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Relaxed Skolam Mean Labeling of 5 – Star Graphs with Partition 3, 2

D.S.T. Ramesh, D. Angel Jovanna

***Abstract**: In this article, our main topic is about the existence of relaxed skolem mean labeling for a 5 – star graph $G = K_{1,\alpha_1} \cup K_{1,\alpha_2} \cup K_{1,\alpha_3} \cup K_{1,\beta_1} \cup K_{1,\beta_2}$ with partition 3, 2 with a certain condition. By using the trial and error method we find the existence of the relaxed skolam mean labeling of 5 - star graph with partition 3, 2 with a specific condition.*

***Keywords**: Star graphs, union of star graphs, mean labeling, relaxed skolem mean labeling, relaxed skolam mean graph.*

I. INTRODUCTION

Labeling of Graphs is an branch of graph theory which is widely used in the area of networking and routing. There are various types of labeling functions introduced by different mathematicians. In this research article we discuss about one of these types of labeling namely Relaxed Skolam Mean Labeling which is derived from Skolam Mean Labeling of Graphs introduced by V. Balaji et.al.[5] in the year 2010. Basic properties of Relaxed Skolam Mean Labeling were already discussed by V.Balaji et.al.[5].

II. PRELIMINARIES

Definition: A graph G = (V, E) with p vertices and q edges is said to be a **skolam mean graph** if there exists a function $f : V \rightarrow \{1,2,3,\dots, p = |V|\}$ such that the induced map $f^* : E \rightarrow \{2,3,\dots, p = |V|\}$ given by

$$f^*(e = uv) = \begin{cases} \dfrac{f(u)+f(v)}{2} & \text{if } (f(u)+f(v)) \text{ is even} \\ \dfrac{f(u)+f(v)+1}{2} & \text{if } (f(u)+f(v)+1) \text{ is even} \end{cases}$$

then, the resulting distinct edge labels are from the set

$\{2,3,\dots,p = |V|\}$.

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* Correspondence Author
Dr. D.S.T.Ramesh*, Associate Professor Department of Mathematics in Nazareth Margoschis College, Pillayanmanai, Tuticorin (Tamil Nadu), India.
D. Angel Jovanna, Research Scholar Department of Mathematics in Nazareth Margoschis College, Pillayanmanai, Tuticorin (Tamil Nadu), India.

Definition 2.2 [5]: A graph $G = (V, E)$ with p vertices and q edges is said to be a relaxed skolam mean graph if there exists a function $f : V \rightarrow \{1,2,3,\dots, p+1 = |V|+1\}$ such that the induced edge map $f^* : E \rightarrow \{2,3,\dots, p = |V|+1\}$ given by

$$f^*(e = uv) = \begin{cases} \dfrac{f(u)+f(v)}{2} & \text{if } (f(u)+f(v)) \text{ is even} \\ \dfrac{f(u)+f(v)+1}{2} & \text{if } (f(u)+f(v)+1) \text{ is even} \end{cases}$$

The resulting distinct edge labels are from the set $\{2,3,\dots,p+1 = |V|+1\}$

Note: There are p vertices and available vertex labels are p + 1 and hence one number from the set $\{1,2,3,\dots,p+1 = |V|+1\}$ is not used and we call that number as the relaxed label. When the relaxed label is p + 1, the relaxed mean labeling becomes a skolam mean labeling.

Result: The three star graph $K_{1,a} \cup K_{1,b} \cup K_{1,c}$ satisfies relaxed skolam mean labeling if $a + b \le c \le a + b + c$.

III. MAIN RESULT

Theorem: The 5 - star graph $G = K_{1,\alpha_1} \cup K_{1,\alpha_2} \cup K_{1,\alpha_3} \cup K_{1,\beta_1} \cup K_{1,\beta_2}$ where $\alpha_1 \le \alpha_2 \le \alpha_3$ and $\beta_1 \le \beta_2$ is a relaxed skolam mean graph if $\beta_1 + \beta_2 - \alpha_1 - \alpha_2 - \alpha_3 = -6$.

Proof: Let $\sigma_1 = \alpha_1; \sigma_2 = \alpha_1 + \alpha_2; \sigma_3 = \alpha_1 + \alpha_2 + \alpha_3$ and $\delta_1 = \beta_1; \delta_2 = \beta_1 + \beta_2$.

Consider the 5 - star graph $G = K_{1,\alpha_1} \cup K_{1,\alpha_2} \cup K_{1,\alpha_3} \cup K_{1,\beta_1} \cup K_{1,\beta_2}$.

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The condition $\beta_1 + \beta_2 - \alpha_1 - \alpha_2 - \alpha_3 = -6$ gives rise to the case $\delta_2 = \sigma_3 + 6$. In this case we will establish that the graph G is relaxed skolam mean.

Let the set of vertices of G be $V = V_1 \cup V_2 \cup V_3 \cup V_4 \cup V_5$ where

$V_k = \{v_{k,i} : 0 \le i \le \alpha_k\}; 1 \le k \le 3$ and $V_4 = \{v_{4,i} : 0 \le i \le \beta_1\}; V_5 = \{v_{5,i} : 0 \le i \le \beta_2\}$

. Let the edge set of G be

$$E = \bigcup_{k=1}^{3} \{v_{k,0}v_{k,i} : 1 \le i \le \alpha_k\} \cup \bigcup_{k=4}^{5} \{v_{k,0}v_{k,i} : 1 \le i \le \beta_{k-3}\}.$$

Case: Let $\delta_2 = \sigma_3 - 6$.

G has $\sigma_3 + \delta_2 + 5 = 2\sigma_3 - 1$ vertices and $\sigma_3 + \delta_2 = 2\sigma_3 - 6$ edges.

We define the rsv function

$f : V \to \{1, 2, ..., p+1 = \sigma_3 + \delta_2 + 5 + 1 = 2\sigma_3\}$ as follows:

$f(v_{1,0}) = 2; \quad f(v_{2,0}) = 4; \; f(v_{3,0}) = 6;$

$f(v_{4,0}) = \sigma_3 + \delta_2 + 5 = 2\sigma_3 - 2;$

$f(v_{5,0}) = \sigma_3 + \delta_2 + 6 = 2\sigma_3$

$f(v_{1,\kappa}) = 2\kappa - 1 \qquad 1 \le \kappa \le \alpha_1$

$f(v_{2,\kappa}) = 2\sigma_1 + 2\kappa - 1 \qquad 1 \le \kappa \le \alpha_2$

$f(v_{3,\kappa}) = 2\sigma_2 + 2\kappa - 1 \qquad 1 \le \kappa \le \alpha_3$

$f(v_{4,\kappa}) = 2\kappa + 8 \qquad 1 \le \kappa \le \beta_1$

$f(v_{5,\kappa}) = 2\delta_1 + 2\kappa + 8 \qquad 1 \le \kappa \le \beta_2$

Here 8 is the relaxed label.

We get labels for edges as follows:

The edge labels of $v_{1,0}v_{1,\kappa}$ is $\kappa + 1$ for $1 \le \kappa \le \alpha_1$ ($2, 3, ..., \alpha_1 + 1 = \sigma_1 + 1$), $v_{2,0}v_{2,j}$ is $\sigma_1 + \kappa + 2$ for $1 \le \kappa \le \alpha_2$ ($\sigma_1 + 3, \sigma_1 + 4, ..., \sigma_1 + \alpha_2 + 2 = \sigma_2 + 2$), $v_{3,0}v_{3,j}$ is $\sigma_2 + \kappa + 3$ for $1 \le \kappa \le \alpha_3$ ($\sigma_2 + 4, \sigma_2 + 5, ..., \sigma_2 + \alpha_2 + 3 = \sigma_3 + 3$), $v_{4,0}v_{4,\kappa}$ is $\sigma_3 + \kappa + 3$ for $1 \le \kappa \le \beta_1$ ($\sigma_3 + 4, \sigma_3 + 5, ..., \sigma_3 + \beta_1 + 3 = \sigma_3 + \delta_1 + 3$) , $V_{5,0}V_{5,\kappa}$ is $\sigma_3 + \delta_1 + \kappa + 4$ for $1 \le \kappa \le \beta_2$ ($\sigma_3 + \delta_1 + 5, \sigma_3 + \delta_1 + 6, ..., \sigma_3 + \delta_1 + \beta_2 + 4 = \sigma_3 + \delta_2 + 4 = 2\sigma_3 - 2$)

The edge labels are therefore

$2, 3, ..., \sigma_1 + 1,$

$\sigma_1 + 3, \sigma_1 + 4, ..., \sigma_2 + 2,$

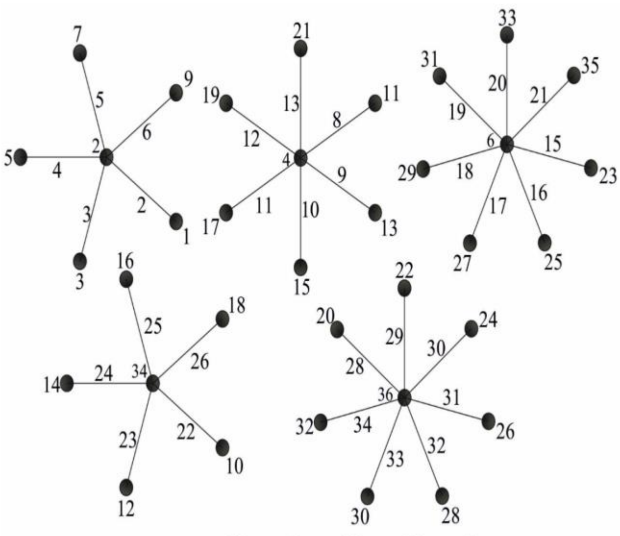
$\sigma_2 + 4, \sigma_2 + 5, ..., \sigma_3 + 3,$

$\sigma_3 + 4, \sigma_3 + 5, ..., \sigma_3 + \delta_1 + 3,$

$\sigma_3 + \delta_1 + 5, \sigma_3 + \delta_1 + 6, ..., 2\sigma_3 - 2$

These edge labels, the images of the rse function of the graph G are therefore distinct. Hence G is a relaxed skolam mean graph.

Example:



$K_{1,5} \cup K_{1,6} \cup K_{1,7} \cup K_{1,5} \cup K_{1,7}$

IV. CONCLUSION

In this research article we concentrated mainly on the existence of relaxed skolam mean labeling of a 5 – star graph $G = K_{1,\alpha_1} \cup K_{1,\alpha_2} \cup K_{1,\alpha_3} \cup K_{1,\beta_1} \cup K_{1,\beta_2}$ with the condition $\beta_1 + \beta_2 - \alpha_1 - \alpha_2 - \alpha_3 = -6$. Trial and error method is used to find the existence of the labeling function.

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AUTHORS PROFILE

Dr. D.S.T.Ramesh, M.Sc., M.Phil., Ph.D., H**e** is an Associate Professor of Mathematics in Nazareth Margoschis College, Pillayanmanai, Tuticorin, Tamil Nadu. He has 33 years of teaching experience. He guided 8 Ph.D."s and 5 M. Phil.. His area of specialization is Graph Theory. He published 51 research papers in National and International Journals. He was invited as a resource person for National conferences organized by Sacred Heart College, Tirupattur and Pope's College, Sawyerpuram.

D. Angel Jovanna, Research Scholar of Mathematics in Nazareth Margoschis College, Pillayanmanai, Tuticorin, Tamil Nadu. Attended 3 International conferences, 1 National level Seminar and 1 National level webinar. Presented papers in three International Conferences and published one paper in Turkish Journal of Mathematics.

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