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(Short Communication)

On a uniformly accelerated frame of reference

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Abstract

Candidates for a uniformly accelerated frame of reference are considered. The simplest of them is uniformly accelerated. This system is described in space-time with non-zero curvature.

Keywords: Frame of reference, Law of dependence of time on acceleration, Uniformly accelerated frames of reference.

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1. Introduction

Einstein introduced the concept of a relativistic frame of reference, considering the frame of reference as a set of rulers and clocks. Minkowski formulates the concept of physical space as a four-dimensional metric space-time [1]. However, the time coordinate in this space is no longer independent of the other coordinates. The course of the clock depends on speed and acceleration:

> *"Thanks to the theory of relativity, it becomes possible to have a four-dimensional interpretation of the "world", since in this theory time loses its independence ...*", Einstein [2].

Here, in the special theory of relativity, space acts as an independent physical object.

In 1907, Einstein considered a uniformly accelerated frame of reference [3] and obtained a non-trivial result - the rate of accelerated clocks in the first approximation linearly depends on the potential

$$\tau = \tau_0 \left(1 + \frac{a_x x}{c^2}\right),$$

where τ_0 is the time at the origin and a_x is the acceleration. This law is reflected in the general theory of relativity [4]:

$$\tau = \frac{t}{c}\sqrt{g_{00}}. \qquad (1)$$

2. Development

Perhaps the simplest example of such a reference system is the accelerated frame of Christian Møller [5]. The system can be obtained by successive infinitesimal rotations of the Minkowski space. Linear element of this system:

$$ds^2 = \left(1 + \frac{\alpha x}{c^2}\right)^2 c^2 dt^2 - (dx^2 + dy^2 + dz^2).$$

See that this metric satisfies the physical law (1). However, if we calculate the acceleration of a test particle in the field of inertia of this system according to the equation of motion

$$\frac{d^2 x^i}{dt^2} = -c^2 \Gamma^i_{00}\,, \qquad (3)$$

get acceleration

$$\frac{d^2 x}{dt^2} = -\left(\frac{ax}{c^2} + 1\right) a\,,$$

which depends on the x coordinate, which means that system (2) is not uniformly accelerated.

3. John Bell

John Bell [6] a system of two identical point rockets launched simultaneously at some distance from each other. At the same time, he suggested that rockets accelerate according to the same law. Bell concludes that the distance between them does not change, and the thread connecting the rockets will undergo Lorentz contraction and will definitely break. Based on this thought experiment, Bell concludes that it is necessary to revise the special theory of relativity. In the new theory, according to Bell [6], only extended bodies under the action of elastic forces undergo Lorentz contraction, while individual bodies keep their distance at any speed.

The attentive reader should be wary of the use of the special theory of relativity in an accelerated frame of reference. However, Bell's error is found already within the framework of the special theory of relativity. Here is a verbatim quote from the classic textbook [7]:

> «Clocks synchronized in their own frame of reference and separated by a distance ΔL turn out to be out of sync in a frame moving relative to them at a speed v, and the amount of mismatch is equal to
>
> $$\delta = -\frac{\Delta L v}{c^2}, \qquad (15.14)$$»

This result is equivalent to law (1). Thus, the Bell system turns out to be non-physical (outlawed).

However, the authors of the preprint [8], agreeing with Bell, go further by suggesting that the principle of equivalence be abandoned and that the special theory of relativity be used to describe gravity. Specifically, the authors describe the Bell system using the metric

$$ds^2 = dt^2 - 2\sinh at\, dxdt - (dx^2 + dy^2 + dz^2).$$

It is clear that here the time is the same in all space, which contradicts the law (1) for accelerated reference systems.

4. Conclusions

The author of these lines spent quite a lot of time searching for a homogeneous frame of reference in flat space, since he considered it important then. However, recently, almost by accident, such a system was found in the pseudo-Riemannian space:

$$ds^2 = \left(1 + \frac{\alpha x}{c^2}\right) dt^2 - (dx^2 + dy^2 + dz^2). \qquad (4)$$

(4)

I will give her convolutions of the curvature tensor and Christoffel symbols:

$$R_{00} = \frac{a^2}{2ac^2x + c^4}, R_{11} = \frac{a^2}{(2ax + c^2)^2};$$

$$R = -\frac{2a^2}{(2ax + c^2)^2};$$

$$\Gamma^1_{00} = \frac{a}{c^2},\ \Gamma^0_{01} = \Gamma^0_{10} = \frac{a}{2ax + c^2};$$

$$\Gamma_{100} = -\frac{a}{c^2},\ \Gamma_{010} = \Gamma_{001} = \frac{a}{c^2}.$$

From these data and formula (3) it can be understood that the acceleration in system (4) is equal to the constant a.

References

[1] Minkowski H. Raum und Zeit. Phis. ZS. 10, 104 (1909).

[2] Einstein A. Über die spezielle und die allgemeine Relativitatstheorie (Gemeinverstdndlich). Druck und Verlag von Friedr. Vieweg & Sohn, Braunschweig, (1920).

[3] Einstein A. Über das Relativitätsprinzip und die aus demselben gezogenen Folgerungen. Jahrb. d. Radioaktivitat u. Elektronik, 4, 411—462 (1907).

[4] Landau L D, Lifshitz E M. The Classical Theory of Fields Vol. 2 (4th ed.). Butterworth–Heinemann (1975).

[5] Moeller C On homogeneous gravitational fields in the general theory of relativity and the clock paradox. Published in Ejnar Munksgaard, (1943).

[6] Bell J.S. Speakable and unspeakable in quantum mechanics. – Cambridge University Press, p.67. (1993).

[7] Panovsky V., Philips M. Classical electrodynamics. Tr. from English.Физматгиз, M.1963.

[8] Gerstein S.S. Logunov A.A. G.S. Bell's Task, Preprint, Institute for High-Energy Physics, p. 96-55, Protvino, 1996.