26th EVGA Working Meeting, 11th-15th June 2023, Bad Kötzting, Germany

Neglected issues of terrestrial datum definition in VLBI

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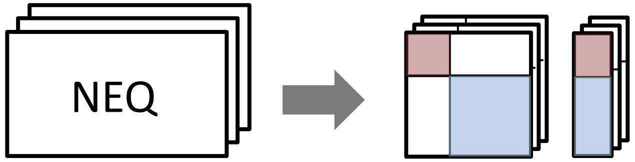
Motivation and current status at Vienna Center for VLBI

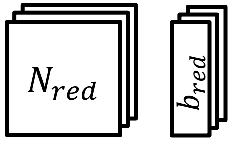
- In progress: Development of a new **state-of-the-art/ stand-alone** Python software for intra-/ inter-technique combination on NEQ level
  - Input: pre-reduced, **datum-free NEQ** (SINEX)
- Current status: VLBI Global solution (multi-session solution) on NEQ level

→ Different methods of datum definition + scaling of datum conditions

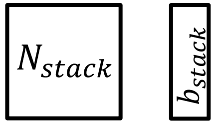
Global solution theory



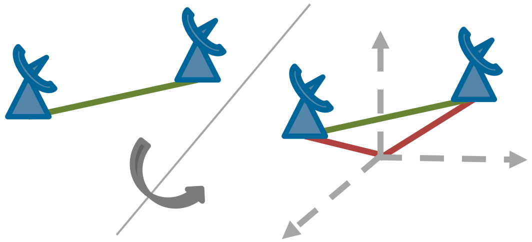
VLBI pre-reduced,
datum-free NEQs



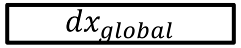
Session-wise reduction of
non-global parameters



Stacking

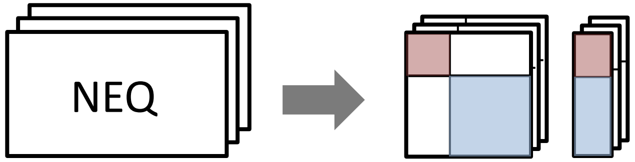


Introduction
of geodetic datum



Global solution

Global solution theory



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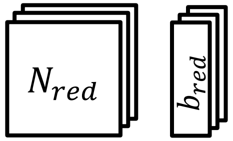
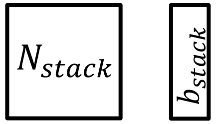


Session-wise reduction of non-global parameters

$d = 6$



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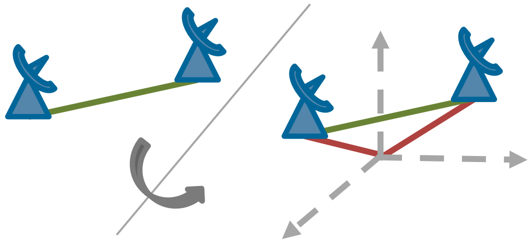
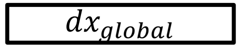


Introduction of geodetic datum

$d = 0 \rightarrow N^{-1}$



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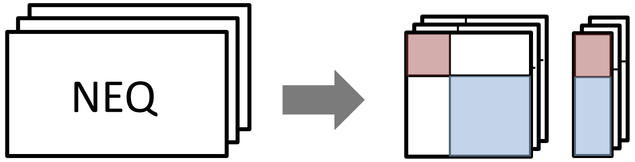


VLBI pre-reduced,
datum-free NEQs

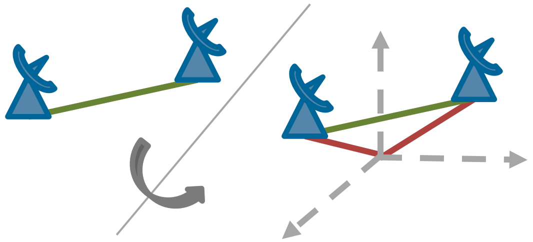
N_{red} b_{red}

Session-wise reduction of
non-global parameters

$d = 6$

N_{stack} b_{stack}

Stacking



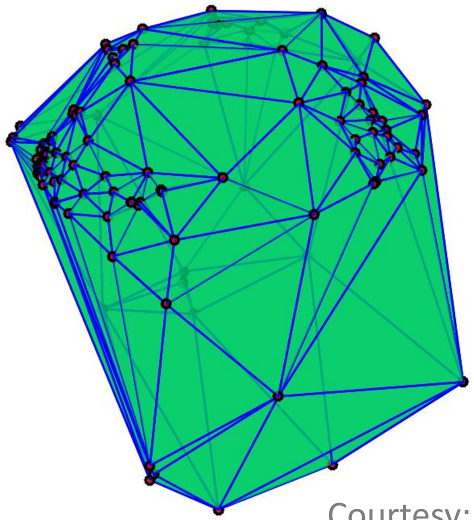
Introduction
of geodetic datum

$d = 0 \rightarrow N^{-1}$

dx_{global}

Global solution

Geodetic datum definition



Courtesy: Armin Corbin

- VLBI based on precise relative relationships within observing network

→ Precise geometric configuration

$d = 6$ → 3 translations, 3 rotations

- Absolute position of point cloud?

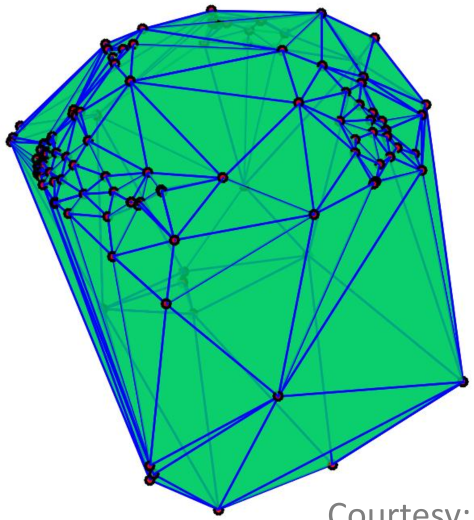
→ No-net-translation (NNT) and no-net-rotation (NNR) approach

→ Introduction of a datum using a conventional frame as target frame

$\sum_{i=1}^{N} \Delta \boldsymbol{x}^i = \boldsymbol{0}$ [m] — No-net-translation constraint

$\sum_{i=1}^{N} (\boldsymbol{r}^i \times \Delta \boldsymbol{x}^i) = \boldsymbol{0}$ [m^2] — No-net-rotation constraint

Geodetic datum definition



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→ $\boldsymbol{B}$

Geodetic datum definition

- Apply datum definition to NEQ: *(e.g., Sillard and Boucher (2001), Angermann et al. (2004), Dach et al. (2015))*
  - *Conditions with infinite weight (Helmert rendering)*

  = forcing translations/ rotations w.r.t. reference frame to be zero

$$N_c = \begin{bmatrix} N & B \\ B^T & 0 \end{bmatrix}$$

  - *Constraints with covariances*

  = loose constraints: lead to a "loose fit" of network within the datum definition

1) $H = (B^T B)^{-1} B^T$

2) $H = B^T$

$$N_c = N + H^T P H$$

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$$1)\ H = (B^T B)^{-1} B^T$$

$$2)\ H = B^T$$

$$\rightarrow N_c = N + H^T P H$$

- Scaling of constraint matrix B

  $\leftrightarrow$ Singular Value Decomposition (with $\lambda_{1-6} = 0$)

  = normalizing column length of B to one

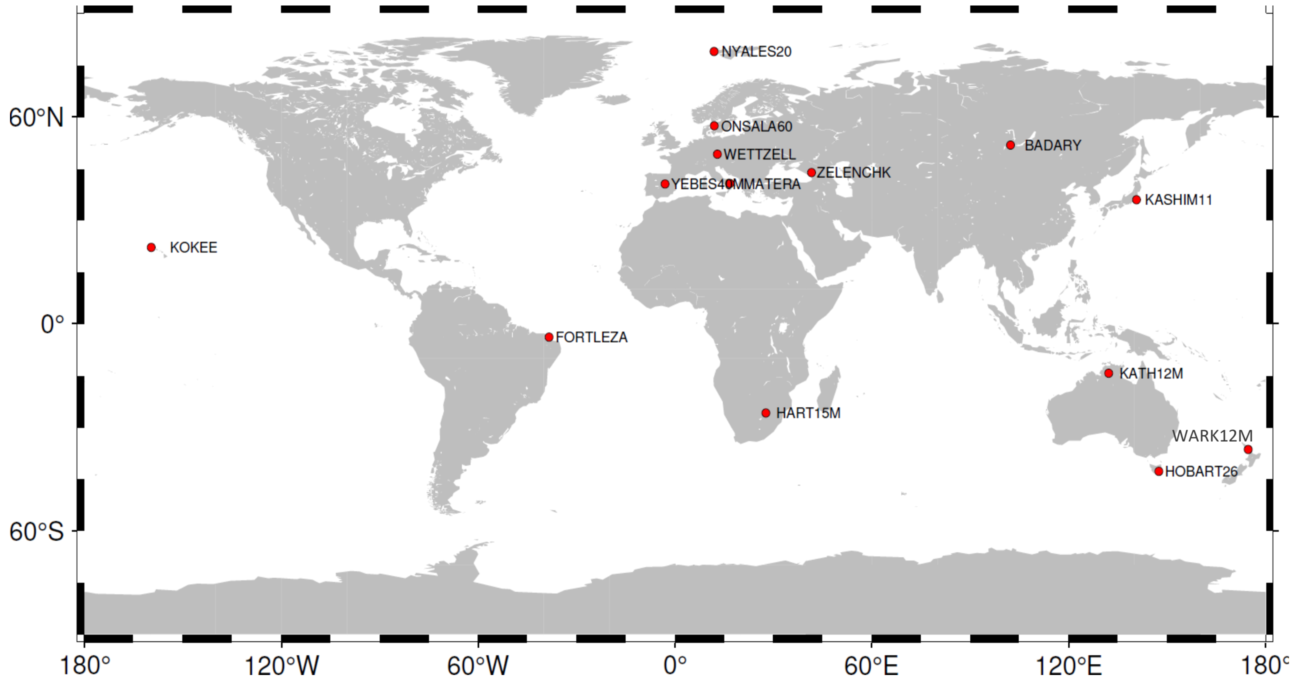
strict:

$$B_{NNT} * \frac{1}{\sqrt{N}} + B_{NNR} * \frac{1}{\sqrt{\sum_{i=1}^{N}(x_{0,i}^2 + y_{0,i}^2 + z_{0,i}^2)}}$$

common:

$$B_{NNR} * \frac{1}{R_E}$$

Experiment setup



CONT17 campaign

- November 28 – December 12 2017
- 15 sessions
- 14 stations

→ multi-session solution (only estimating coordinates)

1) $H = (B^T B)^{-1} B^T$

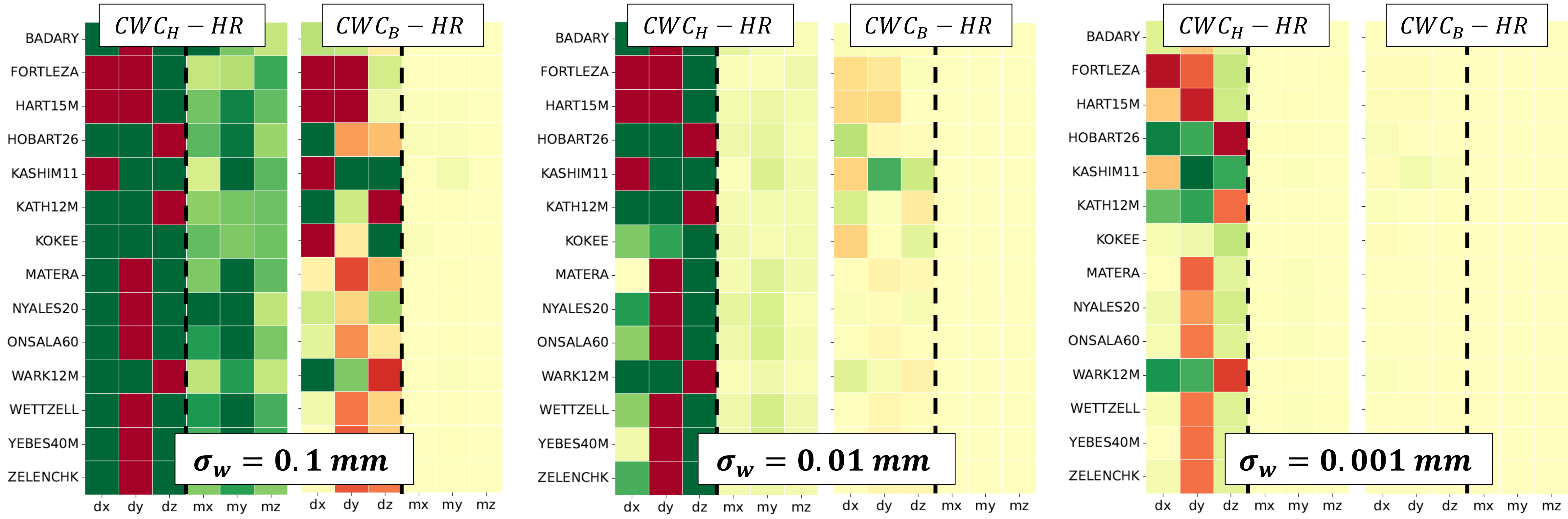
2) $H = B^T$

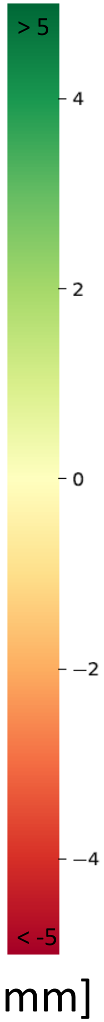
Comparability of methods? (HR, CWC_H, CWC_B)
Influence on results/ fulfillment of conditions?

TRF datum definition methods

$$N_c = N + H^T P H$$

Difference in results (corrections *dx* and accuracies *mx*): $CWC_{H,B} - HR$ (using *common* scaling)





	HR
NNT	5.134781e-16
	6.136584e-17
	1.465841e-16
NNR	-8.412826e-10
	8.076313e-10
	-2.073648e-10

TRF datum definition methods

Fulfillment of NNT/NNR conditions (using *common* scaling)

$$\sum_{i=1}^{N} \Delta x^i = 0 \ [\mathrm{m}] \qquad \sum_{i=1}^{N} \left(r^i \times \Delta x^i \right) = 0 \ [\mathrm{m}^2]$$

No-net-translation constraint — No-net-rotation constraint

	CWC_H ($\sigma_w = 0.1\ mm$)	CWC_B ($\sigma_w = 0.1\ mm$)	CWC_H ($\sigma_w = 0.01\ mm$)	CWC_B ($\sigma_w = 0.01\ mm$)	CWC_H ($\sigma_w = 0.001\ mm$)	CWC_B ($\sigma_w = 0.001\ mm$)
NNT	0.005136	-0.012315	0.000060	-8.622118e-07	6.022829e-07	-8.623666e-09
	-0.004254	0.013166	-0.000056	5.912033e-09	-5.585076e-07	5.729475e-11
	0.001205	-0.004660	0.000018	2.511653e-07	1.794937e-07	2.513280e-09
NNR	47488.227051	132625.811943	601.332719	-11.688458	6.028015	-0.116940
	-35551.871295	-25038.180411	-865.538933	-7.620066	-8.749430	-0.076206
	-152346.044266	-434867.000858	-2272.864043	-39.945308	-22.845439	-0.399512

$\sigma_w = 0.1\ mm$ — $\sigma_w = 0.01\ mm$ — $\sigma_w = 0.001\ mm$

TRF datum definition methods

	HR
N	5.134781e-16
N	6.136584e-17
T	1.465841e-16
N	-8.412826e-10
N	8.076313e-10
R	-2.073648e-10

Fulfillment of NNT/NNR conditions (using *common* scaling)

$$\sum_{i=1}^{N} \Delta x^i = 0 \text{ [m]}$$

No-net-translation constraint

$$\sum_{i=1}^{N} \left(r^i \times \Delta x^i \right) = 0 \text{ [m}^2\text{]}$$

No-net-rotation constraint

	CWC_H ($\sigma_w = 0.1\ mm$)	CWC_B ($\sigma_w = 0.1\ mm$)	CWC_H ($\sigma_w = 0.01\ mm$)	CWC_B ($\sigma_w = 0.01\ mm$)	CWC_H ($\sigma_w = 0.001\ mm$)	CWC_B ($\sigma_w = 0.001\ mm$)	
N	0.005136	-0.012315	0.000060	-8.622118e-07	6.[illegible]22829e-07	-8.623666e-09	N
N	-0.004254	0.013166	-0.000056	5.912033e-0[illegible]	-5.585076e-07	5.729475e-11	N
T	0.001205	-0.004660	[illegible]	[illegible]e-07	1.794937e-07	2.513280e-09	T
N	47488.227051	132625.811943	[illegible]	[illegible]	6.028015	-0.116940	N
N	-35551.871295	-25038.180[illegible]	-865.538933	-7.620066	-8.749430	-0.076206	N
R	-152346.044266	-434867.000858	-2272.864043	-39.945308	-22.845439	-0.399512	R

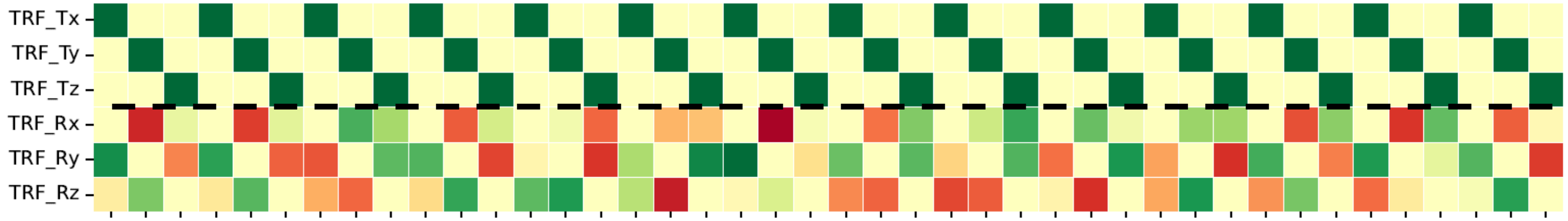
Improvement of condition fulfillment

$\sigma_w = 0.1\ mm$ | $\sigma_w = 0.01\ mm$ | $\sigma_w = 0.001\ mm$

TRF scaling methods

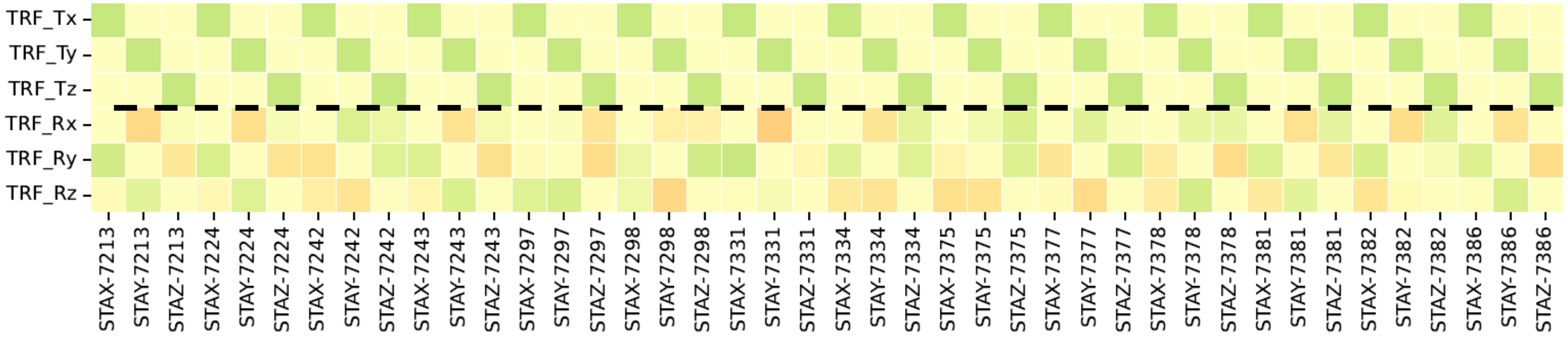
- <u>common</u> scaling of constraint/ condition matrix B

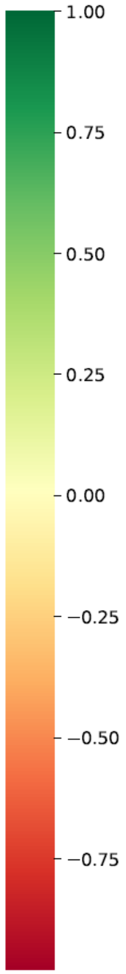
$B_{common} =$



- <u>strict</u> scaling of constraint/ condition matrix B

$B_{strict} =$





TRF scaling methods

strict:	$B_{NNT} * \frac{1}{\sqrt{N}}$	$B_{NNR} * \frac{1}{\sqrt{\sum_{i=1}^{N}(x_{0,i}^2+y_{0,i}^2+z_{0,i}^2)}}$
common:	-	$B_{NNR} * \frac{1}{R_E}$

Fulfillment of NNT/NNR conditions

→ No impact on estimates/ variances, improvement of condition fulfillment

	HR_{none}	HR_{common}	HR_{strict}
NNT	-8.760354e-17	5.134781e-16	-3.365364e-16
	-2.879641e-16	6.136584e-17	4.401861e-17
	-5.637851e-17	1.465841e-16	2.515349e-17
NNR	-3.819878e-11	-8.412826e-10	1.691660e-10
	-1.382432e-10	8.076313e-10	-3.965397e-10
	5.820766e-11	-2.073648e-10	2.692104e-10

	$CWC_{H,none}$	$CWC_{H,common}$	$CWC_{H,strict}$
NNT	0.024658	6.022829e-07	4.303518e-08
	-0.017310	-5.585076e-07	-3.990801e-08
	0.003792	1.794937e-07	1.282590e-08
NNR	202575.854871	6.028015	0.430728
	42155.391884	-8.749430	-0.625234
	-498571.551772	-22.845439	-1.632447

$\sigma_w = 0.001\ mm$

TRF scaling methods

strict:	$B_{NNT} * \frac{1}{\sqrt{N}}$	$B_{NNR} * \frac{1}{\sqrt{\sum_{i=1}^{N}(x_{0,i}^2+y_{0,i}^2+z_{0,i}^2)}}$
common:	-	$B_{NNR} * \frac{1}{R_E}$

Fulfillment of NNT/NNR conditions

→ No impact on estimates/ variances, improvement of condition fulfillment

Improvement of condition fulfillment

	HR_{none}	HR_{common}	HR_{strict}	$CWC_{H,none}$	$CWC_{H,common}$	$CWC_{H,strict}$	
NNT	-8.760354e-17	5.134781e-16	[illegible]	[illegible]	6.022829e-07	4.303518e-08	NNT
	-2.879641e-16	6.136584e-17	4.401861e-17	-0.017310	-5.585076e-07	-3.990801e-08	
	-5.637851e-17	1.465841e-16	2.515349e-17	0.003792	[illegible]	1.282590e-08	
NNR	-3.819878e-11	-8.412826e-10	1.691660e-10	202575.854871	[illegible]	0.430728	NNR
	-1.382432e-10	8.076313e-10	-3.965397e-10	42155.391884	-8.749430	-0.625234	
	5.820766e-11	-2.073648e-10	2.692104e-10	-498571.551772	-22.845439	-1.632447	

$\sigma_w = 0.001\ mm$

Conclusion and Outlook

- Conclusion:
  - Different methods of datum definition → impact on *dx, mx* + fulfillment of datum conditions
  - Different scaling methods → **no** impact on *dx, mx* but on fulfillment of datum conditions
  - Comparability of methods:
    - CWC → very tight constraints necessary

- Outlook:
  - CRF datum definition? Scaling?
  - In progress: Development of a new **state-of-the-art/ stand-alone** Python software for intra-/ inter-technique combination on NEQ level

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References:

Altamimi Z, Boucher C, Sillard P (2002) New trends for the realization of the international terrestrial reference system, Advanced in Space Research 30(2)

Dach R, et al. (2015) Bernese GNSS Software Version 5.2, Astronomical Institute, University of Bern, ISBN: 978-3-906813-05-9

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