

## **Sporadic-E and GNSS Scintillation**

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Multiple recent reports have claimed observations from ground-based GNSS measurements of near-equatorial daytime L1 (1.6 GHz) amplitude scintillation associated with sporadic-*E*. While there is a long history of detecting ground-based VHF scintillation associated with sporadic-*E*, if the L1 observations hold up it could say something new about ionospheric structure. Refractive index fluctuations in the ionosphere decrease as  $1/f^2$ . Moreover, L1 scintillation measurements respond to smaller irregularity length scales transverse to the line of sight (LOS) than VHF, and the irregularity spectral density function (SDF) for electron density fluctuations typically decreases with decreasing scale size. The *E*-region is a thin layer, so there is not a large distance to integrate through to increase the phase effects. Some of the strongest historical VHF scintillation observed to be associated with sporadic-*E* layers had an amplitude scintillation index,  $S_4$ , of 0.4–0.6. Using frequency scaling for power-law spectral indices appropriate to *E*-layer instabilities, this translates to an L1  $S_4$  of 0.02–0.03, which is typically at or below the  $S_4$  noise floor for GNSS receivers. Slant-path enhancements could raise this to 0.09 at low elevations. Could coherent structures, such as discrete edge-diffraction mechanisms, plausibly generate observable  $S_4$  levels at L1? This talk will explore both random and coherent scintillation mechanisms to assess the physical conditions required to generate an  $S_4$  of 0.2 or higher at 1.6 GHz due to *E*-region irregularities. Particularly, are the required electron density fluctuations consistent with what is likely to be associated with sporadic-*E*?

### *Introduction*

Recent articles (Seif et al., 2017; Shaikh et al., 2021) have reported daytime L1 (1.6 GHz) GNSS amplitude scintillation in near-equatorial regions attributed to sporadic-*E* (Es). If L-band scintillation due to Es were solidly confirmed, it could tell us something new about *E*-region irregularities. The purpose of this talk is to convey what might be special about the confirmed existence of ground-based L1 observations of GNSS scintillation from Es based on the propagation physics alone. It also describes some why some caution should be exercised with respect to these observations and identifies corroborating data that would strengthen the case for detection, particularly for  $S_4 > 0.2$ . Note that this talk is concerned only with irregularities that are strictly confined to the *E*-layer, taken as between 90 and 110 km in altitude.

### *Discussion*

Amplitude scintillation normally decreases with increasing frequency. For example, if scintillation is saturated at 250 MHz, there may only be weak scintillation at L1—a common occurrence with nighttime equatorial *F*-region scintillation. First, the ionospheric refractive index perturbations themselves decrease as  $1/f^2$ , where  $f$  is the radio frequency. Also, electron density fluctuation SDFs usually decrease with decreasing scale size. As a consequence of these two factors, the ionospheric phase fluctuations at the Fresnel scale—the relevant scale size transverse to the LOS for diffraction—are usually much smaller at L1 than for VHF. Note that the Fresnel scale at L1 is also  $0.4\times$  the Fresnel scale at 250 MHz for a given ionospheric distance.

For a power-law SDF (Rino, 1979), we can explore the difference quantitatively. Assuming the same LOS through the ionosphere, the frequency scaling is

$$\frac{S_{4w1}^2}{S_{4w0}^2} = \left(\frac{\lambda_1}{\lambda_0}\right)^{p/2+3/2}, \quad (1)$$

in the limit of weak scatter. Here  $S_{4w1}^2$  is the weak-scatter  $S_4$  associated with the radio signal of wavelength  $\lambda_1$ . Likewise,  $S_{4w0}^2$  is the weak-scatter  $S_4$  associated with the radio signal of wavelength  $\lambda_0$ . The amplitude scintillation index,  $S_4$ , is defined by  $S_4^2 = (\langle I^2 \rangle - \langle I \rangle^2) / \langle I \rangle^2$ , where  $I$  is the received signal power and  $\langle \dots \rangle$  denotes averaging. The term,  $p$ , in the exponent is the spectral index of the phase fluctuations. The L1 wavelength is  $\lambda_1 = 0.19$  m; the 250-MHz radio wavelength is  $\lambda_0 = 1.2$  m. For near-equatorial Es,  $p$  is in the range of 3.6 to 4.7 (Yadav et al., 2015). Some of the highest reported VHF observations of scintillation for near-equatorial Es are  $S_{4w0} = 0.4$ – $0.6$  (Rastogi and Mullen, 1981; Yadav et al. 2015), which is still weak scatter. The resulting estimate for  $S_{4w1}$  at L1, or  $S_{4L1}$ , is  $0.02$ – $0.03$ . Even allowing an unusually shallow spectrum of  $p = 2$ , the highest  $S_{4L1}$  under this approach is  $0.06$ . Notably, Seif et al. (2017) use a threshold of  $S_{4L1} = 0.2$  for daytime GNSS scintillation attributed to Es. In contrast, Shaikh et al. (2021) report  $S_{4L1} \sim 0.1$  associated with ionosonde foEs  $> 7$  MHz, which is nearer to the power-law estimate.

A weak link in this scaling argument is the assumption of a maximum 250-MHz  $S_4$  in the historical range of  $0.6$ . There could be higher VHF  $S_4$  values for Es either missed in the literature search or not yet observed. The extrapolation also becomes complicated if the VHF scintillation due to Es were to approach saturation at  $S_4 \sim 1$ , in which case Eq. (1) no longer applies. The key point is that frequency scaling significantly reduces L1  $S_4$  relative to VHF  $S_4$  in the range of these historical VHF observations.

Furthermore, one does not expect the  $E$ -region to produce large scintillation for ground-based observations since it is a relatively thin layer with electron densities normally an order of magnitude lower than the  $F$ -region peak. The relevant strength parameter in a power-law phase screen environment is  $C_k L$ , where  $C_k \propto \langle N_e^2 \rangle$  represents irregularity spectral strength at the 1-km scale and  $L$  is the mean layer thickness. Both of these factors become lower for  $E$ -region irregularities than  $F$ -region irregularities and tend to reduce the anticipated  $S_4$ . While VHF  $S_4$  from nighttime equatorial  $F$ -region structure is commonly saturated, VHF scintillation from the  $E$ -region alone may not be. Again, this line of argument does not necessarily preclude 250-MHz  $S_4 > 0.6$  for Es, but it suggests that it is unlikely in the absence of corroborating evidence of unusually high density and large TEC fluctuations. (Interestingly, the long ray paths in the  $E$ -region for GNSS radio occultations,  $\sim 1000$  km, do reopen the possibility of significant  $C_k L$  due to Es.)

Coherent structures in the  $E$ -region have been long considered a possible source of VHF scintillation (e.g., Basu and Das Gupta, 1969). Here the case for appreciable L1 amplitude scintillation from Es becomes at least more plausible. With coherent structures there is potential for  $S_4$  enhancement at particular scale sizes—e.g., due to focusing effects. Studying general scintillation behavior for arbitrary coherent structures is difficult, but it can be explored with analytically tractable cases. We consider the 1D sinusoidal phase screen of Hewish (1951). In this model, the wave amplitude just beyond the screen is

$$A(x, z = 0^+) = \exp \left[ i \Delta \phi \sin \left( \frac{2\pi}{\lambda} x \right) \right], \quad (2)$$

where  $x$  is the coordinate transverse to the LOS,  $z$  is the distance from the phase screen,  $\Delta\phi$  is the amplitude of the phase perturbation and  $\Lambda$  is the scale size of the phase structure. The corresponding  $S_4(z)$  is (Beach and Lovelace, 1997):

$$S_4^2 = 2 \sum_{n=1}^{\infty} J_n^2(2\Delta\phi \sin(n\pi r_F^2/\Lambda^2)), \quad (3)$$

where  $J_n(s)$  is the Bessel function of the first kind of order  $n$  and  $r_F = \sqrt{\lambda z}$  is the Fresnel scale. This expression for  $S_4$  is valid continuously between weak and strong scatter.

We will explore the behavior of this Eq. (3) model for exceptionally strong  $E$ -region fluctuations of 100% above and below an assumed ambient  $N_e = 10^{12}/\text{m}^3$ , which is an order of magnitude greater than typical  $E$ -region density but perhaps could be found in blanketing sporadic- $E$ . Note that Yadav et al. (2015) only model 6–19%  $N_e$  deviations based on physical inference from VHF scintillation observations of sporadic- $E$ . For a vertical path through a 20 km thick  $E$ -region, these hypothesized  $\pm 100\%$  fluctuations from an exceptionally high  $N_e$  would translate to about 1 TECU or 5.3 rad of L1 phase. At a cutoff elevation of  $30^\circ$ , the additional slant distance increases the effect to 4 TECU or 21 rad. Figure 1 plots the L1  $S_4$  results for several  $\Delta\phi$  values within these limits as a function of sinusoid scale size,  $\Lambda$ , where we have used  $z = 196$  km from  $30^\circ$  elevation in all cases. It shows that irregularities must have a transverse scale size less than 1200–2400 m, depending on  $\Delta\phi$ , to produce an  $S_{4L1} > 0.2$ .

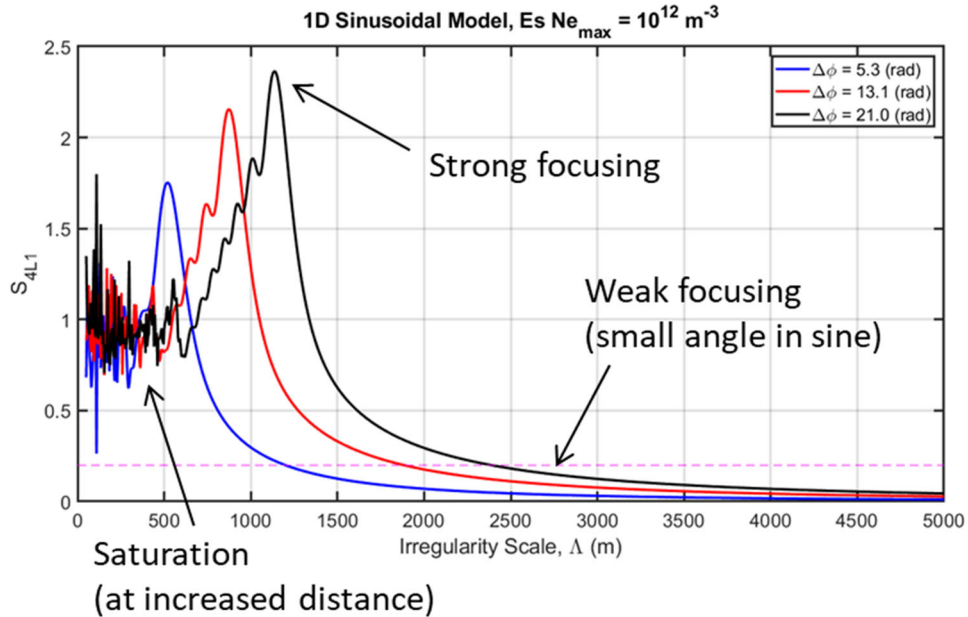


Figure 1. Sinusoidal model results with various propagation regimes identified for the unrealistically large  $N_e = 10^{12}/\text{m}^3$  and 100% modulation. The light dashed horizontal line is  $S_{4L1} = 0.2$ .

Figure 2 plots the  $S_{4L1}$  results as a function of irregularity scale for similar  $\pm 100\%$  fluctuations from a more realistic  $E$ -region  $N_e = 10^{11}/\text{m}^3$ . The resulting peak  $S_{4L1}$  values are lower—notably when  $\Delta\phi = 0.5$  rad, the peak  $S_{4L1}$  does not even reach 1—and the scale sizes must be significantly smaller, 400–750 m, to yield  $S_{4L1} > 0.2$ . Nevertheless, achieving  $S_{4L1} > 0.2$  still appears to be feasible from a propagation perspective.

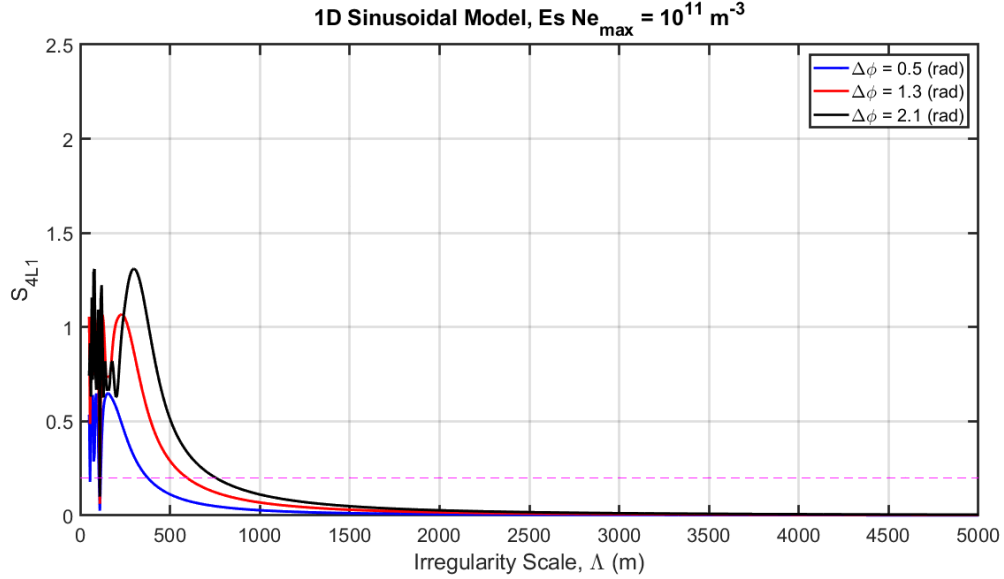


Figure 2. Sinusoidal model results for the more realistic  $N_e = 10^{11}/\text{m}^3$ , but still with 100% modulation.

One may reasonably counter that sporadic- $E$  irregularities are likely not 1D nor a periodic array of sinusoids. Nevertheless, the qualitative result—that coherent irregularities must be smaller than a particular scale size to produce significant L1  $S_4$ —should still hold based on a geometric optics argument. In the geometric optics limit, one may treat coherent irregularities as an array of lenses of with the lens thickness a function of the locally imparted phase to the wave front. Above a certain transverse scale size, the radius of curvature of the lenses is too large to produce appreciable focusing or defocusing; hence, the  $S_4$  is small. In other words, coherent irregularity structures must be smaller than a certain scale size to cause  $S_4$  to register measurably. If the structures deviate somewhat from sinusoidal shape or uniform spacing, we conjecture that the strong focusing peak would be reduced in magnitude and that  $S_4$  variability visible at smaller scales in the plots would be smoothed out. The same arguments also hold at VHF, but two factors make the maximum allowable VHF scale sizes for coherent structures larger: (1) the  $\Delta\phi$  at VHF becomes larger for the same ionospheric density perturbations, and (2) the Fresnel scale,  $r_F$ , increases.

### Conclusions

We have shown that L1 scintillation due to sporadic- $E$  is unlikely to be greater than  $S_{4L1} \sim 0.1$  for power-law SDF irregularities, unless the historically observed VHF upper bound of 250-MHz  $S_4 = 0.6$  is exceeded. Coherent electron density structures can yield higher L1  $S_4$  but the electron density fluctuations must be exceptionally strong and possibly above what is plausible for the  $E$ -region. Coherent structure scale sizes must also be constrained to produce appreciable L1 scintillation. The presence of L1  $S_4$  above 0.2 would imply a maximum allowable irregularity scale size smaller than  $\sim 1$  km. If significant L1  $S_4$  due to near-equatorial sporadic- $E$  were verified, it could say something unique about the presence of sub-kilometer coherent structures with large electron density variations.

To bolster the case for L1 scintillation due to near-equatorial Es, spurious  $S_4$  values should be eliminated. A major source of non-ionospheric  $S_4$  is multipath—typically, though not exclusively, experienced at low elevation angles. An additional noise contribution to  $S_4$  at low elevations is reduced net signal path gain near the horizon in typical ground-based installations. Figure 3 shows the occurrence of L1  $S_4$  values over the course of a day at a fixed near-equatorial GPS site as a function of elevation angle.

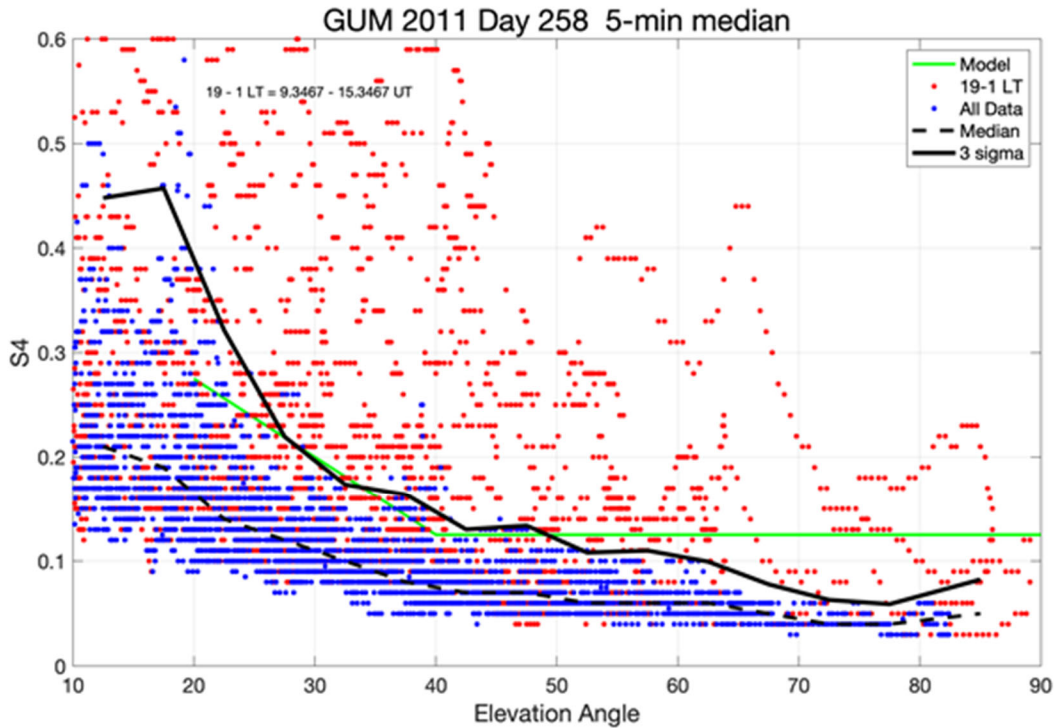


Figure 3. Elevation distribution of 5-minute median GPS L1  $S_4$  observed over the course of a day during the solar cycle 24 maximum at Guam (courtesy K. M. Groves).

The  $S_4$  values in Figure 3 are grouped into nighttime (red) cases, which include post-sunset equatorial  $F$ -region scintillation, and daytime (blue) cases, where little ionospheric scintillation occurs. These daytime background results are typical of most equatorial GNSS installations and indicate a large amount of spurious scintillation that exceeds  $S_4 = 0.2$ , particularly below  $30^\circ$  elevation. Mitigation techniques include elevation masks and comparing  $C/N_0$  time series from one day to the next, noting that multipath patterns will repeat in  $C/N_0$  records with a 4-min/day advance. The more detailed the knowledge of the local multipath environment, the better spurious values can be excluded. A further minor source of non-ionospheric  $S_4$  could be inter-satellite interference (Hajkowicz, 1999; Beach and Baragona, 2007), associated with certain Doppler matching conditions between pairs of satellites.

After taking steps to eliminate potential non-ionospheric  $S_4$  sources, the next corroborating step should be to review high-rate TEC values associated with the  $S_4$  measurements. Time-varying TEC can provide indications of whether coherent structures are present. In general, it is preferable to review time series rather than just statistical measures for this detailed look. Other useful information would include local ionospheric drifts and independent confirmation of sporadic- $E$ —e.g., from a local sounder (Yadav et al., 2015; Shaikh et al., 2021) or GNSS radio occultation (Seif et al., 2017). However, care should be taken to ensure that the radio occultation detections are sufficiently near the ground-based measurement LOS.

Carefully verified observations of significant L1  $S_4$  associated with sporadic- $E$  would be scientifically interesting. It could denote unusually strong coherent structures with spacing of the order of 1 km or less. It should be noted that, in any case, ground-based L1 scintillation observations are a poor

detector for routine Es since unusual propagation conditions are required to exceed the noise-floor  $S_4$  value, which for most receivers is in the range of 0.1 to 0.2.

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