On the Identifiability of Noise Statistics and Adaptive KF Design for Robust GNSS Carrier Tracking

Jordi Vilà-Valls, Pau Closas and Carles Fernández-Prades
Centre Tecnològic de Telecomunicacions de Catalunya (CTTC), 08860, Barcelona, Spain
Email: {jvila, pclosas, cfernandez}@cttc.cat

Abstract—**Carrier synchronization is of paramount importance in any communications or positioning system. Mass-market Global Navigation Satellite System (GNSS) receivers typically implement traditional carrier tracking techniques based on well-established phase-locked loop architectures, which are only reliable in quite benign propagation conditions. Under non-nominal harsh propagation conditions, the signal may be affected by shadowing, strong fading, multipath or severe ionospheric scintillation, and thus, traditional architectures are not valid anymore and there exists an actual need for robust tracking solutions. Several approaches to overcome the conventional PLL limitations have appeared during the last decade, being the Kalman filter (KF) based architectures the most promising research line. The main drawback of standard KFs is the assumption of perfectly known process and measurement noise statistics, a knowledge that is always constrained by the system model accuracy. Beyond heuristic solutions, a general framework for the design of adaptive KFs correctly dealing with both process and measurement noises, that would be of capital importance for the practitioner, has not been established. The main goal of this contribution is to provide a clear answer to this fundamental question. It is shown that the main driver on the KF performance is not the adjustment of the measurement noise but the adequate tuning of the process noise statistics. Within this framework, a comprehensive discussion is given for the correct design of adaptive KF architectures for robust carrier tracking applications, where the key idea is to use two independent noise statistics estimation strategies to sequentially adapt both parameters. The design choice is supported by a discussion on the identifiability of the noise statistics' parameters. Simulation results are provided showing the need of fully adaptive solutions, and the achieved performance gain of KF-based architectures when compared to traditional tracking loops.**

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1. INTRODUCTION

The problem under study concerns the derivation of efficient and robust methods for carrier phase tracking in Global Navigation Satellite Systems (GNSS). The satellite-based positioning systems were initially designed to operate in clear sky propagation conditions, therefore not taking into account the harsh propagation conditions appearing in real-life scenarios. In such scenarios, the signal may be affected by high dynamics, shadowing, strong fadings, multipath effects or ionospheric scintillation. Among these propagation conditions, the dense urban multipath case (with possible non-line-of-sight conditions) and the ionospheric scintillation are certainly the most challenging ones. These scenarios must be considered as a benchmark on the performance for the correct robust carrier tracking filter design. Notice that synchronization is typically carried out following a two-state approach: acquisition and tracking. The first stage provides a coarse estimate of the synchronization parameters, and the second one refines those estimates, filtering out noise and tracking any possible time-variation [1]. In this contribution, the focus is on the carrier phase tracking stage, thus acquisition is not considered and a perfect time-delay synchronization is assumed.

State-of-the-Art

The carrier phase tracking techniques implemented in conventional GNSS receivers rely on well known phase-locked loop (PLL) architectures, which are only reliable under quite benign propagation conditions, and thus they are not suitable to cope with challenging real-life scenarios. The problem of standard PLLs is the well-known noise reduction versus dynamic range trade-off (*i.e.*, a small bandwidth is needed to be able to filter out the noise and track signals with low carrier to noise ratios (C/N_0), and a large bandwidth has to be used to cope with high dynamics), which is mainly driven by the bandwidth and order of the loop. These techniques have been shown to deliver poor estimates or even lose lock under harsh propagation conditions [2, 3]. Several improved PLL-based techniques have been proposed in the literature: cooperative loops [4], switching architectures [5], adaptive bandwidth approaches [6, 7] or noise reduction techniques [2]. Refer to the recent and up-to-date survey on robust carrier tracking techniques [8] and the references therein for a complete overview of PLL-based architectures.

The main drawback of all the PLL-based architectures is the inherent suboptimality, being usually heuristically tuned and still used in practice because of their simplicity. In terms of performance, they have been clearly overcome by Kalman filter (KF)-based tracking techniques [3, 9–12], where the filter is automatically adjusted so as to minimize the mean square error, and optimally designed from a statistical filtering approach. Moreover, the KF has an inherent adaptive bandwidth (i.e., time-varying Kalman gain) and a flexible architecture, which is much more convenient than the standard PLLs, inherited from the analog era.

Generally speaking, there is not a perfect methodology providing good performances in any scenario, and the KF is not an exception. An optimal KF design, where the Kalman gain is recursively adjusted as a function of the estimation error covariance and the system working conditions, needs an exact knowledge of the process and measurement noise statistics. Therefore, an optimal and correct behavior is restricted by the accuracy of the dynamic model and the *a priori* fixed system parameters. In practical real-life applications the possibly time-varying system working conditions may not be fully known, and those quantities need to be estimated or somehow adjusted to provide a robust solution. Some strategies appear in the literature to overcome the standard KF limitations. The so-called Adaptive KFs (AKFs), try to sequentially adapt the filter parameters (*e.g.*, the process and measurement noise covariance matrices) to the actual working conditions, usually using an heuristic/ad-hoc approach [13, 14], innovations-based solutions for one of the two parameters of interest [13, 15] or the C/N_0 estimates to easily adjust the measurement noise statistics [16, 17]. A very recent attempt to take into account both noise statistics in an adaptive manner [18] is still rather heuristic, scenario dependant, needs a proper tuning and is far from being an optimal adaptive solution. To sum up, a general framework for the correct and optimal design of adaptive KFs dealing with both process and measurement noises does not exist, being a fundamental missing gap in the carrier tracking literature.

Contribution

In this paper, the main goal is to propose a general optimal framework for the correct design of adaptive KFs, providing a clear answer to this fundamental problem. The design choice and the corresponding proposed architecture are supported by a discussion on the identifiability of the noise statistics' parameters within the KF framework, which determines the two fundamental design rules. The key idea is to use two independent noise statistics estimation strategies to sequentially adapt both parameters, where the proposed architecture must use a C/N_0 estimator to adjust the filter to the system noise and a covariance estimation method for the process noise (i.e., system dynamics). It is shown that the main driver on the KF performance is not (only) the adjustment of the measurement noise but the correct tuning of the process noise statistics, being a crucial point in the AKF design. Simulation results are provided to show the need of fully (optimal) adaptive solutions, and the expected performance gain of correctly designed AKF-based architectures when compared to traditional tracking loops.

2. SIGNAL MODEL

GNSS signal model

The baseband analytic representation of a generic GNSS transmitted signal can be expressed as

$$s(t) = P_x(t)d(t-\tau(t))c(t-\tau(t))e^{j(2\pi f_d(t)+\theta_e(t))}, \quad (1)$$

where $P_x(t)$, $d(t)$ and $c(t)$, stand for the signal amplitude, the navigation message and the spreading code, respectively. The synchronization parameters are the code delay, $\tau(t)$, carrier Doppler frequency shift, $f_d(t)$, and carrier phase, $\theta_e(t)$. The digitized signal (sampling period T_s) at the output of the radio frequency front-end feeds the digital receiver's channels. The goal of each channel is to acquire and track the signal of a single satellite. After the acquisition stage, which provides the first code delay and Doppler shift estimates, $\hat{\tau}(t)$ and $\hat{f}_d$, the sampled signal is correlated with a locally-generated replica and then accumulated over the integration period T_s. The samples at the output of the correlators are [19]:

$$y_k = A_k d_k R(\Delta\tau_k)\frac{\sin(\pi\Delta f_{d,k}T_s)}{\pi\Delta f_{d,k}T_s}e^{j(2\pi\Delta f_{d,k}T_s+\Delta\theta_k)} + n_k, \quad (2)$$

where k stands for the discrete time $t_k = kT_s$, A_k is the signal amplitude at the output of the correlators after accumulation over T_s, d_k is the data bit, $R(\cdot)$ is the code correlation function and $\{\Delta\tau_k, \Delta f_{d,k}, \Delta\theta_k\}$ are the code delay, Doppler shift and carrier phase errors, respectively.

Taking into account the problem at hand (*i.e.*, study of robust carrier phase estimation techniques), a simplified signal model with a perfect code delay estimation, $\Delta\tau_k = 0$, can be considered. Moreover, it is also considered that $R(0) = 1$, no data bits are present in the received signals (this is the case when using pilot signals or data wipe-off techniques), and that the attenuation factor is negligible, $\text{sinc}(\Delta f_{d,k}T_s) \approx 1$. Under these assumptions, the samples at the input of the carrier phase tracking stage are

$$y_k = \alpha_k e^{j\theta_k} + n_k, \quad (3)$$

where the amplitude, α_k, may suffer from fades due to scintillation, shadowing and multipath. Carrier phase includes both the phase variations due to the receiver's dynamics, $\theta_{d,k}$, and the other phase impairments, $\theta_{i,k}$, which may come for instance from receiver phase noise or scintillation phase variations. Therefore, $\theta_k = \theta_{d,k} + \theta_{i,k}$, and the Gaussian measurement noise is $n_k \sim \mathcal{N}(0, \sigma^2_{n,k})$.

State-space model formulation

In the carrier phase tracking estimation problem, considering that the input to the tracking block is given by (3), the parameter of interest is the time-varying phase θ_k. In order to write a state evolution equation, the state to be tracked has to be defined along with a proper modeling of its time evolution. In standard KF formulations, the carrier phase is usually modeled using a Taylor approximation of the time-varying evolution driven by relative dynamics between the satellite and the receiver, where the order depends on the expected receiver's dynamics. For instance, the 2^{nd} order approximation, considering only a Doppler frequency shift, $f_{d,k}$ (Hz), is given by

$$\hat{\theta}_k = \theta_0 + 2\pi f_{d,k}kT_s, \quad (4)$$

where θ_0 (rad) is a random constant phase value. Or the common 3^{rd} order case, including a possible Doppler rate, is written as

$$\hat{\theta}_k = \theta_0 + 2\pi\left(f_{d,k}kT_s + \frac{1}{2}\dot{f}_{d,k}k^2T_s^2\right), \quad (5)$$

with $\dot{f}_{d,k}$ (Hz/s) the Doppler frequency rate (i.e., the Doppler dynamics). In the first case, the state to be tracked is given by $\mathbf{x}_k^\top \doteq \left[\hat{\theta}_k \ f_{d,k}\right]$, while for the third order approximation is $\mathbf{x}_k^\top \doteq \left[\hat{\theta}_k \ f_{d,k} \ \dot{f}_{d,k}\right]$. Using this particular carrier phase model, the 3^{rd} order case process equation (phase expressed in cycles, $rad/2\pi$) is

$$\mathbf{x}_k = \underbrace{\begin{pmatrix} 1 & T_s & T_s^2/2 \\ 0 & 1 & T_s \\ 0 & 0 & 1 \end{pmatrix}}_{\text{transition matrix}} \mathbf{x}_{k-1} + \mathbf{v}_k, \quad (6)$$

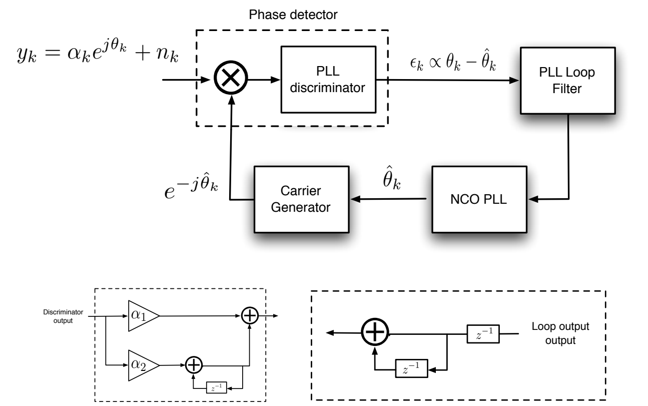

Figure 1. Basic PLL architecture (top), 2^{nd} order PLL loop filter (bottom - left) and NCO (bottom - right).

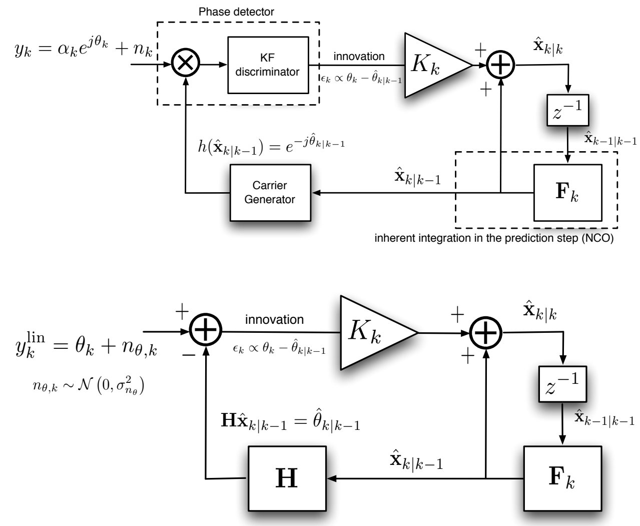

Figure 2. Standard KF-based carrier phase tracking architecture (top) and the linearized KF equivalent (bottom).

where the transition matrix is commonly denoted $\mathbf{F}_k$, and the process noise $\mathbf{v}_k \sim \mathcal{N}(0, \mathbf{Q}_k)$ models possible uncertainties or mismatches on the dynamic model and the phase errors introduced by non-nominal propagation conditions. The process noise covariance matrix $\mathbf{Q}_k$ is usually *a priori* designed according to the problem at hand and depending on the system working conditions. Equations (6) and (3) define the standard 3^{rd} order KF state-space formulation.

It is important to notice that this example is only one of the possible formulations (which will be considered in this contribution as the *standard* KF carrier tracking formulation), and thus different carrier phase models can be considered leading to different state evolution equations. This flexibility is one of the major advantages of the state-space problem formulation. For instance, the 3^{rd} order model was used in [20, 21] for carrier tracking under scintillation conditions; the code delay was included in the state-space formulation for joint code and carrier tracking in [22]; the state-space was augmented to include the scintillation phase evolution in [23], and both scintillation amplitude and phase in [24].

3. Standard PLL and KF-based Carrier Tracking Techniques

Legacy PLL Architectures

The standard PLL architecture is built up with three main blocks: a phase detector based on a discriminator, a filter, and a carrier generator, the latter being driven by a numerically controlled oscillator (NCO) (standard architecture sketched in Figure 1, where α_1 and α_2 are the loop filter coefficients). The phase detector produces an error signal which is proportional to the carrier phase error, which is driven to zero by the filter loop. In the absence of data bits, the optimal maximum likelihood (ML) estimator is the four quadrant arctangent discriminator, while the two quadrant arctangent discriminator is the preferred option when data is present in the signal. As already pointed out in the introduction, the main problem of the standard PLLs is the existing noise reduction versus dynamic range trade-off, which may lead the filter to lose lock, therefore somehow limiting the applicability of these architectures in challenging scenarios. This trade-off is mainly driven by the bandwidth of the PLL: a small bandwidth is need to filter out the noise and a large bandwidth is mandatory to cope with fast phase variations. In modern digital receivers standard PLL architectures, or more advanced solutions such as cooperative loops (FLL-assisted PLL) [4] or adaptive bandwidth PLL solutions [6], are still the methods of choice, mainly because of its implementation and tuning simplicity. Recall that the PLL bandwidth is *a priori* heuristically fixed by the user, being the only parameter that needs to be specified. Therefore, its design and implementation simplicity turns to be the main drawback in time-varying scenarios. To summarize, these methods are far from being optimal, must be heuristically tuned and usually lack of adaptativity to time-varying scenarios. Moreover, their performance has been clearly overcome by KF-based strategies, supporting the idea that more advanced signal processing techniques should be envisaged and taken into account for the near-future GNSS receivers' architecture.

Standard KF-based Carrier Tracking

Using the state-space formulation given in Section 2, it is easy to construct a KF to solve the carrier phase tracking problem. The standard KF formulation does not apply the filter equations to the state-space model given by (3) and (6), because the measurement equation is nonlinear, therefore, a suboptimal nonlinear solution (i.e., extended KF) should be implemented. The strategy is to mimic the conventional PLL architecture and use a phase detector plus filter plus carrier generator structure. In this case, the measurements y_k in (3) go through a phase detection stage (carrier compensation plus discriminator) to obtain the noisy phase measurements (the four quadrant arctangent discriminator, which is the Maximum Likelihood estimator in the absence of data bits, is usually considered). The relation between the KF-based carrier tracking solution, the standard KF equations and the traditional PLL architecture is easily seen by comparing the KF block diagram sketched in Figure 2, the PLL architecture in Figure 1 and the theoretical formulation in Algorithm 1 (which considers a linearized model KF formulation as shown in the bottom plot in Figure 2, where $y_k^{\text{lin}} = \theta_k + n_{\theta,k}$, and $\mathbf{H} = [1\ 0\ 0]$ for the state formulation in (6)).

From the KF filter equations in Algorithm 1, it is easy to see that the measurement noise variance must be specified for the

Algorithm 1 Standard Carrier Tracking KF formulation

Require: $\hat{\mathbf{x}}_0$, $\mathbf{P}_{x,0|0}$, $\mathbf{Q}_k$ and $\sigma^2_{n_\theta,k}$ $\forall\, k$.

1: Set $k \Leftarrow 1$

Time update (prediction)

2: Estimate the predicted state:

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{F}\hat{\mathbf{x}}_{k-1|k-1}.$$

3: Estimate the predicted error covariance:

$$\mathbf{P}_{x,k|k-1} = \mathbf{F}\mathbf{P}_{x,k-1|k-1}\mathbf{F}^\top + \mathbf{Q}_k.$$

Measurement update (estimation)

4: Estimate the predicted measurement:

$$\hat{y}_{k|k-1} = \hat{\theta}_{k|k-1} = \left[\hat{\mathbf{x}}_{k|k-1}\right]_1.$$

5: Estimate the innovation covariance matrix:

$$\sigma^2_{y,k|k-1} = \mathbf{H}\mathbf{P}_{x,k|k-1}\mathbf{H}^\top + \sigma^2_{n_\theta,k}.$$

Estimate the Kalman gain

$$\mathbf{K}_k = \mathbf{P}_{x,k|k-1}\mathbf{H}^\top\left(\sigma^2_{y,k|k-1}\right)^{-1}.$$

6: Estimate the updated state

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k\left(y_k - \hat{\theta}_{k|k-1}\right).$$

7: Estimate the corresponding error covariance:

$$\mathbf{P}_{x,k|k} = \mathbf{P}_{x,k|k-1} - \mathbf{K}_k\mathbf{H}\mathbf{P}_{x,k|k-1}.$$

8: Set $k \Leftarrow k+1$ and go to step 2.

Kalman gain computation, $\mathbf{K}_k$. This parameter is no longer the variance of the measurement noise in (3), but the variance of the new *linearized* measurement at the output of the discriminator. An expression for the approximated variance of the phase noise at the output of the ATAN discriminator [25] is given by

$$\sigma^2_{n_\theta,k} = \frac{1}{8\pi^2\,(C/N_0)_k\,T_s}\left(1 + \frac{1}{2\,(C/N_0)_k\,T_s}\right). \qquad (7)$$

In this basic architecture the process noise covariance $\mathbf{Q}_k$ must also be specified according to the expected dynamics. Notice that both measurement noise variance and process noise covariance matrix are system parameters, therefore, an optimal KF operation is constraint by the complete knowledge of the system working conditions (i.e., known $\sigma^2_{n_\theta,k}$ and $\mathbf{Q}_k$ $\forall\, k$).

4. On the Identifiability of Noise Statistics Parameters

In standard KF-based tracking architectures, both measurement noise variance, $\sigma^2_{n,k}$ (or equivalently $\sigma^2_{n_\theta,k}$), and process noise covariance matrix, $\mathbf{Q}_k$, are assumed to be perfectly known, which is not realistic in practical implementations and leads to poor performances in unknown, time-varying scenarios. In real-life applications, the two main reasons supporting that standard KF tracking approach is suboptimal are:

1. Using a discriminator inside the filter architecture is actually an approximation. Moreover, a discriminator is a nonlinear function, and therefore the Gaussian assumption is no longer guaranteed.

2. The full knowledge about the system working conditions (i.e., system noise statistics) is mandatory for an optimal KF operation. But those noise statistics (both the distribution and its parameters) may be unknown to a certain extent.

In this contribution, only the latter is considered. It is clear from the discussion that those two quantities must be somehow adjusted according to the actual working conditions to obtain a robust tracking solution able to cope with time-varying scenarios. When considering the noise statistics estimation problem, some issues on the identifiability of the statistics' parameters and their correct estimation may arise.

Some preliminary considerations:

- Only the additive white Gaussian noise case is considered, therefore only the measurement and process noise covariance matrices need to be estimated.
- The first problem on the identifiability refers to the joint estimation of both system noise parameters, that is, $\mathbf{Q}_k$ and $\sigma^2_{n,k}$ (or, equivalently, $\sigma^2_{n_\theta,k}$). System parameters, including both noise statistics and other possible system model parameters, are gathered in vector $\boldsymbol{\Psi}_k$.
- The second problem on the identifiability refers to the joint state and system parameters estimation. That means considering an augmented state, $\tilde{\mathbf{x}}_k = \{\mathbf{x}_k, \boldsymbol{\Psi}_k\}$, and a single KF to track the full set.
- Both previous points on the identifiability are treated within the Kalman filter framework, and therefore the conclusions could be different if other approaches are used.

I - Joint Parameters Estimation

From the design point of view, the first problem that comes to mind within the noise statistics estimation context is to consider if only one or both system parameters need to be estimated. This choice mainly depends on the assumptions and the problem at hand, but from a theoretical standpoint the more general case implies the estimation of both measurement and process noise statistics' parameters. The main question that arises here is the identifiability when considering the joint $\{\mathbf{Q}_k, \sigma^2_{n_\theta,k}\}$ estimation problem. All the information available to infer these parameters has to be obtained from (or is contained in) the filter itself. Within the KF formulation (see Algorithm 1), it is clear that both prediction and measurement update steps are always interconnected. For instance, the mathematical evidence of such interconnection can be seen in the following equations. The Kalman gain, $\mathbf{K}_k$, can be written as

$$\begin{aligned}\mathbf{K}_k &= \mathbf{P}_{x,k|k-1}\mathbf{H}^\top\left(\sigma^2_{y,k|k-1}\right)^{-1}\\ &= \left(\mathbf{F}\mathbf{P}_{x,k-1|k-1}\mathbf{F}^\top + \mathbf{Q}_k\right)\mathbf{H}^\top\left(\sigma^2_{y,k|k-1}\right)^{-1}\\ &= \left(\mathbf{F}\mathbf{P}_{x,k-1|k-1}\mathbf{F}^\top + \mathbf{Q}_k\right)\mathbf{H}^\top\left(\mathbf{H}\mathbf{P}_{x,k|k-1}\mathbf{H}^\top + \sigma^2_{n_\theta,k}\right)^{-1}\end{aligned}$$

and therefore, at time k, it depends on both noise statistics parameters $\mathbf{Q}_k$ and $\sigma^2_{n_\theta,k}$. Estimation and prediction error covariances are:

$$\begin{aligned}\mathbf{P}_{x,k|k} &= \left(\mathbf{I} - \mathbf{K}_k\mathbf{H}\right)\mathbf{P}_{x,k|k-1}\\ &= \left(\mathbf{I} - \mathbf{K}_k\mathbf{H}\right)\left(\mathbf{F}\mathbf{P}_{x,k-1|k-1}\mathbf{F}^\top + \mathbf{Q}_k\right),\\ \mathbf{P}_{x,k|k-1} &= \mathbf{F}\mathbf{P}_{x,k-1|k-1}\mathbf{F}^\top + \mathbf{Q}_k\\ &= \mathbf{F}\left(\left(\mathbf{I} - \mathbf{K}_{k-1}\mathbf{H}\right)\mathbf{P}_{x,k-1|k-2}\right)\mathbf{F}^\top + \mathbf{Q}_k.\end{aligned}$$

As the Kalman gain depends on both noise statistics, the same happens with the estimation and prediction error covariance matrices. The same dependence can be written for the predicted and updated state estimates. Therefore, all the computations within the filter depend on both parameters to be estimated. Intuitively it seems difficult to distinguish between errors (i.e., errors introduced by a wrong process noise covariance or a wrong measurement noise variance) using only the information given by the filter, so a mismatch on the estimation may occur because we do not control the coupling between both noises. From our knowledge, very few studies have discussed such identification problem. This concept has been briefly introduced in [26], stating that it is impossible to distinguish between overspecification of the model error and underspecification of the measurement error, and vice versa. To illustrate the mathematical results given in [26] which support this statement, let us assume that the estimation problem at hand can be represented by a very simple scalar state-space model of the form

$$x_k = x_{k-1} + v_k, \quad \text{with} \quad v_k \sim \mathcal{N}(0, Q), \tag{8}$$

$$y_k = x_k + n_k, \quad \text{with} \quad n_k \sim \mathcal{N}(0, R), \tag{9}$$

and that the process and measurement noise variances are somehow misspecified (the tilded variables represent the incorrect version of the true/correct ones, that is, $\tilde{Q} = \alpha Q$ and $\tilde{R} = \beta R$). Applying the standard Kalman filtering theory to this system and considering that the filter has attained the steady-state regime ($P_{k|k-1} = P_p$ and $P_{k|k} = P_e$), the correct and misspecified prediction and estimation error variances are given by

$$P_p = P_e + Q \ \ ; \ \ \tilde{P}_p = \tilde{P}_e + \tilde{Q} \tag{10}$$

$$P_e = \frac{\tilde{P}_p^2 R + \tilde{R}^2 P_p}{(\tilde{P}_p + \tilde{R})^2} \ \ ; \ \ \tilde{P}_e = \frac{\tilde{R}\tilde{P}_p}{\tilde{P}_p + \tilde{R}}. \tag{11}$$

Solving this system of four equations, one can find the values of P_p, P_e, $\tilde{P}_p$ and $\tilde{P}_e$ only in terms of $\{\alpha, \beta, Q, R\}$. While the misspecified variances $\tilde{P}_p$ and $\tilde{P}_e$ depend linearly on α and β (i.e., depend separately of $\tilde{Q}$ and $\tilde{R}$), the correct values P_p and P_e are functions of α/β. This leads to the following design rule:

> **First design rule**: using standard noise statistics' estimation methods, only one of the two parameters can be correctly estimated while considering the other one known.

II - Joint State and Parameters Estimation

The problem at hand is the estimation of both the original states of the system $\mathbf{x}_k$ (*i.e.*, the carrier phase and Doppler terms) and other unknown parameters of the state-space model $\mathbf{\Psi}_k$. From a pragmatic point of view, this can be accomplished using two different approaches: either using a single filter to estimate both unknowns, or separate (probably interacting) methods each one dealing with a single set (i.e., $\mathbf{x}_k$ or $\mathbf{\Psi}_k$):

1. First approach: joint state and parameters estimation. This approach implies to define an augmented state which gathers all the unknowns, $\tilde{\mathbf{x}}_k = \{\mathbf{x}_k, \mathbf{\Psi}_k\}$, and use a single KF to track the complete set. *A priori*, the joint estimation should improve the performance with respect to the second approach, which couples different methods.

2. Second approach: parallel (interacting) methods. In this case the main idea is to decouple the inference of the state evolution from the parameters estimation, that implies using parallel methods possibly interacting with each other. For instance, using a standard KF-bases solution for carrier tracking and one or more parallel methods (coupled with the KF core) to estimate the other state-space model parameters. From the previous discussion on the joint parameters estimation and the consequent "first design rule", it can be concluded that only one of the two covariance matrices will be estimated together with the state evolution.

The main question is to decide which of the two approaches is preferable, and if there exist problems on the identifiability of the noise statistics parameters when using the first one. Consider a constant and diagonal process noise covariance matrix, $\mathbf{Q} = \text{diag}(\sigma_1^2, ..., \sigma_N^2)$, being N the state dimension, and the system parameters vector $\mathbf{\Psi} = [\sigma_1^2, ..., \sigma_N^2]^\top$. Note that the joint estimation of $\mathbf{x}_k$ and $\mathbf{\Psi}$ leads to a nonlinear state-space model, thus an EKF-type (linearized) solution must be considered. Assume that at time k, the state and process noise covariance parameters are independent and jointly Gaussian, that is, the joint probability distribution can be written as (i.e., $\mathbf{y}_{1:k}$ refers to $[y_1, ..., y_k]$)

$$p(\tilde{\mathbf{x}}_k|\mathbf{y}_{1:k}) = \mathcal{N}\left(\left[\begin{array}{c}\hat{\mathbf{x}}_{k|k}\\ \hat{\mathbf{\Psi}}_{k|k}\end{array}\right], \left[\begin{array}{cc}\mathbf{P}_{x,k|k} & \mathbf{0}\\ \mathbf{0} & \mathbf{P}_{\psi,k|k}\end{array}\right]\right). \tag{12}$$

where the independence is clear in the formulation from the null cross-covariance matrix. Using the standard state evolution equation

$$\mathbf{x}_{k+1} = \mathbf{F}_k\mathbf{x}_k + \mathbf{v}_k \ \ \text{with} \ \ \mathbf{v}_k \sim \mathcal{N}(0, \mathbf{Q}(\mathbf{\Psi})_k), \tag{13}$$

the predictive probability distribution at time $k+1$ is

$$\begin{aligned} p(\tilde{\mathbf{x}}_{k+1}|\mathbf{y}_{1:k}) &= p(\mathbf{x}_k|\mathbf{y}_{1:k-1})p(\mathbf{\Psi}|\mathbf{y}_{1:k-1}) \\ &= \mathcal{N}\left(\mathbf{F}_k\hat{\mathbf{x}}_{k|k}, \mathbf{F}_k\mathbf{P}_{x,k|k}\mathbf{F}_k^\top + \mathbf{Q}(\hat{\mathbf{\Psi}}_{k|k})\right) \\ &\quad \times \mathcal{N}\left(\hat{\mathbf{\Psi}}_{k|k}, \mathbf{P}_{\psi,k|k}\right), \end{aligned} \tag{14}$$

from which is clear that the state and parameters predicted values are dependent and no longer jointly Gaussian. But the standard EKF prediction and update steps formulation does not propagate the third-order cross-conditional moment, as it relies on the Gaussian assumption and the propagation of only the first two moments of the distribution, and thus the interconnection between states and parameters estimates is lost. In fact, the EKF only takes into account the linear dependency among variables, which is null in this case, and does not take into account the actual nonlinear dependency. To sum-up, although extended state-spaces can be considered to perform a joint estimation, the elements of the process noise covariance matrix $\mathbf{Q_k}$ cannot be jointly estimated along $\mathbf{x_k}$ within the KF framework, since KFs neglect the dependency between $\mathbf{x_k}$ and the noise statistics parameters. Because the prediction step of the EKF is computed using only the first two moments, and $\mathbf{x_k}$ and $\mathbf{Q_k}$ are not jointly Gaussian, $\mathbf{Q_k}$ is not identifiable [27,28]. From these results, it can be stated that the joint state and Gaussian noise covariance estimation is not a valid approach, thus the second approach must be adopted for a proper filter design.

> **Second design rule**: the state and system model parameters must be estimated using separate methods.

5. ADAPTIVE KF DESIGN FRAMEWORK IN GNSS RECEIVERS

This section presents the general framework for the correct design of AKF solutions to overcome the standard KF limitations. The key point is to take into account the two design rules given in the previous section: *i*) only one of the two noise statistics parameters can be correctly estimated together with the system states, and *ii*) these parameters and states cannot be jointly estimated in a single filter. Which covariance must be estimated (i.e., process or measurement noise)? How can the other one be adjusted without breaking those design rules? The measurement noise or observational errors are usually much better known and characterized than the system or model errors, and therefore it is reasonable to consider a known measurement noise variance and the process measurement covariance matrix estimation problem as a key design point. But even if it is better characterized, the measurement noise statistics parameters are not fully known and must also be adjusted. In the sequel, both separate approaches (*i.e.*, separate from the KF core, in accordance with the second design rule in Section 4) to deal with unknown measurement and process noise statistics are detailed.

Measurement noise statistics estimation

As it has already been pointed out, it is impossible to obtain simultaneously correct estimates for both noise statistics parameters. Taking advantage of the problem at hand, in the case of GNSS receivers a C/N_0 (carrier to noise density ratio) empirical estimator is always available. Keeping in mind the standard KF architecture, the linearized measurement noise variance (7) is computed from the actual C/N_0, therefore, it is straightforward to obtain an estimate of the measurement noise variance by using this empirical estimate:

$$\hat{\sigma}^2_{n_\theta} = \frac{1}{8\pi^2 \widehat{C/N_0} T_s}\left(1 + \frac{1}{2\widehat{C/N_0} T_s}\right). \tag{15}$$

This is one of the key points to obtain a fully adaptive KF, because the measurement error can be sequentially adapted to the actual working conditions without compromising the correct adjustment of the system model using a covariance estimation method, thus fulfilling the first design rule in Section 4. This approach has been already adopted in [17] and [18].

Process noise statistics estimation

The main goal of this section is to provide an up-to-date state-of-the-art of possible alternatives to solve the Gaussian process noise covariance estimation problem. A plethora of alternatives exists in the literature for the estimation of a Gaussian noise covariance matrix within the KF framework. The interest of the Signal Processing and Control community on the covariance estimation problems for linear dynamic Gaussian systems begins in the mid ’60s. Several contributions appeared from different points of view. An old survey paper [29] classified the possible alternatives into four general classes: Bayesian, maximum likelihood (ML), covariance matching and correlation methods. All the traditional on-line state-of-the-art techniques are included therein. A more general classification (vis-a-vis the Merha’s paper) is to consider two main groups: *on-line* and *off-line* methods [30]. The four groups introduced by Mehra (adaptive methods) lie into the on-line methods group. In the second group we can include *subspace* and *prediction error estimation* methods. [30] provides a good insight of the pros and cons of the different alternatives. To solve the recursive Bayesian estimation problem we need on-line noise statistics estimation methods to be embedded in the filter structure. That’s the reason why most of the effort in the field has gone to this direction.

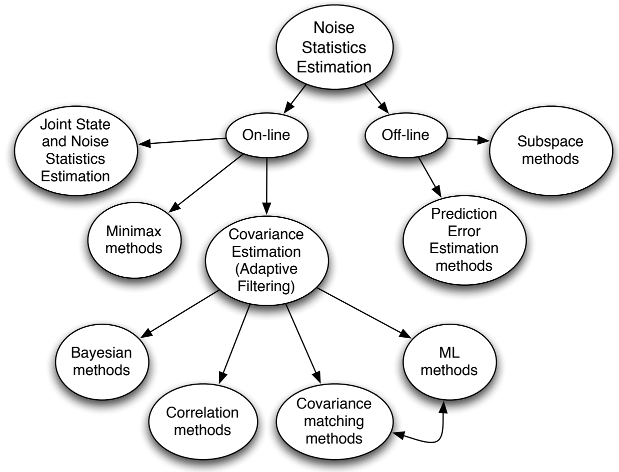

Figure 3. Noise statistics estimation state-of-the-art diagram

The ML methods find the estimates that maximize the likelihood function [31–33], following different approaches: joint state/parameters ML solution, marginal ML solution (only for the parameters) and conditional mode estimates [29]. In general the ML estimates may not accept an analytic expression and simulation based solutions are needed. An interesting tool is the Expectation-Maximization (EM) algorithm [34] which gives an iterative solution to compute the ML estimates. Covariance matching methods compute the process and measurement noise covariances from the residuals of the state estimation problem. The idea is to make these residuals consistent with their theoretical covariances [35]. This method was corrected by Leathrum [36], and compared to Maybeck’s ML estimator [32] in [37], showing that both are equivalent if the system noise is zero-mean. The correlation methods, pioneered by Mehra [29] and Bélanger [38], are the most popular for the estimation of Gaussian noise covariance matrices. These methods are based on the correlation function of the innovation sequence and the derivation of a set of equations which relate this function to the unknown parameters [39].

Within the correlation methods, Odelson *et al.* [40] derived the Autocovariance Least Square (ALS) method, being one of the most promising covariance estimation techniques derived in the last decade. This method has been extended to consider correlated process and measurement noises in [41], time-varying nonlinear systems using an EKF-type linearization approach [42], and optimal weighting of the least-squares solution [43]. Concerning the ML estimation and the EM solution, some interesting contributions are [44, 45] and [46]. Other new approaches include robust M-estimation least-squares [47, 48], variational Bayesian approximations [49], robust generalized ML [50] and generalized multiple model adaptive estimation [51]. In [28], the authors compare the ALS method [40] and the method introduced in [52], which is a joint state and parameters estimation method taking into account the propagation of third order cross-covariance within the KF.

Adaptive KF design architecture

From the previous discussion, all the necessary elements to come up with a powerful AKF design framework are available. The general architecture is based on three main blocks: *i*) the core of the filter follows the standard KF-based carrier tracking architecture introduced in Section 3, *ii*) the measurement noise variance is adjusted from the receiver carrier to noise density estimator, and *iii*) the KF process noise covariance matrix is adjusted using one of the methods presented in Section 5.

In a more general case where the loss of Gaussianity caused by the discriminator needs to be avoided, the same design framework also applies. In this case, the standard KF core must be replaced by an EKF directly operating with the received signal complex samples and the carrier to noise density estimator is used to tune the measurement noise variance at the input of the carrier tracking stage. The proposed AKF design architecture is sketched in Figure 4.

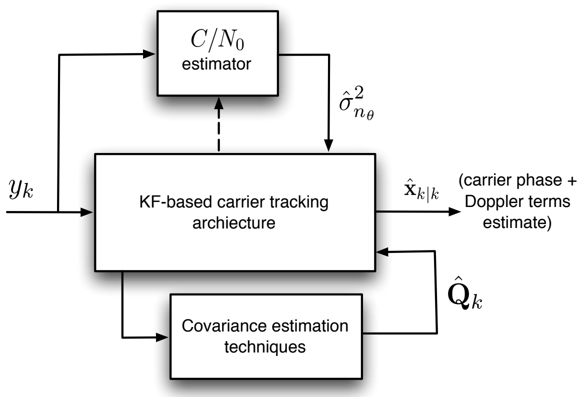

Figure 4. Adaptive KF design framework graph.

6. COMPUTER SIMULATIONS

The main target scenarios of using advanced signal processing techniques for carrier tracking are the harsh propagation environments. As already pointed out, in these cases the signal may be affected by high dynamics, strong fadings, urban multipath effects or ionospheric scintillation. Among these propagation conditions, the ionospheric scintillation is certainly the most challenging one due to the combination of both fading and rapid phase changes. Moreover, the fact that scintillation effects are typically unnoticed for mass-market GNSS receivers has led this effect to receive rather little attention in the signal processing literature. These are the reasons for considering the following numerical example of carrier tracking under scintillation conditions as a performance benchmark for the AKF design.

Harsh propagation scenario: ionospheric scintillation

Ionospheric scintillation refers to the disturbances caused by electron density irregularities along the propagation path through the ionosphere. In particular, these harsh propagation conditions affect GNSS signals with amplitude fades and carrier phase variations, that usually happen in a simultaneous and random manner. But in some cases there exists a correlation between both disturbances, the so-called canonical fades [53], which is a very challenging scenario from a carrier tracking point of view. In the sequel, a parsimonious signal model to represent the behavior of scintillation onto the GNSS received signal samples is briefly introduced. Refer to [24] for a detailed description. In terms of the complex-valued baseband received signal, $x(t)$, the scintillation can be modeled as a multiplicative channel,

$$x(t) = \xi_s(t)s(t) + w(t), \quad (16)$$

where $s(t)$ is the complex-valued baseband equivalent of the transmitted signal, $w(t)$ is the noise term, and the complex-valued stochastic process representing the presence of scintillation is defined as

$$\xi_s(t) = \rho_s(t)e^{j\theta_s(t)}, \quad (17)$$

with the corresponding envelope and phase components, $\rho_s(t)$ and $\theta_s(t)$. Some recent contributions [54, 55] have introduced a method called the Cornell Scintillation Model (CSM) to synthesize realistic scintillation, embedded in the so-called Cornell Scintillation Simulation Matlab toolkit[2]. Notice that for the simulation of a scintillation data set only two parameters must be specified, $\{S_4, \tau_0\}$, which determine the intensity and correlation of the scintillation complex gain components, respectively. Generally speaking, higher S_4 and lower τ_0 lead to more severe scintillation, where the ranges of possible values are $0 < S_4 \leq 1$ and $0.1 \leq \tau_0 < 2$ seconds, respectively [54]. Three main scintillation regions are considered: weak ($S_4 \leq 0.3$), moderate ($0.3 < S_4 \leq 0.6$) and strong/severe ($0.6 < S_4$).

Numerical results

In order to show the need of fully adaptive solutions illustrative numerical results are given for a carrier phase tracking example where the signal of interest is corrupted by severe scintillation. The following parameters were used: simulation time $T_{sim} = 10$s, integration time $T_s = 1$ms, $C/N_0 = 35$ dB-Hz, $f_{d,0} = 10$ Hz (Doppler) and $\dot{f}_{d,0} = 1$ Hz/s (Doppler rate). To obtain statistically significant results, the root mean square error (RMSE) was used as a measure of performance, and obtained from 200 Monte Carlo runs.

In order to show the importance of correctly adjusting in a time-varying manner the process noise statistics, a trajectory with a time-varying process noise covariance matrix $\mathbf{Q_k}$ (varying between $\mathbf{Q_{max}}$ and $\mathbf{Q_{min}}$) was considered for two different scenarios: the first one taking into account only the phase evolution due to the receiver dynamics, and the second one using the same dynamics but affected by severe scintillation in the period $5s < t \leq 8$s, with $S_4 = 0.8$ and $\tau_0 = 0.1$.

The RMSE obtained with a standard KF considering different values of $\mathbf{Q}$, for both scenarios, is given in Fig. 5. It is clear from the results obtained in the dynamics-only case (Fig. 5 - top) that the optimum is to consider the correct covariance matrix. To be robust against unexpected phase variations it is always better to overestimate the noise covariance, while underestimating usually leads the filter to loss of lock (divergence). Considering that a part of the signal is affected by severe scintillation (Fig. 5 - bottom), both the performance and adaptability of the filter also depend on the value given to the process noise covariance, and the optimum may not be the same as for the dynamics-only case.

Using the same dynamics, Fig. 6 compares the RMSE obtained with a standard KF (with fixed $\mathbf{Q}$ and underestimated σ_n^2), and an AKF only estimating the measurement noise variance and considering different values of $\mathbf{Q}$. In this scenario, a strong fading in $[2-4]$s was considered.

[2]This software was used in the computer simulations to generate the desired scintillation effect and then assess the performance of the proposed methods. The toolkit is available at `http://gps.ece.cornell.edu/tools.php`

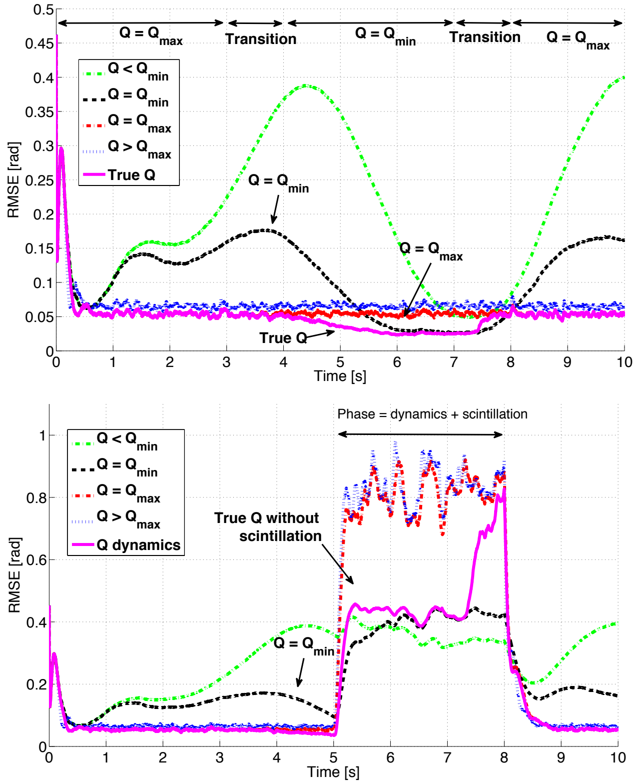

Figure 5. RMSE for different standard KF considering different values of **Q**, for both dynamics only (top) and dynamics + scintillation (bottom) scenarios.

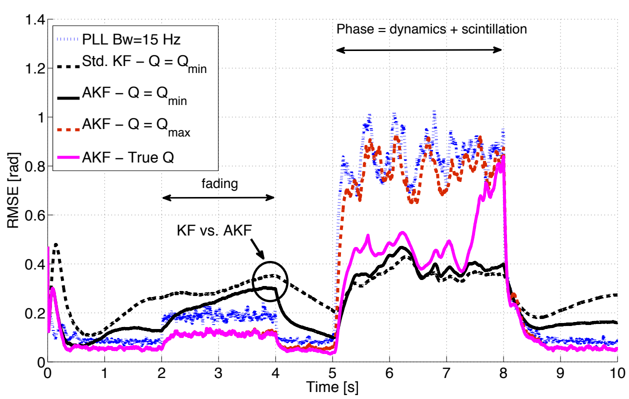

Figure 6. Comparison between the std. KF and an AKF adjusting the measurement noise.

The results show the expected behavior of the AKF, always performing slightly better than the KF, but again prove the need of correctly adjusting the process noise covariance matrix. These results suggest that the fully adaptive KF design detailed in this paper must be adopted. The general AKF design framework is an appealing solution, not only to deal with scintillation but also for more general robust carrier tracking applications in harsh propagation conditions.

7. Conclusions

This paper presented a filter design framework/methodology to deal with robust carrier phase tracking under harsh propagation conditions. A discussion on the identifiability of the noise statistics parameters motivated the design choice. The adaptive KF-based approach is based on a filter that sequentially adjusts both process and measurement noise statistics. The key point being the use of two parallel noise statistics estimation strategies coupled with the standard KF. The need of estimating both statistics was supported by computer simulation in a GNSS carrier phase tracking application, using a simulated trajectory with different scenarios. Future work goes towards analyzing the impact of using different covariance estimation methods and carrier to noise density estimators.

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BIOGRAPHY

Jordi Vilà-Valls *received the Ph.D. degree in Electrical Engineering from the Grenoble Institute of Technology (INPG), Grenoble, France, in 2010. He was a Postdoctoral Research Associate with the Signal Processing and Communications (SPCOM) group, UPC (2010-12); and with the Signal Processing for Communications and Navigation (SPCOMNAV) group, UAB (2012-13). In 2014 he joined the CTTC as a Research Associate at the Statistical Inference Department. His primary areas of interest include statistical signal processing, estimation and detection theory, nonlinear Bayesian estimation, robustness and adaptive methods; with applications to positioning, localization and tracking systems, communications, aerospace and seismology.*

Pau Closas *received the M.Sc. and Ph.D. in Electrical Engineering from the Universitat Politècnica de Catalunya (UPC) in 2003 and 2009, respectively. He holds a M.Sc. degree in Advanced Mathematics and Mathematical Engineering from UPC since 2014. In 2003, he joined the Department of Signal Theory and Communications, UPC, as a Research Assistant. During 2008 he was Research Visitor at the Stony Brook University (SBU), New York, USA.*
In 2009 he joined the CTTC, where he currently holds a Senior Researcher position and is the Head of the Statistical Inference for Communications and Positioning Department in the Communications Systems Division. His primary areas of interest include statistical and array signal processing, estimation and detection theory, Bayesian filtering, robustness analysis, and game theory, with applications to positioning systems, wireless communications and mathematical biology. Pau Closas is the recipient of the EURASIP Best PhD Thesis Award 2014 and the 9^{th} *Duran Farell Award for Technological Research from UPC for sustained research in advanced GNSS receiver design.*

Carles Fernández–Prades *received the M.Sc. and Ph.D. (cum-laude) degrees in Electrical Engineering from the Universitat Politècnica de Catalunya (UPC) in 2001 and 2006, respectively. In 2001, he joined the Department of Signal Theory and Communication at UPC as a Research Assistant, getting involved in European and National research projects both with technical and managerial duties. He also was Teaching Assistant in the field of Analog and Digital Communications (UPC 2001-2005). In 2006 he joined CTTC, where he currently holds a position as Senior Researcher and serves as Head of the Communications Systems Division. His primary areas of interest include signal processing, estimation theory and nonlinear Bayesian filtering, with applications in positioning, communications systems, software radio and the design of RF front-ends.*