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# A possible variant of the PRS solution

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## Summary

It may be advantageous to eliminate the astrometric unknowns of the PRS stars, rather than the set zero points.

## The PRS solution

The observation equation for star $i$ in set $j$ is essentially of the form

$$\underline{a}'_{ij}\, \underline{x}_i + c_j = h_{ij} \qquad (1)$$

where $\underline{x}_i$ is a 5-vector of astrometric parameter corrections for star $i$, $c_j$ is the set zero point, and $h_{ij}$ the abscissa residual from Step 1.

For the whole mission there are about 1830 sets and 1000 PRS stars, so the total number of unknowns is 1830 + 5 × 1000 = 6830 (plus some global parameters which are of no consequence here). The number of unknowns can be reduced by eliminating, in the course of accumulating the normals, all 'local' unknowns, i.e. those which appear only in a limited sequence of successive observations.

### 'Old method'

If observations (abscissae) are chronologically ordered, i.e. set by set, then the $c_j$ may be regarded as 'local' and eliminated. This results in a system of normal equations for the 5000 astrometric parameters ($\underline{x}_i$).

### 'New method'

If observations have been sorted after star numbers before the PRS solution, one may instead regard the astrometric parameters as 'local', and eliminate then by processing the observations one star at a time. The resulting normals then only have 1830 unknowns ($c_j$).

## Discussion

The two methods are mathematically equivalent, since the full normals, obtained from (1), are the same; they differ only by the order in which unknowns are eliminated. The pseudosolution approach for fixing the system orientation and rotation proposed in NDAC/LO/018 is equally applicable to both methods. The algorithm for forming and solving the normals is the same, only that star and set indices ($i$ and $j$) are interchanged.

Having to sort the set results (abscissae) according to star numbers may seem like a great disadvantage with the new method. But such a sorting is anyway required for Step 3 and double-star detection, so it can just as well be done before the PRS solution as immediately after (before Step 3).

On the other hand there may be important advantages with the new method:

(1) The resulting system of normals is smaller – 1830 instead of 5000 for a 2.5 year mission. For provisional PRS solutions after (say) 0.5 and 1.0 year, the system is proportionally smaller. Thus many more PRS solutions can easily be made, using different PRS stars and/or at different stages of the mission. Provisional positions will be more accurate, and solutions for the attitude and slit errors will benefit from this.

(2) We are not necessarily limited to only 1000 PRS stars. It is rather likely that perhaps 2000 - 3000 FK5 stars are good enough to serve as PRS stars, and we can then easily include them without increasing the number of unknowns in the resulting normals. The processing time increases <u>linearly</u> with the number of PRS stars, instead of cubically, as with the old method. It may even be possible to use 50 - 60,000 PRS stars in the final solution, once all slit errors and multiple stars have been eliminated.

(3) The power of detecting faulty observations (abscissae), remaining slit errors and unexpected double stars is greatly enhanced. When processing the observations of star $i$, part of it consists in forming the 'subnormals' for $\underline{x}_i$:

$$\left( \sum_j \underline{a}_{ij} \underline{a}'_{ij} \right) \underline{x}_i + \sum_j \underline{a}_{ij} c_j = \sum_j \underline{a}_{ij} h_{ij} \qquad (2)$$

If the second term is neglected (small $c_j$), we have a 5 × 5 system that can be solved for $\underline{x}_i$ and for which residuals can be analyzed (exactly as in Step 3). This should eliminate many erroneous observations and dubious stars already <u>before</u> they are added to the normals for $c_j$. With the old method, however, one must actually go through a full pass of the PRS solution (with 5000 unknowns) before having any sensible residuals to examine!

(4) There is no need for a special process backsolving the PRS astrometric parameters, since this is now identical to Step 3. In contrast, with the old method, $c_j$ must be backsolved after $\underline{x}_i$, by some special process or by 'Step 1-after-Step 2'. In other words, the new method is more economical in terms of computation, data storage and programming, by solving <u>only</u> for the unknowns immediately required, viz. the set zero point.

The 'new method' is actually the original Step 2 proposed by me in 1976 (Oct 19), except that at that time all programme stars were used as PRS stars.