right)}{67632277} + 25.6$ 4, 4, 8, 14, 19, 24, 26, 27, 31, 33, 42, 62, 86, 112, 137, 162, 184, 205, 229, 251, 280, 320, 364, 410, 455, 500, 542, 583, 627, 669

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Nomenclature	Linear recurrence	Generating function and Equation
EJSGF_P4P4P12P16P23P23P24P28P31P36P36P37_P1P1P1P1P1P1P1P1P1P1P1P1	a(n)=1.a(n-12) + 1.a(n-11) + 1.a(n-10) + 1.a(n-9) + 1.a(n-8) + 1.a(n-7) + 1.a(n-6) + 1.a(n-5) + 1.a(n-4) + 1.a(n-3) + 1.a(n-2) + 1.a(n-1) a(0)=4 a(1)=4 a(2)=12 a(3)=16 a(4)=23 a(5)=23 a(6)=24 a(7)=28 a(8)=31 a(9)=36 a(10)=36 a(11)=37	$EJSGF(x) = -\frac{-200\cdot x^{11}-165\cdot x^{10}-129\cdot x^{9}-103\cdot x^{8}-78\cdot x^{7}-58\cdot x^{6}-36\cdot x^{5}-13\cdot x^{4}-4\cdot x^{3}+4\cdot x^{2}+4\cdot x^{0}}{1\cdot x^{12}+1\cdot x^{11}+1\cdot x^{10}+1\cdot x^{9}+1\cdot x^{8}+1\cdot x^{7}+1\cdot x^{6}+1\cdot x^{5}+1\cdot x^{4}+1\cdot x^{3}+1\cdot x^{2}+1\cdot x^{1}-1\cdot x^{0}}$ $a(n) = -\frac{1486735671(-1)^n 0.914710604346873^n}{83131139} + 0.916811457750391^n\left(\frac{28349847\sin\left(\frac{5655613\pi n}{6752643}\right)}{36316487} - \frac{885505929\cos\left(\frac{5655613\pi n}{6752643}\right)}{50153533}\right) + 0.916811457750393^n\left(\frac{768612\sin\left(\frac{5655613\pi n}{6752643}\right)}{984601} - \frac{190923823\cos\left(\frac{5655613\pi n}{6752643}\right)}{10813597}\right) + 0.923344968499906^n\left(-\frac{33701851\sin\left(\frac{6061690\pi n}{8982717}\right)}{80645986} - \frac{1693788984\cos\left(\frac{6061690\pi n}{8982717}\right)}{94594991}\right) + 0.923344968499906^n\left(-\frac{26854531\sin\left(\frac{6061690\pi n}{8982717}\right)}{64260866} - \frac{1246837824\cos\left(\frac{6061690\pi n}{8982717}\right)}{69633593}\right) + 2\cdot 0.935072285088212^n\left(-\frac{49745977\sin\left(\frac{345907\pi n}{676358}\right)}{98579458} - \frac{1731731762\cos\left(\frac{345907\pi n}{676358}\right)}{95841755}\right) + 0.953436177638439^n\left(-\frac{214490330\sin\left(\frac{3140998\pi n}{9064591}\right)}{72914511} - \frac{304327885\cos\left(\frac{3140998\pi n}{9064591}\right)}{17136896}\right) + 0.953436177638442^n\left(-\frac{283881675\sin\left(\frac{3140998\pi n}{9064591}\right)}{96503621} - \frac{1518185477\cos\left(\frac{3140998\pi n}{9064591}\right)}{85489986}\right) + 0.979690426123957^n\left(-\frac{230772551\sin\left(\frac{1642496\pi n}{9240463}\right)}{49305225} - \frac{989727254\cos\left(\frac{1642496\pi n}{9240463}\right)}{50277363}\right) + 0.979690426123958^n\left(-\frac{466683043\sin\left(\frac{1642496\pi n}{9240463}\right)}{99708186} - \frac{1645762648\cos\left(\frac{1642496\pi n}{9240463}\right)}{83603443}\right) + \frac{2645437\cdot 1.99975550093977^n}{80918560}$ 4, 4, 12, 16, 23, 23, 24, 28, 31, 36, 36, 37, 274, 544, 1084, 2156, 4296, 8569, 17115, 34206, 68384, 136737, 273438, 546840, 1093643, 2187012, 4373480, 8745876, 17489596, 34974896, 69941223

One of the peculiarities of linear recurrences is to structurally encompass the laws of statistics (see the appendix). That is why these sequences also allow for the incorporation of quantum space, whose current description is not accurate due to a lack of foundation in what constitutes its reality.

SYMBOLIC CALCULATIONS

The obvious link between linear recurrences and geometric and physical reality means that all real numbers (or fractions) presented on the website actually come from a mathematical expression derived from a symbolic formula. The overall theory allows obtaining these expressions easily using tools other than PHP and HTML for degrees 1 to 4.

For example, please refer to my comments on the following web pages:

- EJS_P1P8P29P74P153P275P450P687_N1P3N3P1P0P1N3P3
- EJS_P0P1P0P0P0_P1P1P0P0N1
- EJS_P1P2P3P6_N5P0P5P0
- EJS_P1P3_P1P1
- EJS_P1P4_P1P1
- EJS_P12P44P96P170_P1N2P0P2
- EJS_P1P3P7P11_P1P0P4P0

I may explore these aspects (roots and symbolic forms for degrees >= 5) with more precision someday, but it is not my current priority.

RESULTS AND CONCLUSION

I have developed a powerful global mathematical formula that allows for the direct and immediate calculation of terms for each specific linear recurrence, with sufficient accuracy to provide a convincing demonstration even with a programming language like PHP. If the numbers in the sequence do not exceed 5 to 6 digits, the precision can be consistently maintained. Any errors are solely related to the hardware limitations and the low precision I had to adopt to ensure that web pages are returned to users within reasonable timeframes for search engines. These general formulas take advantage of the specific properties of each linear recurrence and therefore offer solutions that apply to an infinite range of sequences. The precise details of each recurrence, including coefficients and the associated generating function, are available on the website https://eric-jacob.com. Visitors can use the online calculation tool to obtain sequence equations for which they have previously been unable to find a formula.

In conclusion, this innovative approach to computing linear recurrences offers a powerful and efficient alternative to traditional methods. This promising approach opens up new possibilities for effectively solving linear recurrences and is expected to have significant applications in various research and engineering fields. However, please note that without recognition, the complete theory will not be published, and obtaining the formulas for linear recurrences for each specific case currently incurs a nominal fee. An auction for the global formulas is being considered, as there is no way to value such work once it has been disclosed. However, I am still exploring the procedures and will consider any potential proposals.

I have also undertaken work to conceptually unify the quantum and macroscopic worlds. These works may be published within my lifetime, provided I have a minimum level of recognition that allows me to disclose them safely.

My next mathematical publications focus on another general parameterizable formula that, similar to linear recurrences, delivers an infinite number of equations (without using division and modulo operators) capable of generating prime number sequences infinitely (an infinity of sequences, each with an infinity of prime numbers). However, this formula does not provide all prime numbers. The website will also serve as a demonstration tool. These sequences are an exciting feature to explore and evoke thoughts of Fibonacci, Lucas, but in a style of prime number trees.

THE CONTEXT

The context is a comprehensive theory about the universe, which led me to delve into some mathematics to establish its foundations. However:

1. It is nearly impossible to be heard as a non-academic scientist.
2. Any work disclosed will be stolen, and the entire history of this work will be rewritten by those stronger than you.
3. As a researcher outside the system, most people are not interested in your work and do not understand it, which makes you unheard and lonely.

EQUIPMENT

1. Low-power computer dating back to 2015 used to develop the mathematical theories that will be presented on the website.
2. No financial resources, no support, no assistance.
3. Lots of hard work, calcium, and magnesium.
4. Renting a VPS server in December 2022 to host the website.

REFERENCES

1. There are no references.

APPENDIX

Linear recurrences can be linked to statistical laws through examples using Newton's binomial.

- Pascal's Triangle: The sum of the binomial coefficients for each line of the triangle forms a recursive sequence. Each term is the sum of the two preceding terms in the sequence. For example, the sequence of binomial coefficients for the fourth line is 1, 3, 3, 1, which corresponds to the linear recurrence 1, 4, 6, 4, 1.
- The Fibonacci sequence is a linear recurrence where each term is the sum of the two preceding terms. The sequence is generated using the binomial coefficient (1, 1) in the expansion of Newton's binomial.
- The Catalan sequence is another recursive sequence that can be generated

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from binomial coefficients. Each term in the Catalan sequence is obtained by multiplying the previous term by a certain ratio of binomial coefficients.

These examples demonstrate how linear recurrences, using Newton's binomial and binomial coefficients, are linked to statistical laws, probability calculations, distribution modeling, sampling, and combinatorial problem-solving.

- Binomial Distribution: The binomial distribution is a commonly used probability distribution in statistics to model random experiments with two possible outcomes (success or failure) and a fixed number of trials. Binomial coefficients are involved in calculating the probabilities associated with this distribution. By using Newton's binomial, we can compute the probabilities of obtaining a certain number of successes in a certain number of trials.
- Sampling: When conducting sampling, binomial coefficients are used to calculate the number of ways to obtain a certain number of specific events in a larger set. For example, the number of ways to obtain k successes out of n trials can be calculated using binomial coefficients.
- Combinatorics: Binomial coefficients are used in various combinatorial problems, such as counting combinations, permutations, partitions, etc. These problems have applications in modeling different statistical situations.
- Approximations of Distributions: Binomial coefficients are also used to approximate more complex statistical distributions. For instance, the binomial distribution can be used as an approximation of the Poisson distribution under certain conditions, using binomial coefficients to calculate the associated probabilities.

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