

**THE METHODOICAL SYSTEM OF TEACHING THE SUBJECTS DETERMINED THE  
REALIZATION OF THE SUB-STANDARDS FOR THE "POINT IN SPACE, STRAIGHT LINE,  
PLANE" EDUCATIONAL UNIT (CLASS X)**

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**Abstract**

In the article, the content of the skills intended to become the subject of students in teaching materials for the "Point in space, straight line, plane" educational unit, which is included in the content of the Mathematics subject in the X classes of general education schools, is presented. Attention is directed to the place of the content elements that perform the "enabling function" in the formation of the mentioned skills in the "Point in space, straight line, plane" educational unit and to the technology of turning them into action in the students' cognition. "Point, straight line and plane in space" of the transformed elements in the educational unit "Point, straight line and plane" The mutual position of straight lines and planes in space", "Parallelism of a straight line and a plane. Perpendicularity of a straight line to a plane. The angle between a straight line and a plane", "Theorem of three perpendiculars", "Angle between two planes". "Dihedral angles. Perpendicular planes", "Parallel planes. Projections and Problem-Solving" are appropriate to make it into a system on topics. We should note that the task system, which serves to organize and manage the educational process for mastering the content elements presented by the students, should contain verbs that are required to be performed on the topics. It included examples of tasks in the work's content.

**Keywords:** Teaching unit; space; plane in space; straight line in space; perpendicular; dihedral angle; projection; paradigm; content lines; system; subsystem; assignment; organization and management of the educational process; methodical system.

The actuality of the subject. There are no recommendations that are useful to be adopted for guidance in the teacher's activity in the currently used textbook complex and various educational literature about teaching units, basic standards, sub-standards, and defined topics in the subject of mathematics. This is very useful to teach teaching mathematics according to the curriculum model. We do not deny that the teaching process in the subject is logical for continuous improvement, most didactic problems are eternal. Of course, as the experience of using the curriculum is formed, the methodical system of the teaching process adequate for it should be improved. Based on this logic, "A point in space, a straight line,

Interpretation of the generalizations formed in the research. The analysis of the materials got from scientific sources provides a basis for such a generalization that in the content of mathematical subjects taught education schools, the materials related to the concepts of numbers, function, and space have had a superior "enabling function" and along the line of development of these concepts (from simple to complex) brought together in a dialectical unity. Even many pedagogues (didactics, methodists) who have gained the image of "classical status" have supported such an idea that general mathematical education should refer directly (sometimes directly) to the development trend and level of the existing mathematical science, as well as to the logic of the development of the concepts of numbers and functions should be adequate. [5-8; 12]. The con-

tent lines of the Mathematics subject based on the curriculum model currently applied to education schools are precisely the modernization of this idea. [2; 57].

Our generalization, derived from the research materials we collected on the mathematics subject curriculum, is that In teaching the topics covered in the "Point in space, straight line, plane" educational unit, students are expected to become subjects of the following skills:

- models the concept of point, straight line, and plane in space in a real situation;
- shows that he has mastered the concept of a plane in space by proving the theorem and solving problems with a geometric representation;
- geometrically describes the mutual position of straight lines in space;
- expresses the geometric properties of straight lines in words and describes them geometrically;
- expresses the geometric properties of the straight line and the plane according to their mutual situations, and describes them geometrically;
- geometrically describes the angle between a straight line and a plane;
- finds the distance from a point in space to the vertices and sides of polygons on the plane;
- models mutual situations of planes in real situations;
- expresses propositions about the perpendicularity of planes in words and proves theorems, models their situations in real situations;

- shows that the angle between two planes is a dihedral angle with geometrical representations, determines its size with the corresponding linear angle;
- explains the parallelism of planes with examples from real-life situations, and models with real objects;
- expresses propositions about the perpendicularity of planes in words and proves theorems with a geometric representation, models their situations in real situations [3-4]

Mastering the materials of this educational unit enables students' active mathematical vocabulary to be enriched with the following concepts: space, plane, point, straight line, coplanar points, collinear points, projection, dihedral angle, linear angle, and considerations related to them.

It is recommended to take advantage of virtual tools (important links for teaching the course) and various worksheets as an additional resources in teaching the materials for the mentioned educational unit. For the sake of justice, it should be said that it is possible to get examples of tasks of speech from scientific sources produced by educational scientists of Azerbaijan and other nationalities (the presented list of literature can be selected as an address for this purpose).

First, let's present the interpretation of the generalization on the topic "Point, straight line and plane in space", and let's continue this process on the following topics. In the first lesson on "point, straight line, a plane in space", examples of the model of point, straight line, and plane in space are given. It created the habit of clear visualization of space in students based on the application of visual aids in the teaching process. What is important is that the students understand that three points that are not on a straight line are necessary for the existence of a plane, to conclude the possibility of passing a plane from a straight line and a point taken outside of it, two intersecting straight lines, and they can identify points, straight lines and planes in space be able to depict or show geometrically any spatial figure.

Students should know that point, straight line, we also considered plane as spatial figures. The axioms and theorems known from planimetry are true on every plane of space.

It should be clear to them that the following additional axioms are accepted in space:

Axiom 1. For an arbitrary plane, there are points belonging to this plane and points not belonging to it.

Axiom 2. If two different planes have a common point, then they intersect in a straight line passing through this point.

Axiom 3. One and only one plane passes through three points not in a straight line. [3; 44].

It should also be clear to students that one and only one plane passes through a straight line and a point not on the straight line; one and only one plane passes through two intersecting straight lines; two different parallel straight lines pass through one and only one plane.

With the help of practical works and visual representations, every student should consciously gain the conclusion that the plane can be defined by a straight

line and a point not on it, by two intersecting straight lines, by two parallel straight lines.

In teaching the subject, they should learn that points on a straight line are called collinear and points on a plane are called coplanar points. In the textbook [3; 45] and methodical sources [5-8], it is recommended to offer students the following tasks for this stage:

1. Points B, C, and D are on a straight line (collinear), points A, B, C, and D are on a plane (coplanar), and point E is not on this plane. How many planes pass through the points?

a) A, B, and C; b) B, C, and D; c) A, B, C, and D  
d) A, B, C and E

2. How many planes can be passed through three straight lines intersecting in pairs?

3. How many planes can be passed through four different points? See all cases.

And so on.

It is enough to devote 1 hour to the teaching of the topic "The mutual position of straight lines and planes in space". It is necessary to make the information with the following content an active component of students' knowledge and experience. Of course, this stage of the educational process should be of a visual and practical nature.

Because of the teaching of the subject, it should be clear to the students: Two straight lines in space can be parallel (can overlap in special cases). Two straight lines can intersect in space. If two straight lines intersect or are parallel, then they lie in a plane. This corresponds to the intersection and parallelism of straight lines in planimetry. If two intersecting straight lines are intersected by a third straight line at different points, these straight lines lie on the same plane. If two straight lines in space intersect at a point with a third straight line, these straight lines may or may not lie in the same plane. Two straight lines that are not parallel in space may not intersect. Two straight lines that are not parallel and do not intersect are called cross-straight lines. It is impossible to pass a plane through both of the two cross straight lines. The angle between the two cross-straight lines is called the angle between two intersecting straight lines parallel to them. If one of two straight lines intersects the plane of the other at a point that does not belong to this straight line, then these straight lines are intersected.

parallelism of the straight line to the plane. We present the sign in question to students as a theorem in the following formula.

*Theorem.* If a straight line not on a plane is parallel to any straight line on that plane, then it is also parallel to the plane itself.

It is acceptable to present the proof form of the theorem to the students as follows.

α the straight line that is not on the plane should be parallel to the straight line on that plane (a colored image is brought to the eyes of the students), and let's pass the plane through straight lines. and planes will intersect along a straight line. If a straight line intersects the plane, the point of intersection must be on the straight line. This is impossible as it is. So,  $a \parallel b \Rightarrow a \parallel \alpha$



three perpendicular theorems, and based on this visualization, the theorem is presented to them in words in the following text:

*Theorem.* If a straight line on a plane is perpendicular to the projection of a slope drawn on the plane, it is also perpendicular to the slope itself.

A short version of the theorem is presented to the students. The version is as follows: and if  $a \perp BC$   $BC \perp BA$   $a \perp AC$

The theorem is proved in the conditions of students' cognitive activity. The teacher voices his opinion that the inverse theorem of this theorem is also true, and together with the students, the inverse theorem is formulated as follows:

*Inverse theorem.* If a straight line on a plane is perpendicular to the slope drawn to the plane, it is also perpendicular to its projection.

Following the text and color image, the short form of the theorem is determined and its proof is offered to the students as an independent work. [10; 71-72 ]

To better master the three-perpendicular theorem and its inverse theorem, it is recommended that students be encouraged to work on sample tasks such as the following:

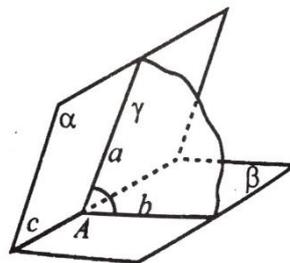
*Example 1.* The length of the perpendicular drawn from the vertex of a right angle to the plane of the triangle is 7.2 units, and the length of the height of the triangle drawn from the vertex of the right angle to the hypotenuse is 9.6 units. Find the distance from point M to the hypotenuse of the triangle.  $ABCCM$

*Example 2.* A perpendicular of length 15 units is raised to the plane of the triangle from the vertex of the large angle of the triangle with equal sides. Find the distance from the endpoint of the perpendicular to the larger side. 10, 17, 21 [3;52]

It is necessary to take advantage of teaching materials related to the theorem of three perpendiculars and to take advantage of examples of educational and varied creative tasks.

"The angle between two planes. Dihedral angles. 2 hours can be devoted to teaching the subject of "perpendicular planes". It is advisable to start the teaching process by considering the mutual situations of the planes. It should be noted that the mutual position of the planes can be: parallel, intersecting, and overlapping planes. Based on this basis, the dihedral angle is defined as follows.

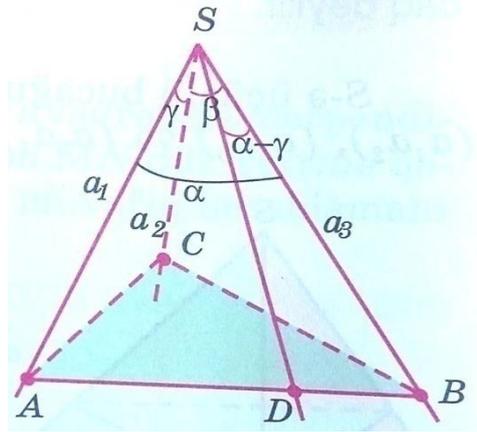
*Definition.* The figure formed by two half-planes with common boundaries is called a dihedral angle.



In the interpretation based on the definition, students understand: Half-planes are called faces of a dihedral angle, and their common border is called a tongue of a dihedral angle; 4 dihedral angles are obtained from the intersection of two planes; if we take any point on the tongue of a dihedral angle and draw perpendicular rays from this point, each of which is in separate half-planes, the angle obtained is called the linear angle of the dihedral angle; a dihedral angle is measured by its linear angle; the degree measure of a linear angle indicates the degree measure of a dihedral angle; all exterior angles of a dihedral angle are coincident with a parallel transposition (degrees are equal, straight lines perpendicular to the same straight line are parallel); the value of a linear angle does not depend on the position of its vertex; Dihedral angles have sizes from  $0^\circ$  to  $180^\circ$  [9; 238-239]

After the student has completed several examples of tasks related to the concept of dihedral angles, they are presented with information about the concept of trihedral angles.

Students are suggested to take a triangle and a point outside its plane and draw its rays. The teacher directs their attention to the received image and calls out the definition given below. *Definition.* A figure formed by plane angles that have a common vertex and are not located on one plane is called a trihedral angle. Plane angles are called the faces of a trihedral angle, their sides are called the tongues of a trihedral angle, and the common vertex is the vertex of a trihedral angle. Each tongue is also the tongue of a dihedral angle.  $ABCSSA, SB, SC \angle ASC, \angle ASB, \angle BSC$



The teaching process is continued on the following theorems.

*Theorem 1.* The sum of the plane angles of a trihedral angle is smaller than  $360^\circ$

*Theorem 2.* Each plane angle of a trihedral angle is smaller than the sum of the other two plane angles.

It is necessary to take advantage of teaching materials related to the concepts of "angle between two planes", "dihedral angle", and "trihedral angle" and the examples of educational and varied creative tasks. Let's give some examples.

1. Is there a trigonometric angle whose degrees of plane angles are given as follows?

a), b), c),  $30^\circ$   $120^\circ$   $60^\circ$

2. A point is taken on one face of a dihedral angle equal to the other face. Find the distance of this point from the line.  $45^\circ - a$

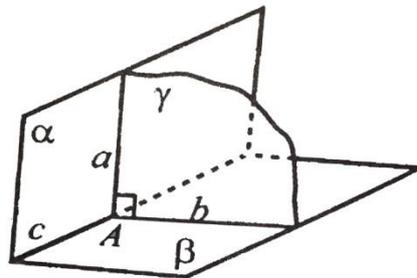
3. Each of the two plane angles of the trigonal angle is the third. Find the dihedral angle opposite the third plane angle.  $60^\circ$

4. A triangle with sides  $ABC$  is given. passes through the plane forming an angle with the plane of the triangle. find the distance from its vertex to its plane.  $AB = 18, BC = 12, AC = 10$   $45^\circ$   $\alpha$   $\beta$

The process of teaching the subject continues with the introduction of perpendicular planes.

They are introduced with such a definition.

*Definition.* If the dihedral angle obtained by the intersection of two planes is a right angle, these planes are called perpendicular planes.



Based on this definition, the teacher presents the sign of the perpendicularity of planes.

*Theorem (Sign of perpendicularity of planes).* If a plane passes through a straight line perpendicular to another plane, then these planes are perpendicular.

The work of mastering the perpendicularity of planes continues with the organization of students' activities on relevant tasks. Examples of such tasks include:

1. A point is at a distance from two perpendicular planes. Find the distance from this point to the line of intersection of the planes.  $a$   $b$

2. draw the plane and the straight line perpendicular to this plane. draw three planes passing through the straight line and perpendicular to the plane.  $NMLAM$   $AMNML$

3. Which idea presented below is true and which idea is false?

a) From any point taken on a straight line, only one perpendicular can be drawn to this straight line.

b) If point  $A$  is on the plane and point  $B$  is on the plane, then no other point of the straight line  $AB$  is on the plane.  $\alpha\beta\alpha$

c) If each of the two intersecting planes is perpendicular to the third plane, then these planes are also perpendicular to each other.

d) Only one perpendicular plane can be drawn from a given point on a straight line to this straight line.

e) If a plane is perpendicular to one of two intersecting straight lines, it is also perpendicular to the other.

4.. If the distance of a point taken on one face of a dihedral angle from the thread is twice the distance from the other face, find the value of this dihedral angle

5. If an isosceles right triangle is folded along its height and its planes form a dihedral angle, and its sides

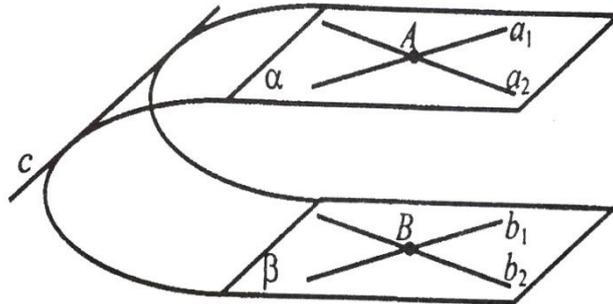
are perpendicular to each other, then it forms an angle. Prove it.  $ABCBDABDCBDDADCBAABC60^0$

6. A triangle with sides is given. by passing through the plane that makes an angle with the plane of the triangle, find the distance of its vertex from the plane.  $AB = 9, BC = 6, AC = 5, \angle C = 45^0$  [3; 5-7; 10]

“Parallel planes. It is recommended to devote 3 hours to teaching Projections and Problem-Solving. First, attention is paid to the sign of parallelism of the

planes. This sign is formulated as a self-contained theorem below:

*Theorem.* (Sign of parallelism of planes). If two intersecting straight lines of one plane are correspondingly parallel to two intersecting straight lines of the other plane, then these planes are parallel to each other.



The theorem is proved by the converse hypothesis with the participation of students as an active party. In the process of teaching this subject, students' attention is directed to five more theorems and their proof methods. The theorems in question are:

*Theorem.* If two parallel planes intersect with a third plane, then the lines of intersection are parallel.

*Theorem.* Parts of parallel straight lines between parallel planes are equal.

*Theorem.* Two straight lines perpendicular to the same plane are parallel.

*Theorem.* Two planes perpendicular to the same straight line are parallel.

*Theorem.* A straight line perpendicular to one of two parallel planes is also perpendicular to the other.

The concepts of the distance between two parallel planes and two intersecting straight lines are also included in the teaching of this subject.

It is explained to the students that the distance between two parallel planes is equal to the length of the perpendicular drawn from an arbitrary point of one of these planes to the other.

The concept of the distance between two intersecting straight lines is explained as follows: A parallel plane can be drawn from each of the two intersecting straight lines. For example, let's pass a plane parallel to a straight line. For this, let's draw a straight line that intersects or is parallel to the straight line. and if we pass a plane through the intersecting straight lines, this plane is parallel to the straight line. the distance from an arbitrary point of the straight line to this plane is equal to the distance between the intersecting straight lines. if there is a segment AB perpendicular to its plane and the common perpendicular of the cross straight lines.  $ba \perp a_1, a_1 \perp b, a \perp ab, b \cap a_1 = B, \alpha \perp ab$

Continuing the teaching of the subject, the students are told that projection is used to depict spatial figures on a plane. The explanation is as follows: let us take an arbitrary straight line that intersects its plane.

The point of intersection of the plane of the line parallel to the straight line from point A of the figure will be the image of point A. By constructing the image of each point with this rule, the image of the figure is obtained. In this case, parallel parts of the figure are described by parallel parts, and the ratio of parallel straight-line parts is preserved. In a special case, when the image is perpendicular to the straight line plane, it is an orthogonal projection of the figure. The relation is used to find the length of the projection of the piece. In the general case, the formula is true if the angle between the plane of the polygon and the plane of projection is Here is the area of the polygon,  $\alpha \perp A' \perp \alpha, AB = A'B' \cos \varphi, S_p = S_f \cdot \cos \varphi, S_p -$

The work of mastering the concepts of parallel planes and projection continues with the organization of students' activities on relevant tasks. The following can be an example of such tasks[3; 8;10]:

1. The straight line AB is parallel to its plane and perpendicular to its plane. CD lies on a straight line plane.  $\alpha \perp \beta$

1) Take a picture according to the condition;

2) Which is true?

a) b) c) d)  $\alpha \parallel \beta, \alpha \perp \beta, AB \parallel CD, CD \perp \alpha$

2. Reflect on the given conditions in the picture.

$\alpha$  and their planes intersect along a straight line.

The points of intersection of the straight line at point E are on the same plane.  $\beta \perp \alpha, CD \perp \alpha, E$

3. In a cube with a tongue, draw a plane through the middle of the two neighboring sides of the upper seat and the center of the lower seat. Calculate the perimeter of the section.  $a$

4. Perpendicular in length and inclined in length are drawn between two parallel planes. On each plane is the distance between their ends. Find the distance between the perpendicular and the middle of the slope.  $4m, 6m, 3m -$

5. The length of two straight line segments remaining between two parallel planes and dir. The ratio between their projections on one of the planes is

6:7. Find the distance between these planes.  $51 \text{ см}$   $53 \text{ см}$  –

6. The seat of an equilateral triangle is on the plane. a plane parallel to the plane intersects its side at point D and its side at point E. Prove that the triangle is also equilateral.  $ABCBC\alpha\alpha\beta ABACADE$

7. The ends of a straight line segment are at a distance from the plane and (. Find the distance of the middle point of the segment from the plane.  $a > b$ )

a) If the fabric does not cut the plane; b) If it cuts the fabric plane.

8. The orthogonal projection of an isosceles triangle on the plane is an equilateral triangle. find the area of if  $CAB\alpha AD \perp \alpha, AE \perp BC, \angle CAB = 48\text{см}^2, AE = 16\text{см} \angle BCD$  –

The authors of the "Mathematics-10" methodical manual for teachers [4] recommended defining the criteria for the summative assessment of students' mastery levels in the "Point in space, straight line, plane" educational unit in the following content, which we appreciate the presented system of criteria:

1. Presents the propositions that define the plane in words and geometrically.

2. Presents the mutual position of points and straight lines in space in words, geometrically, and by problem-solving.

3. Describes the relationship between a straight line and a plane in words and geometrically.

4. Describes the definitions and theorems related to the perpendicularity of a straight line to the plane verbally and geometrically and apply them to problem-solving.

5. Applies the theorem and propositions about the perpendicularity of two planes to problem-solving.

6. Applies theorems and propositions about the parallelism of two planes to problem-solving.

7. Draws orthogonal projections of figures on a plane and solves problems.

In our opinion, these presented criteria are correctly formulated to determine the level of realization of the enabling functions of the "Point in space, straight line, plane" educational unit. [3; 63-64]

The result. 1) In the process of teaching the topics covered in the "Point in space, straight line, plane" educational unit, the selection and application of task types should be focused on for students to become subjects of the verbs based on the expected results;

2) In the process of teaching materials for the "Point in space, straight line, plane" educational unit, the inclusion of necessary terms in the content of students' active mathematical vocabulary should be considered an important didactic requirement;

3) It is recommended to use virtual tools (important links for teaching the course) and various worksheets as an additional resources in teaching

materials for the "Point in space, straight line, plane" educational unit;

4) Since the materials of the "Point in space, straight line, plane" educational unit belong to the "Geometry" content line, in the real pedagogical process, they are used both in full (with the content covered by the subject of Mathematics as a whole) and in parts belonging to each content line (the other four contents). dialectical unity ("system-structure" approach-whole-part relations) between the elements of the line) should be expected;

5) The expectation of the "opportunity-movement-quality" paradigm in selecting tasks and making them into a system has a positive effect on the efficiency of the educational process.

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