Complex Fuzzy Abstraction: The Brain Logic

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Abstract:

Taking abstraction as the starting point, we build a complex, *self-organizing fuzzy logic system*. Such a system, being built on top of abstraction as the base, turns out to be just a special outcome of the laws of abstraction. As the system is self-organizing, it runs automatically towards optimization. Using such a system in neural networks, we may come as close as possible to the workings of the human brain. The *abstract fuzzy optimization* is seen to follow a Gaussian distribution.

Introduction:

There are quite a large number of differences between the way present day computers work and the way the brain works. The very way in which basic processing is done by computers varies vastly from the way the brain processes information. Computers, working on binary logic, can take into account only *high* or *low* states. They can have a

number of inputs that are processed by a single processing unit. After being processed, the input(s) can have a single output or a number of outputs. On the other hand, brain cells seem to be able to process information using abstract fuzzy logic that leads to more energy optimization. In fact, estimates suggest that present day computers use up ten million times more energy to process the same amount of information that the human brain does.

This does not take into account, however, the accuracy and speed of decision making by the two systems for large enough data inputs. The basic difference between the today's computers and the brain lie in the fact that while computers have only two logical states, the brain uses can have many such states and in-between states simultaneously generating patterns similar to attractor maps. Decisions are asymptotic functions of such maps in the case of the brain, while in computers the decisions are only approximate points in the output space. As such, all waypoints in the input system itself can act as decision making units inside the brain. The way in which this happens is no longer linear (as in computers), but nonlinear. This nonlinear information processing by the brain is vastly superior to the linear processing used by computers in arriving at decisions.

Abstraction lies at the heart of the complex, self-organizing fuzzy logic that is used by the brain. In fact, this logic is the direct fallout and a special case of abstraction. The *Theory of Abstraction* and the principle of *Zero Postulation* describes directivity towards optimized solutions and cluster formation in the decision making space as inherent properties of the system itself. They must also be able to describe the formation of various structures and patterns in the decisions that the system can arrive at. As such, it is of great interest for us to investigate how these structures are formed in the decision space. A theory that is able to describe the world in totality has to keep the number of basic postulates it depends upon to zero or near zero.

Reductionism hits a dead end in this regard. On the other hand, abstraction as the starting point of building up a theory may be seen to be of fitting use. It would be much more than a new way of tackling the problem. Even abstract postulates do away with the shackles that bind our theories into the system and bar them from being total descriptions of the system. The abstraction we are talking about here may be defined as, "Postulation of non-postulation" or, in other words, "A system of postulation that gives equal weights to all possible solutions inside the system and favors none of such solutions over others."

Abstraction automatically gives rise to optimized solutions within the universal set of all possible solutions, as has been shown in this book. It is these optimized solutions that make up and drive the non-abstract parts of the world, while the non-optimized solutions remain 'hidden' from the material world, inside the abstract world. Zero postulation or abstraction as the basis of theory synthesis allows us to explore even imaginary and chaotic non-favored solutions as possibilities. With no postulation as the fundamental basis, we are thus able to pile up postulated results or favored results, but not the other way round. We keep describing such implications of abstraction in this book. We deal with the abstraction of observable parameters involved in a given system and formulate a similar basis of understanding them.

Let us consider the example of a three-point isolated system. Let the points be 'A', 'B' and 'C'. Let A and B be decision points, whereas, C be situated anywhere on the straight line joining A and B. The decision flow of both A and B tends to move in all possible directions. These possible directions include the directions towards each other. Thus, at point C, for obvious reasons, an additional effect will be felt due to the tendency of decision to flow from A to B and from B to A, as compared to all other directions.

The points A, B and C being considered parts of an isolated system and all three points being assumed fundamentally similar (with the only difference that A and B contain decisions, while C is empty), the factors R and S must be equal. Thus, we have:

$$\frac{F}{T} = c\frac{\lambda}{D}$$

D being considered the *decision distance* between A and C and x the distance between A and B (say), the *decision distance* between B and C is $D - x$. This *decision distance* can be any length of any given dimensions (as determined by the scaling-ratio of observations, the simplest of dimensions being two) between two points in the decision space.

The effect on C due to the decision-point A can thus be written as,

$$\frac{F_A}{T_A} = c\frac{\lambda_A}{x}$$

Similarly, the effect on the empty point C due to the decision-point B is,

$$\frac{F_B}{T_B} = c\frac{\lambda_B}{D - x};$$

where F_A and F_B are the respective values of flows towards the point C due to A and B, respectively. T_A and T_B are the respective values of time and λ_A and λ_B are the respective values of the differences in concentrations of decisions between A and B.

Substituting x in the above two equations, we have,

$$c\frac{T_A\lambda_A}{F_A} = D - c\frac{T_B\lambda_B}{F_B} \quad \ldots(1)$$

Considering the points to be having equal factors, i.e., considering $\lambda_A = \lambda_B = \lambda$ (say),$F_A = F_B = F$ (say) and $T_A = T_B = T$ (say), equation (1) reduces to,

$$c\frac{T\lambda}{F} = D - c\frac{T\lambda}{F}$$

i.e.,

$$\frac{F}{T} = 2c\left(\frac{\lambda}{D}\right) \quad \ldots(2)$$

Equation (2) describes fundamentally the effect (i.e., the flow F in time T) of two decision-points having same factorial conditions regarding one or a number of already existing decisions.

Pattern Generation:

Considering a collection of such points and applying a statistical approach, the logistic equation for $\left(\frac{F}{T}\right)$ can be written as,

$$2c\left(\frac{\lambda}{D}\right)_{t+1} = 2Kc\left(\frac{\lambda}{D}\right)_t\left[1 - 2c\left(\frac{\lambda}{D}\right)_t\right]$$

i.e.,

$$\left(\frac{\lambda}{D}\right)_{t+1} = K\left(\frac{\lambda}{D}\right)_t \left[1 - 2c\left(\frac{\lambda}{D}\right)_t\right] \quad \ldots (3)$$

where K is a constant.

Also, the quadratic map can be written as,

$$2c\left(\frac{\lambda}{D}\right)_{t+1} = K - \left(2c\frac{\lambda}{D}\right)_t^2$$

i.e.,

$$2c\left(\frac{\lambda}{D}\right)_{t+1} = K - 4c^2\left(\frac{\lambda}{D}\right)_t^2 \quad \ldots (4)$$

All trajectories described by the quadratic map become asymptotic to $-\infty$ for

$$K < -0.25 \text{ and } K > 2$$

As we deal with the flow of a given decision towards one given point or the effects on a given point, the expression for the attractor for each such point can be written as,

$$\left(2c\frac{\lambda}{D}\right)^* = \left(1 - \frac{1}{K}\right) \quad \ldots (5);$$

where $O < K < B$.

$\left(2c\frac{\lambda}{D}\right)^*$ is a point in the desired dimensional plot into which the trajectories seem to crowd. As we do not need to deal with more than one attractor or periodic point, the trajectories will tend to revisit only the attractor point concerned, to the desired level of accuracy of observations and calculations.

For $K \geq 3$, the trajectory behaviour becomes increasingly sensitive to the value of K. There are a few more points to be noted regarding the dependence of the trajectory behavior on the values of K:

1. For $K \leq 1$, the attractor is a fixed point and has a value Ψ.

2. For $1 < K < 3$, the attractor is a fixed point and its value is > 0 but < 0.667.

3. For $3 \leq K \leq 3.57$, period doubling occurs, with the attractor consisting of $2, 4, 8,$ etc., periodic points as K increases within that range.

4. For $3.57 < K \leq 4$, we have the region of chaos, where the attractor can be erratic (chaotic with infinitely many points) or stable.

For all calculations, the desired conditions may be placed at the attractor. A trajectory never gets completely and exactly all the way into an attractor though, but only approaches it asymptotically. In the region of chaos, we apply the method of searching for windows or zones of K-values for which iterations from any initial conditions will produce the periodic attractor, instead of a chaotic one. For the logistic equation (3),

the most common such zone lies at $K \approx 3.83$ and for the quadratic map(4), at $K \approx 1.76$.

Let us consider a given representation with fractal dimension D_F. The fractal dimension is purely geometrical, i.e., it only depends on the shape of the representation. A suitable probability measure $d\mu$, according to the particular phenomenon considered is assigned to the given representation. A coarse grained probability density, as the decision of the hypercube Λ_i of size l is defined as,

$$P_i(l) = \int_{\Lambda_i} d\mu\,(x)$$

where $i = 1,2,3, \dots, N(l)$.

The information dimension D_I is such that,

$$\sum_{i=1}^{N(l)} P_i \ln(P_i) \simeq D_I \ln(l) \qquad \dots(6);$$

where $D_I \leq D_F$.

The number of boxes containing the dominant contributions to the total decision and thus relevant part of the information, is,

$$N_R(l) \propto l^{-D_I} \qquad \dots(7).$$

For each box Λ_i, $D_I = D_F$ for a uniform distribution. When $D_I < D_F$, the measure itself may be called fractal since it is singular with respect to the uniform distribution,

$$P^* = \frac{1}{N(l)} \propto l^{D_F}$$

For each box Λ_i .Thus, $\frac{P_i}{P_i^*}$ can diverge in the limit of vanishing l.

Simulations of the decision-information scaling yields,

$$\langle P_i(l)^q \rangle \equiv \sum_{i=1}^{N(l)} P_i(l)^{q+1}$$

$$\propto l^{q.d_{q+1}} \qquad \ldots (8).$$

The d_q are the Renyi dimensions which generalize the information dimension $D_I = d_1$ as well as the fractal dimension $D_F = d_0$. If the d_q's are not constant, anomalous scaling is to be employed and, as the order q varies, the amount of the difference $D_q - D_F$ gives a first rough measure of the heterogeneity of the probability distribution.

The moment generic observables A computed on scale l is such that,

$$\langle A(l)^q \rangle \propto l^{g(q)} \qquad \ldots (9)$$

Anomalous scaling, i.e., a non-linear shape of the function $g(q)$ is the more common situation, where one does not require unnecessarily to consider only a finite number of

scaling components. In some cases, one may observe strong time variations in the degree of chaoticity. This intermittency phenomenon involves an anomalous scaling with respect to time-dilations identifying the parameter e^{-t} with the parameter l used in spatial dialations of the decision space. A measure of the degree of intermittency requires the introduction of infinite sets of exponents which are analogous to the Renyi dimensions and can be related to a multifractal structure given by the dynamical system in the functional trajectory space.

The Grassberger-Procaccia correlation dimension ν is defined by considering the scaling of the correlation integral,

$$C(l) = \lim_{M\to\infty} \frac{1}{M^2} \sum_i \sum_{j\neq i} \theta\,(l - |x_i - x_j|)\,;$$

where θ is the Heaviside step function and $C(l)$ is the percentage of pairs (x_i, x_j) with distance $|x_i - x_j| \leq l$.

In the limit $l \to 0$,

$$C(l) \propto l^{\nu}.$$

In general,

$$\nu \leq D_F.$$

ν is a more relevant scaling index than D_F since it is related to the point probability distribution on the attractor, while D_F cannot take into account an eventual homogeneity in the visit frequencies.

Let us define the number of points in an F-dimensional spherical representation of the decision space, with radius l and centre at x_i as,

$$n_i(l) = \lim_{M\to\infty} \frac{1}{M-1} \sum_{j\neq i} \theta\,(l - |x_i - x_j|) \qquad \ldots(10).$$

We must introduce a whole set of generalized scaling exponents

$$\langle n(l)^q \rangle = \lim_{M\to\infty} \frac{1}{M} \sum_{i=1}^{M} n_i(l)^q \propto l^{\emptyset(q)}$$

where $\emptyset(1) = \nu$.

Considering a uniform partition of decision space into boxes of size l it is convenient to introduce the probability $P_K(l)$ that a point x_i falls into the K^{th} box. In this case, the moments of P_K can be estimated by summing up the boxes,

$$\langle p(l)^q \rangle = \sum_{K=1}^{N(l)} P_K(l)^{q+1} \propto l^{q.d_{q+1}} \qquad \ldots(11)$$

A moment of reflection shows:

$$\emptyset\,(q)/q = d_{q+1}$$

because of the ergodicity $n_i(l) \sim P_K(l)$, if x_i belongs to the K^{th} box and since one can use either an 'ensemble' average (weighted sum over the boxes) or a 'temporal' average (sum of the time evolution $x(l)$).

The fractal dimension for $q = -1$ is,

$$D_F = d_0 = -\emptyset(-1)$$

while the correlation dimension is,

$$\nu = d_2 = \emptyset(1)$$

According to the Theory of Physical Abstraction, each point x should have the same singularity structure,

$$\Delta V_x(r) \propto r^h, h = \frac{1}{3} \quad \ldots (12)$$

In other words $\varepsilon(x)$ tends to be smoothly distributed in a region of R^3. The eddy turn-over time and the error deviation per unit decision at scale r are defined as,

$$t(r) \sim \frac{r}{\Delta V(r)} \quad \ldots (13)$$

and

$$E(r) \sim \Delta V(r)^2 \quad \ldots (14)$$

The transfer rate of error deviation per unit decision from the eddy at scale r to smaller eddies is then given by

$$\tilde{\varepsilon}(r) = \frac{E(r)}{t(r)} \sim \frac{\Delta V(r)^3}{r} \qquad \ldots (15)$$

Since

$$\varepsilon_x(r) = \left(\frac{1}{r^3}\right) \int_{\Lambda_x(r)} \varepsilon(y) d^3y,$$

[$\Lambda_x(r)$ is a cube of edge r around x we have,

$$\int_{\Lambda_x(r)} \varepsilon(y) d^3y \sim r^3 \qquad \ldots (16)$$

$r \to 0$ means r in the initial range and the regions containing a large part of $\varepsilon(x)$ are a physical approximation of a fractal structure. In this β —model approach,

$$\int_{\Lambda_x(r)} \varepsilon(y) d^3y \propto \begin{cases} r^{D_F} & if\ x \in S \\ 0 & if\ x \notin S \end{cases}$$

in an equivalent way

$$\Delta V_x(r) \propto \begin{cases} r^h & if\ x \in S \\ 0 & if\ x \notin S; \end{cases}$$

where $h = (D_F - 2)/3$

At scale r, there is only a fraction,

$$r^{3-D_F} \propto \frac{r^{-D_F}}{r^{-3}}$$

occupied by active eddies.

The transfer error deviation from the eddy at scale l_n (active eddy) to the scale l_{n+1} is $\varepsilon_n \propto \frac{v_n^3}{l_n}$.

Since, the error deviation transfer rate is constant in the cascade process, for $\beta = 2^{D_F-3}$, we have,

$$\varepsilon_n = \beta \varepsilon_{n+1}, \frac{v_n^3}{l_n} = \beta \frac{v_{n+1}^3}{l_{n+1}} \quad \ldots (17)$$

Iterating, we have,

$$v_n \propto l_n^{1/3} (l_n / l_0)^{\frac{D_F-3}{3}}$$

Each eddy at scale l_n is divided into eddies of scale l_{n+1} in such a way that the energy transfer for a fraction β of eddies increases by a factor $\frac{1}{\beta}$, while it becomes zero for the other ones.

In order to generalize the β-model, we have at scale l_n, N_n active eddies. Each eddy $l_n(k)$ generates active eddies covering a fraction of volume $\beta_{n+1}(k)$. k labels the mother-eddy and $k = 1, \ldots, N_n$.

Since the rate of energy transfer is constant among mother-eddies and their effects, we have,

$$\frac{\nu_n(k)^3}{l_n} = \beta_{n+1}(k)\frac{\nu_{n+1}(k)^3}{l_{n+1}} \quad \ldots(18)$$

The iteration of ν_n gives an eddy generated by a particular history of fragmentations $[\beta_1, \ldots, \beta_n]$, such that,

$$\nu_n \propto l_n^{1/3}\left(\prod_{i=1}^{n}\beta_i\right)^{-1/3} \quad \ldots(19)$$

The fraction of volume occupied by an eddy generated by $[\beta_1, \ldots, \beta_n]$ is $\prod_{i=1}^{n}\beta_i$, such that,

$$\langle|\Delta V(l_n)|^P\rangle \propto l_n^{P/3}\int\prod_{i=1}^{n} d\beta_i \;\beta_i^{(1-P/3)}P(\beta_1, \ldots, \beta_n)$$

With no correlation among different steps of the fragmentation, i.e., with $P(\beta_1, \ldots, \beta_n) = \prod_{i=1}^{n}P(\beta_i)$, the exponent concerned,

$$\zeta_P = \frac{P}{3} - \ln_2\{\beta^{(1-P/3)}\} \quad \ldots(20)$$

For a given transport of decision, between an initial and a final point, let the trajectory of the initial point $x_o = x(o)$ be denoted by,

$$x(t) = f^t(x_o)$$

Expanding $f^t(x_o + \delta x_o)$ to linear order, the evolution of the distance to a neighbouring trajectory $x_i(t) + \delta x_i(t)$ is given by the Jacobian matrix J,

$$\delta x_i(t) = \sum_{j=1}^{d} J^t(x_o)_{ij} \ \ \delta x_{oj}\,,$$

$$J^t(x_o)_{ij} = \frac{\delta x_i(t)}{\delta x_{oj}} \qquad \ldots(21)$$

A trajectory of a decision as moving on a flat surface, as is the simplest decision space (it being a plane), is specified by two position coordinates and the direction of motion. The Jacobian matrix describes the deformation of an infinitesimal neighborhood of $x(t)$along the transport. Its eigenvectors and eigenvalues give the directions and the corresponding rates of expansion or contraction. The trajectories that start out in an infinitesimal neighborhood separate along the unstable directions (those whose eigenvalues are greater than unity in magnitude), approach each other along the stable directions (those whose eigenvalues are less than unity in magnitude), and maintain their distance along the marginal directions (those whose eigenvalues equal unity in magnitude).

Holding the hyperbolicity assumption (i.e., for large n the prefactors a_i, reflecting the overall size of the system, are overwhelmed by the exponential growth of the unstable

eigenvalues Λ_i, and may thus be neglected), to be justified, we may replace the magnitude of the area of the ith strip $|B_i|$ by $\frac{1}{|\Lambda_i|}$ and consider the sum,

$$\lceil n = \sum_{i}^{n} \frac{1}{|\Lambda_i|};$$

where the sum goes over all periodic points of period n. We now define a generating function for sums over all periodic orbits of all lengths,

$$\lceil z = \sum_{n=1}^{\infty} \lceil n \, z^n \qquad \ldots(22)$$

For large n, the nth level sum tends to the limit $\lceil n \to e^{-n\gamma}$, so the escape rate γ is determined by the smallest $z = e^{\gamma}$ for which equation (22) diverges,

$$\lceil z \approx \sum_{n=1}^{\infty} (ze^{-\gamma})^n = \frac{ze^{-\gamma}}{1 - ze^{-\gamma}} \qquad \ldots(23)$$

Making an analogy to the Riemann zeta-function, for periodic orbit cycles,

$$\lceil z = -z \frac{d}{dx} \sum_{p} \ln\left(1 - t_p\right);$$

$\lceil(z)$ is a logarithmic derivative of the infinite product

$$\frac{1}{\zeta(z)} = \prod_{p}\left(1 - t_p\right), t_p = \frac{z^{n_p}}{|\Lambda_p|} \qquad \ldots (24)$$

This represents the dynamical zeta function for the escape rate of the trajectories of decision-transport.

Abstraction says that points inside the decision space cluster to form *decision directions* of a given property, at the desired scaling-ratio. Let us consider one such system of decision making, inside which its constituent points have the tendency to form clusters.

Prediction:

In such transactions, the family of evolution-maps f^t form a group. The evolution rule f^t is a family of mappings of strips of transport B, that we may consider, such that,

1) $f^0(x) = x$

2) $f^t[f^{t'}(x)] = f^{t+t'}(x)$

3) $(x, t) \to f^t(x)$ from $B \times R$ into B is continuous;

where t represents a time interval and $t \in R$.

For infinitesimal times, we may write the trajectory of a given transaction as,

$$x(t + \tau) = f^{t+\tau}(x_0)$$

$$= f[f(x_0, t), \tau] \qquad \ldots (25)$$

The time derivative of this trajectory at point $x(t)$ is,

$$\left.\frac{dx}{d\tau}\right|_{\tau=0} = \partial_\tau f[f(x_0,t),\tau]|_{\tau=0} = \dot{x}(t) \qquad \ldots(26)$$

The vector field is a generalized velocity field,

$$\dot{x}(t) = v(x)$$

If x_q represents an equilibrium point, the trajectory remains stuck at x_q forever. Otherwise, the trajectory passing through x_0 at time $t = 0$ may be obtained by,

$$x(t) = f^t(x_0) = x_0 + \int_0^t d\tau\, v[x(\tau)], x(0)$$

$$= x_0 \qquad \ldots(27)$$

The Euler integrator, which advances the trajectory by $\delta\tau \times \text{velocity}$ at each time step is,

$$x_i = x_i + v_i(x)\delta\tau.$$

This may be used to integrate the equations of the dynamics concerned.

In our decision/perception plane a fuzzy set $\mathrm{L_F}$ may be defined as:

$\mathrm{L_F}$: L→[0,1], …(28).

where L is a domain of elements (universe of discourse).

For every particular value of a variable $L_i \in L$ the degree of membership to fuzzy set L_F is $L_F(L_i)$.

Equation (28) describes how we can incorporate a fuzzy complex number or FCN in our decision/perception plane.

L_F in the universe of discourse L is defined by the complex membership grade function $\mu L_F(L_i)$. The complex membership grade function or CMG is defined as:

$$\mu L_F(L_i) = L_F(L_i) e^{ic} \quad \ldots(29).$$

The Cartesian representation of CMG for $\mu L_F(L_i) = \mu L_F(c_i + ir_i)$ is:

$$\mu(c_i, r_i) = \mu(c_i) + ir_i \quad \ldots(30)$$

And, the polar representation is:

$$c_i e^{isr} \quad \ldots(31),$$

the scaling factor **S** being in the interval $(0, 2\pi]$.

The degree of fulfillment or DOF of any given proposition follows CMG and lies in the interval [0,1].

According to the definition of transformation of coordinates:

$$\mu(c_i, r_i) \Leftrightarrow c_i e^{isr}$$

The operators $\wedge$ and $\vee$ defining t-norm and s-norm respectively and $\mathrm{L_i}$ being the set of fuzzy numbers concerned, the fuzzy set of a function of $\mathrm{L_i}$ has the membership function:

$$\mu(c_i^{'}, r_i^{'}) = \vee_{c_i^{'} = f(\mathrm{L_i})}[\mu(c_1, r_1) \wedge \mu(c_2, r_2) \wedge \mu(c_3, r_3) \ldots \wedge \mu(c_n, r_n)]$$

Using Lyapunov exponents for the measure L, and replacing $2c\left(\frac{\lambda}{D}\right)$ by a quantity $'\tau'$, we have:

$$\frac{d}{d\tau} f^n(L) = \frac{\delta n}{\delta o}$$

i.e.,

$$\frac{\delta n}{\delta o} = \prod_{i=1}^{n} f^{'}(L_i) \qquad \ldots(32).$$

$$b = \frac{1}{n} \log_e \left(\frac{\delta n}{\delta o}\right)$$

i.e.,

$$b = \frac{1}{n} \sum_{i=1}^{n-1} \log_e |f^{'}(L_i)| \qquad \ldots(33).$$

where b is a constant (the local slope of all possible measures), and

$$\Psi = \lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \log_e |f'(L_i)| \qquad \ldots(34).$$

where Ψ is a constant.

Distribution:

Signal processing time in *abstract fuzzy optimization* seems to follow a Gaussian curve.

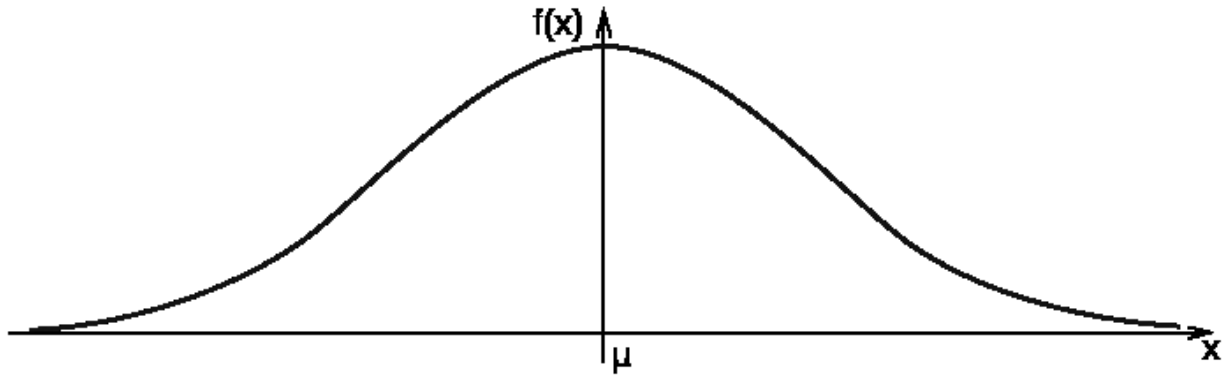

Fig. 1: A Gaussian Curve.

For such a curve, if the height of the peak is R, the mean is μ and the standard deviation σ, then,

$$f(C, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{C^2}{2\sigma^2}}$$

μ being taken to be zero in the super-complex plane of decision making.

For the simplest case of a two dimensional decision plane,

$$L_i = \frac{1}{2\pi\sigma^2} e^{-\frac{(C^2+R^2)}{2\sigma^2}} \qquad \ldots (35).$$

A two-dimensional elliptical Gaussian function for such a case, may be expressed as:

$$f(C,R) = L_i e^{-a(C-C_0)^2+2b(C-C_0)(R-R_0)+c(R-R_0)^2}$$

where a, b and c forms a positive definite matrix as:

$$\begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

A measure for precision in any given direction of decision making is given by the covariance matrix,

$$V = \frac{\sigma^2}{\sqrt{2}P_C P^2} \begin{bmatrix} \frac{3}{2c} & 0 & \frac{-1}{a} \\ 0 & \frac{2c}{a^2} & 0 \\ \frac{-1}{a} & 0 & \frac{2c}{a^2} \end{bmatrix}$$

Where, the precision of the system is represented by P.

Depending upon the value of precision involved, the number of activated states follows a sigmoid distribution:

$$n(t) = \frac{1}{1+e^{-t}}$$

Conclusion:

Natural processes, including decision making follows non-linear pathways that give rise to emergence phenomena. Patterns arise in the whole that cannot be wholly attributed to the sum of the parts. The whole decision making process is way more than the sum of the individual processes involved. From the *Theory of Abstraction*, we know how *information energy* changes with changes in the scaling ratio. The same can be observed in the decision making process too. The difference in dissipation energy information (and as such deviation in a given direction of decision making), which tends to infinity as the number of constituent points inside it tends to infinity. In this respect, at large enough scaling-ratios, the universe seems to work in a similar way as the brain does.

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