HIPPARCOS

Selection of Scanning Law Parameters (K, ξ)

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Summary

A criterion is given for selecting the K-value for best homogeneity of sky coverage. The scanning law $K = 6.4$, $\xi \simeq 43^{\circ}$ is proposed as the "best" choice for a 2.5 year mission.

1. Definition of Scanning Law

The scanning law is composed of two independent motions: the motion of the z axis on the sky, and the spinning motion about the z axis. Here we are only concerned with the first component, given in ecliptical coordinates by $\lambda_z(t)$, $\beta_z(t)$ for $t_o \leq t \leq t_o + T$.

The position of the +z axis, (λ_z, β_z), can also be given with respect to the sun $(\lambda_s, 0)$ by means of the two angles ξ (revolving angle) and ν (phase angle). The relevant transformation is

$$\beta_z = \arcsin(\sin\xi \sin\nu) \tag{1}$$

$$\lambda_z = \lambda_s + \arctan(\tan\xi \cos\nu) \tag{2}$$

In the following we assume that ξ is constant ($< \frac{1}{2}\pi$) while $\nu(t)$ is given by the quasi-uniform scanning law,

$$\nu = \bar{\nu} - \cos\bar{\nu} / (K \tan\xi) \tag{3}$$

$$\bar{\nu} = \bar{\nu}_o + K(\lambda_s - \lambda_{so}) \tag{4}$$

where subscript $_o$ signifies initial conditions (at $t = t_o$). The speed of the z axis on the sky is approximately

$$v = \dot{\lambda}_s \, (K^2 \sin^2\xi + \cos^2\nu)^{\frac{1}{2}} \tag{6}$$

v is not strictly constant, because of the term $\cos^2\nu$, and because of the annual variation of $\dot{\lambda}_s$. The extreme values are

$$v_{min} = 0.016915\ |K|\ \sin\xi \qquad \text{[rad/day]} \quad (\text{summer, } \nu = \pm\tfrac{1}{2}\pi) \tag{7}$$

$$v_{max} = 0.017490\ (K^2\sin^2\xi + 1)^{\frac{1}{2}} \qquad \text{[rad/day]} \quad (\text{winter, } \nu = n\pi) \tag{8}$$

Note that K may be positive or negative depending on the sense of revolution; also the spin may be in either sense about the z axis. Our only convention is that the +z axis is on the sunny hemisphere ($\xi < \frac{1}{2}\pi$).

2. Best values of K

For a given mission length (T years) there are certain discrete K-values which minimise the variations of the scan pattern with longitude. These values satisfy the criterion

"the two integers KT and 2T are relatively prime" (9)

E.g. for T = 2.5, the values K = 5.6, 6.4, 6.8, 7.2, etc would be satisfactory; while for T = 3.0, K = $5\frac{2}{3}$, $6\frac{1}{3}$, $7\frac{2}{3}$, etc would do.

Figs. 1 - 3 will illustrate this condition. To interpret the figures, two things must be remembered:

Firstly, that the scanning pattern is repeated periodically along the ecliptic with period $2\pi/K$ in longitude. The nodes along the periphery in Fig. 1 mark the corresponding longitudes of the z axis, i.e. its positions at a certain, but arbitrary, phase angle (modulo 2π). Successive nodes, which are also equidistant in time (interval = 1/K year), are labelled 0, 1, 2, ... and symbolically connected by arcs which however do not show the actual path of the z axis on the sky.

Secondly, the motion of the -z axis is just as important as that of the +z axis. It is marked in the figures by nodes labelled -0, -1, -2, ... and connected with dashed arcs. It is recalled that a star will be scanned by the two FOV's when its angle from the +z axis, Θ, is in the interval $\frac{1}{2}\pi - \frac{1}{2}w$ to $\frac{1}{2}\pi + \frac{1}{2}w$, w being the transverse width of the FOV's. At that time the angle from the star to -z is however $\pi - \Theta$, which is in the same interval. Thus the -z axis plays exactly the same role for the observability as +z, and both must be plotted to give a correct impression of the sky coverage.

Figure 1 shows the scanning repetition pattern for T = 2.5, K = 6.4, which combination satisfies (9). It is seen that the nodes are uniformly distributed over all longitudes, and with no two nodes at the same position (except for the first and last nodes which should coincide).

For comparison, Figure 2 shows the pattern for T = 2.5, K = 6.2, which does not satisfy (9) although the K-value is in the intervals permitted by the ITT specifications. It should be remarked that the "gaps" in Fig. 2, e.g. between nodes 0 and 13, do not produce any gaps in the sky coverage. However, for stars in the ecliptical zone (roughly $|\beta| < \frac{1}{2}\pi - \xi$) and at right angle to these gaps, the available observing time will be reduced and the scans across any given object will be strongly concentrated in two position angles only. The latter effect may be detrimental to the detection and resolution of slit ambiguities and double stars, although it does not deteriorate the geometrical improvement factors. Thus, K = 6.4 will give better uniformity of observing conditions versus longitude than K = 6.2.

Figure 3 shows the case T = 3.0, $K = 6\frac{1}{3}$, which again is in accordance with (9).

If the mission may be extended beyond the nominal 2.5 years, it would seem unsatisfactory that the choice of K should depend on the initially perhaps uncertain parameter T. Small modifications to K can however be made at any time of the mission, if the accumulated phase angle error is bounded,

$$\left| \int_{t_o}^{t} 2\pi(K - \bar{K})\, dt \right| < \varepsilon \qquad \text{for all } t \in [t_o, t_o+T] \tag{10}$$

where, for instance, $\varepsilon \simeq 0.8$ rad. $\bar{K}$ is the average K-value of the mission. The similarity between K = 6.4 and K = 6.333 for T = 2.5 and 3.0, respectively, makes it possible to use e.g. K = 6.4 for the first two years, and then K = 6.2 for another year, so as to make $\bar{K}$ = 6.333 for the resulting 3-year mission. This is shown in Figure 4. As will be shown below, this change of K can be made without changing the revolving angle.

3. Choice of ξ

For given K, there is a certain minimum revolving angle, $\xi_{min}(K)$, which makes the loops of the z axis path overlap (with some margin), thus guaranteeing a three-fold complete sky coverage per semester. It can be seen that the condition for loop overlap is (with margin)

$$\lambda_s(\nu = 3\pi) - \lambda_s(\nu = 0) \leq 2\xi, \tag{11}$$

from which we obtain approximately, in degrees,

$$\xi_{min} = \frac{1}{K}[270^\circ - 57.3^\circ/(K \tan \xi_{min})] \tag{12}$$

Eqn (12) can be solved by successive approximations. ($K > 0$ is assumed in (12); for negative K substitute $|K|$.)

Since (12) roughly gives $\xi_{min} = 260^\circ/K$ for the K-values of interest, the ITT specification, which is equivalent to

$$\frac{270^\circ}{K} \leq \xi \leq \frac{280^\circ}{K} \tag{13}$$

will satisfy $\xi > \xi_{min}$ with ample margin.

Table 1 gives $\xi_{min}(K)$ for a range of K-values, and also $275^\circ/K$ and the corresponding extreme speeds of the z axis according to (7) - (8).

4. Conclusions and Recommendations

The ITT specification that the fractional part of K should be in the range 0.10 - 0.23 or 0.77 - 0.90 was based on the consideration that "simple" values like K = 6.0 or 6.5 should be avoided. Now we know however exactly which values should be preferred, and it turns out that some of the most interesting K-values fall outside the given ranges. The specification should therefore be modified to comply with (9), with a tolerance similar to (10) for its variations. The other ITT specification corresponding to (13) can however be retained.

If a revolving angle in excess of 40° is desirable and deemed feasible from other viewpoints, then the only good choice for K is 6.4, with $\xi \simeq 43^\circ$. This scanning law, shown in Figure 5, is therefore recommended.

Table 1. Minimum revolving angle (ξ_{min}) versus K, value according to the rule $\xi = 275^o/K$, and the corresponding range of z axis speed (v_{min}, v_{max}) for revolving angle $275^o/K$.

$\|K\|$	ξ_{min}	$\xi = 275^o\|K\|^{-1}$	v_{min}	v_{max}
5.8	44.84°	47.41°	4.139°/day	4.395°/day
5.9	44.06	46.61	4.155	4.412
6.0	43.31	45.83	4.171	4.428
6.1	42.59	45.08	4.186	4.443
6.2	41.89	44.35	4.201	4.458
6.3	41.21	43.65	4.215	4.472
6.33333	40.99	43.42	4.219	4.476
6.4	40.55	42.97	4.228	4.485
6.5	39.92	42.31	4.240	4.497
6.6	39.30	41.67	4.252	4.510
6.7	38.71	41.04	4.264	4.521
6.8	38.13	40.44	4.275	4.532

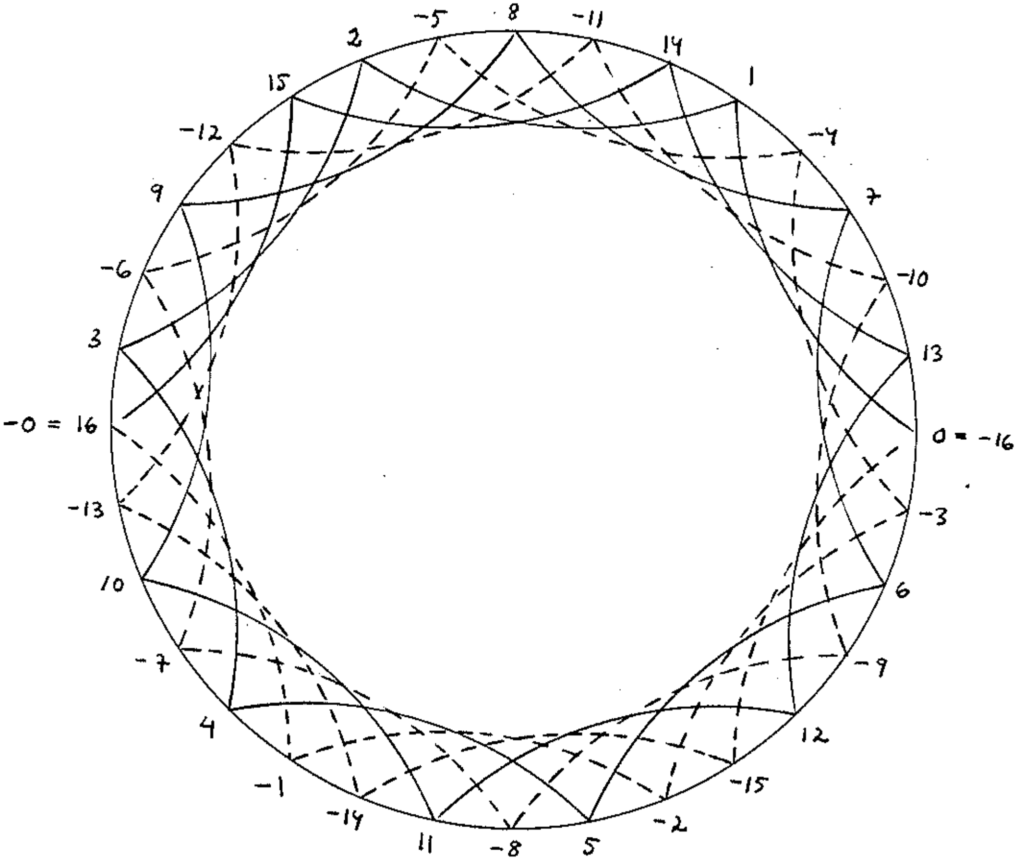

FIGURE 1. Repetition pattern for the scanning law with
T = 2.5, K = 6.4 (see Section 2, p.2 for explanation)

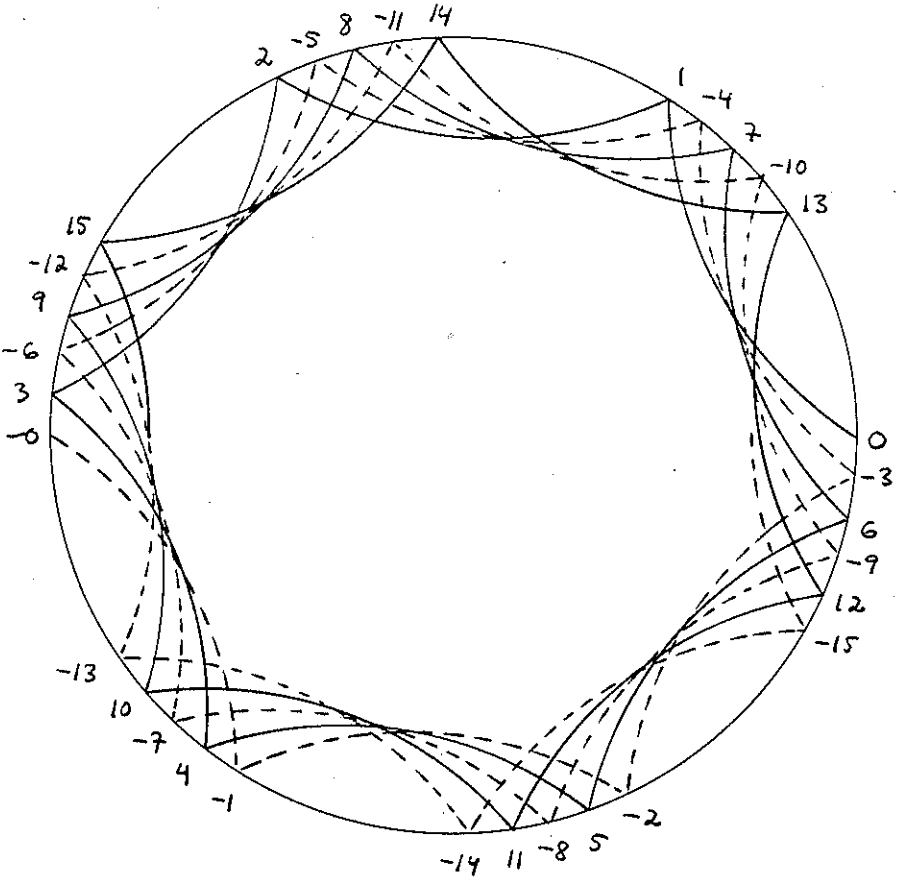

FIGURE 2. Repetition pattern for scanning law with
T = 2.5, K = 6.2

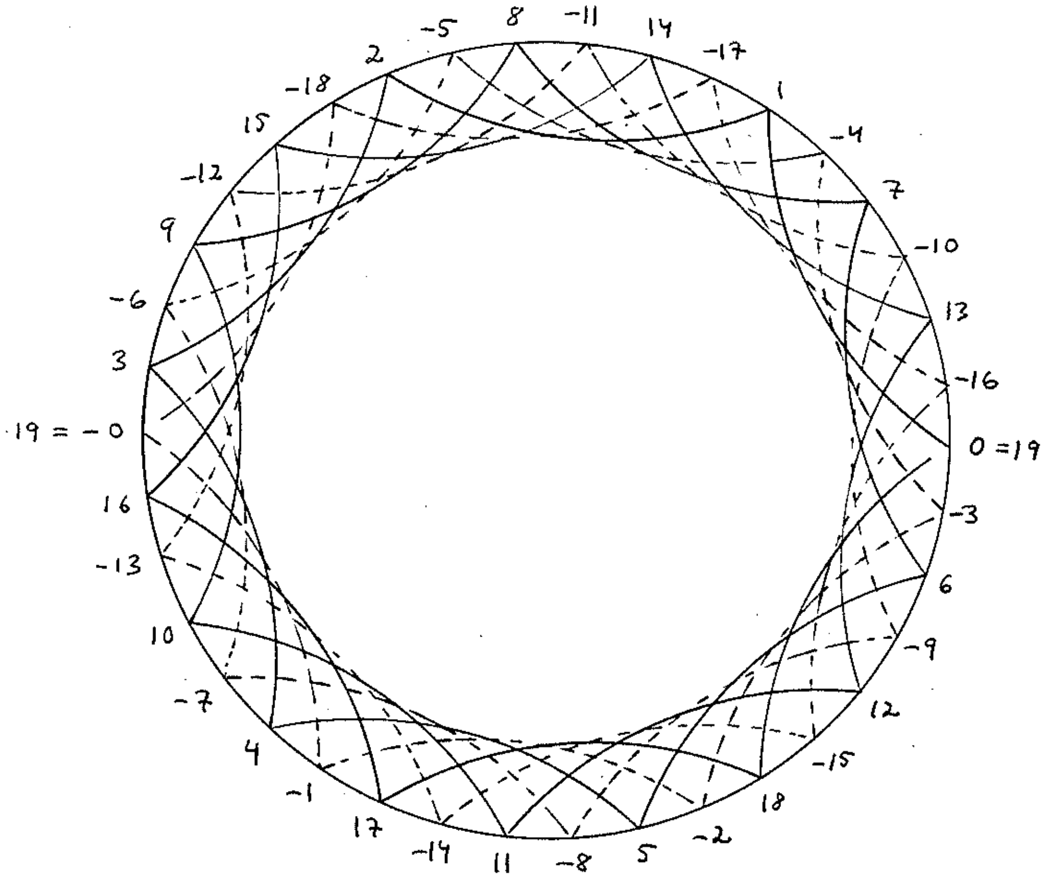

FIGURE 3. Repetition pattern for scanning law with
T = 3.0, K = 6.33333

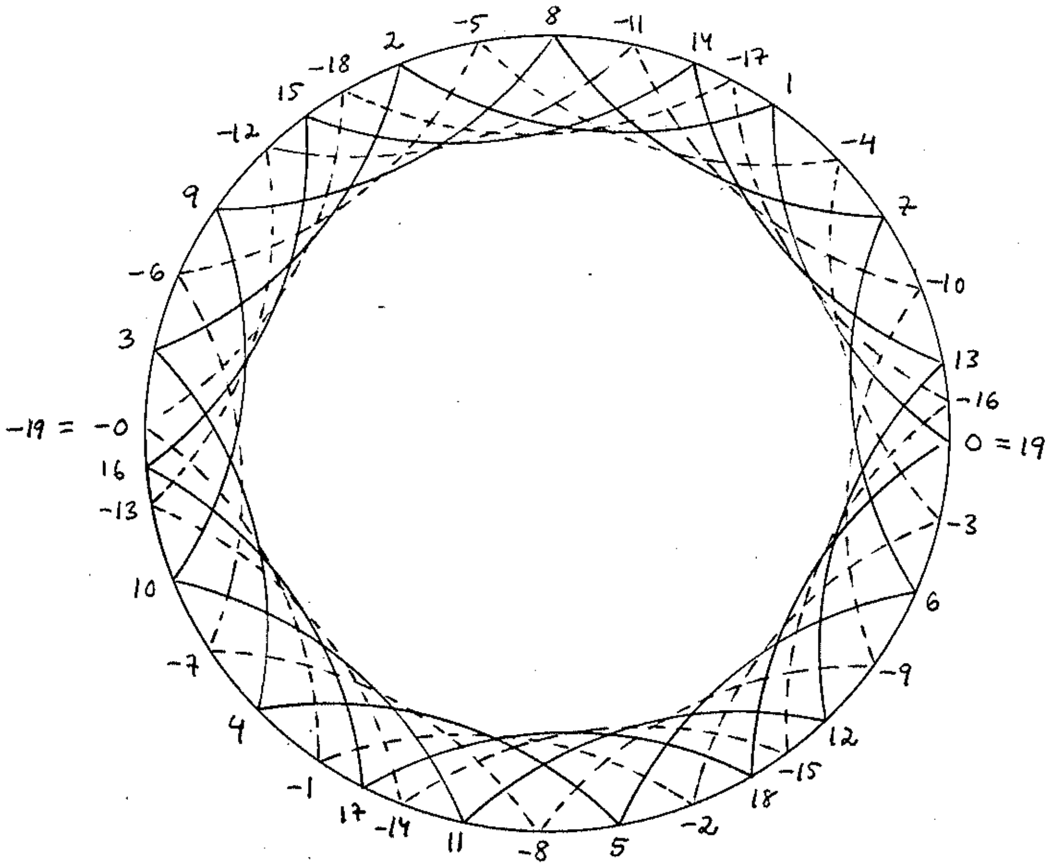

FIGURE 4. Repetition pattern for scanning law with
$K = 6.4$ for $0 \leq t - t_o < 2$ years and
$K = 6.2$ for $2 \leq t - t_o < 3$ years
$\bar{K} = 6.33333$, $T = 3.0$

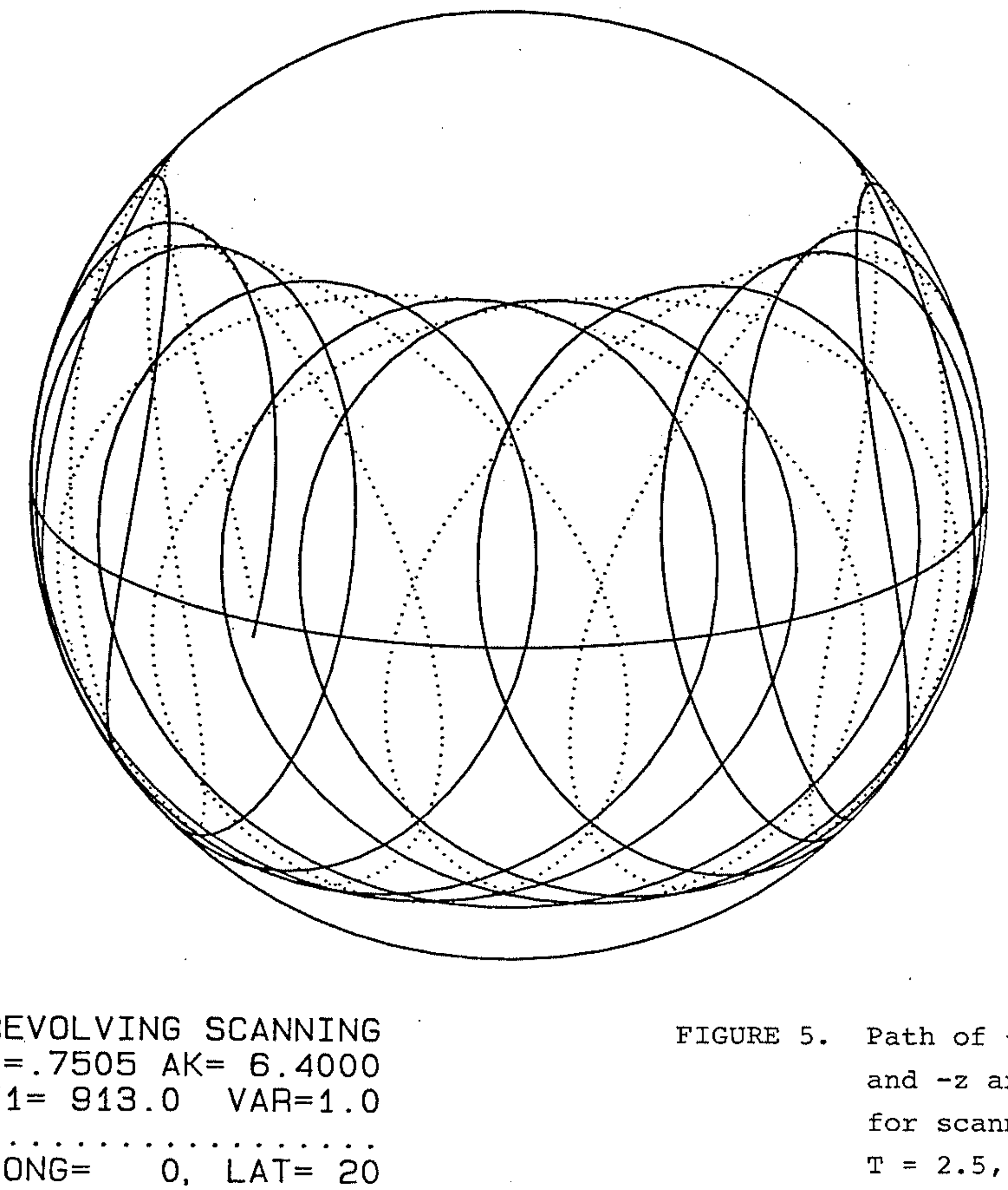

FIGURE 5. Path of +z axis (solid) and −z axis (dotted curve) for scanning law $T = 2.5$, $K = 6.4$, $\xi = 43^\circ$