

Juttner Distribution From Photon Absorption and the Maxwell-Boltzmann Factor

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In (1), the Juttner distribution for a particle is obtained by considering reaction balance using photon absorption and emission by a particle. It is stated in (1) that the Maxwell-Boltzmann factor $\exp(-E/T)$ is not explicitly used. The photon is associated with blackbody radiation and the Bose-Einstein form for $\langle nk \rangle$ is used. We argue that the Bose-Einstein form is linked to a reaction balance form with: $P(e_i) / \{1 + P(e_i)\} = \exp(-e_i/T)$, where e_i is photon energy. This exact ratio appears in a particle-photon balance equation in (1) ensuring that $p(\text{particle } e_2) = p(\text{particle } e_1) \exp(-(e_2 - e_1)/T)$ or that $p(\text{particle } e) = C \exp(-e/T)$. In other words, the implicit assumption in the photon absorption-emission equilibrium problem is: $p(\text{particle } e_1)p(\text{particle } e_2) = p(\text{particle } e_1 + e_2)$ which is equivalent to the Maxwell-Boltzmann factor.

Method of (1)

In (1) a particle with energy e_1 absorbs a photon in blackbody radiation. In equilibrium this process will balance with the emission of a photon according to the equation ((6)) in ((1)) i.e.

$$p(e_1)\langle nk \rangle = p(e_2)\langle nk+1 \rangle \quad ((1))$$

Where $e_2 = e_1 + \hbar k$ i.e. e_2 emits a photon and e_1 absorbs one.

$$\text{Using } \langle nk \rangle = 1 / \{\exp(\hbar k/T) - 1\}: \quad p(e_2) = p(e_1) \exp(-\hbar k/T) \quad ((2))$$

In (1), e_2 and e_1 are then set to differ by v and $v+dv$, where v is velocity and a differential equation obtained.

We suggest one may interchange the values of e_1 and k in ((2)) to obtain:

$$p(e_2) = p(e = \hbar k) \exp(-e_1/T) \quad ((3))$$

This suggests that $p(e) = \exp(-e/T)$. Using $e = m_0 c^2 / \sqrt{1 - v^2/c^2}$ yields the Juttner distribution. We further suggest that the reaction balance approach used is equivalent to the Maxwell-Boltzmann factor and so even though this factor is not introduced explicitly, the balance approach is the same.

Maxwell-Boltzmann Distribution and Reaction Balance

The Maxwell-Boltzmann distribution may be directly obtained using reaction balance with energy conservation i.e.

$$E_1 + e_2 = e_3 + e_4 \quad ((4a)) \quad \text{and} \quad p(e_1)p(e_2) = p(e_3)p(e_4) \quad ((4b))$$

It is assumed in this case that $p(e_i)$ essentially carries no more information than e_i . This assumption is directly linked to the MB factor. Thus:

$$\ln(p(e_i)) = -e_i/T \text{ or } p(e_i) = C \exp(-e_i/T) \quad ((5))$$

Another way to express the principle behind the MB distribution is: $p(e_1)p(e_2)=p(e_1+e_2)$ ((6))

We argue that ((6)) is the underlying assumption in the system with the various particle photon emissions and absorptions. To see this more explicitly, one may consider reaction balance for a boson i.e. the photon. In such a case, ((4b)) needs to be modified because there is an enhancement factor for e_1 to scatter into e_2 , namely $p(e_2)+1$. Thus:

$$e_1+e_2 = e_3+e_4 \quad ((6a)) \quad n(e_1)(1+n(e_3)) n(e_2) (1+n(e_4)) = n(e_3) (1+n(e_1)) n(e_4) (1+n(e_2)) \quad ((6b))$$

In other words, $p(e_i)$ in ((4b)) is replaced by: $n(e_i) / \{1+n(e_i)\}$.

$p(e_i)$ in ((4b)), however, is $\exp(-e_i/T)$ so now: $n(e_i) / \{1+n(e_i)\} = \exp(-e_i/T)$.((7))

((7)), however, is the exact factor which appears in the photon-particle balance equation ((1)). Thus it seems that the Maxwell-Boltzmann factor does appear, although in a somewhat roundabout fashion.

As a result ((1)) is equivalent to the statement: $p(e_2)=p(e_1)p(e=\hbar\omega)$ i.e. $p(e_i+e_j) = p(e_i)p(e_j)$ which we argue is the underlying idea of the Maxwell-Boltzmann distribution i.e. $p(e)=C \exp(-e/T)$.

Conclusion

In conclusion, in (1) reaction balance of particle-photon absorption-emission is used to derive the Juttner distribution i.e. Maxwell-Boltzmann distribution for a particle with $e=m\omega c/\sqrt{1-v^2/c^2}$. It is stated in (1) that the MB factor is not used explicitly. The Bose-Einstein distribution, however, is used to describe the photons i.e. $p(e_1) \langle n_k \rangle = p(e_2) \langle n_{k+1} \rangle$ where $e_2=e_1+\hbar\omega$. We argue that in such a picture $p(e_i)$ for a particle essentially follows the rules $e_1+e_2=e_3+e_4$ and $p(e_i)p(e_j)=p(e_i+e_j)$ which is equivalent to the Maxwell-Boltzmann factor. We show that $\langle n_k \rangle / \langle n_{k+1} \rangle$ is equivalent to the MB factor $\exp(-\hbar\omega/T)$ which shows that $p(e_1)p(e=\hbar\omega) = p(e_2)$ which is the underlying MB form.

References

- 1.Li,C. and Li, J. and Yang, Y. First Principle Derivation of Single Photon Entropy and Maxwell-Juttner Distribution
Entropy 2022, 24(11), 1609; <https://doi.org/10.3390/e24111609>
<https://www.mdpi.com/1099-4300/24/11/1609>