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Three Methods in Reliability Assessment of Engineering Structure

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Abstract: ***The work attempts to choose the handy methods for analyzing structural reliability. Comparative analysis of the methods was performed on an example of dome truss susceptible to stability loss from the condition of node snapping. In the reliability analysis of structure the load magnitudes (P), the axial stiffness of bars (EA), coordinate nodes (Z) are represented by random variables. The criterion of structural failure is expressed by the condition of non-exceeding the admissible load multiplier. The Hasofer-Lind reliability index was determined. In analysis were used three approaches differing way of defining the limit state function: Approach 1 – using of implicit limit state function, Approach 2 – using of explicit neural state functions, Approach 3 – using of the Hybrid Monte Carlo method.***

Keywords: ***Reliability, Neural Networks, Limit State Function, Form Method, Hybrid Monte Carlo Method***

I. INTRODUCTION

The reliability of technical systems, including building structures, is the property of the object that tells whether it works correctly for the required time and under certain operating conditions. The most advanced reliability analysis methods are probabilistic methods. These methods are based on the assumption that input variables are random character. Basic information about structural reliability theory can be found in textbooks and monographs [1]–[5]. Application of probabilistic methods in civil engineering was presented in books e.i. [6] and [7]. At present, the assessment of structural safety is a research area for many researchers. They propose new and more effective methods of analyzing reliability structure. Among probabilistic methods, they gained great popularity: approximation like FORM, SORM and simulation like Monte Carlo. These methods are used in issues of statics, stability and dynamics [8]–[14]. There are also several software packages for reliability analysis, e.g. STRUREL [15], COSSAN [16], NUMPRESS Explore [17].

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II. PROCEDURE FOR RELIABILITY INDEX DETERMINATION

A. Approach 1 – connection between FEM program and reliability software

The first method, used in the paper, was FORM (First Order Reliability Method). This method belongs to the group of fully probabilistic approximation methods of reliability analysis. To describe the problem, full information about the probability distributions of all random variables is used. The measure of reliability is the Hasofer-Lind reliability index.

Approach 1 presented connection between external program KRATA and reliability software – NUMPRESS Explore. First stage was stability analysis in Finite Elements Method software – KRATA. It allows to determine coordinates of the limit point and in consequence the limit function as the condition of the non-exceeding of the admissible vertical load multiplier of node.

Next step was implementation of random variables with parameters: mean value, standard deviation and probability density function, to reliability software NUMPRESS Explore. Then user have to define a limit state function. In our case this function using external variable as realization of numerical procedure from KRATA software. It is so called implicit limit state function of random variables. Last stage of analysis is choice of reliability method. In analysis only FORM method was used. The reason for choosing this method was too long calculations time for e.g. Monte Carlo method. A detailed description of this method is presented in the paper [9]. The course of action in the applied this approach presents block diagram (Fig. 1).

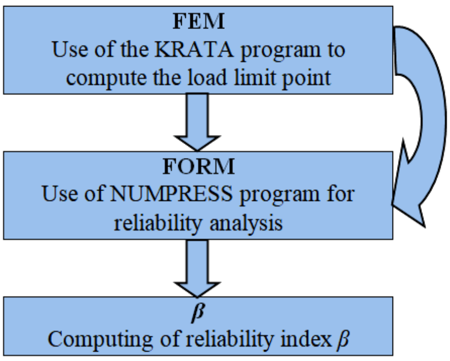

Fig. 1. Scheme of Applied Approach 1 to Reliability Analysis.

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B. Approach 2 – explicit neural state function

In the second method, Approach 2, neural networks replace polynomial state functions in the approximation methods. Assumed random variables were treated as elements of input vectors to NN. The neural network output was a scalar and responded to the limiting load. Scheme of application this approach presents Fig. 2.

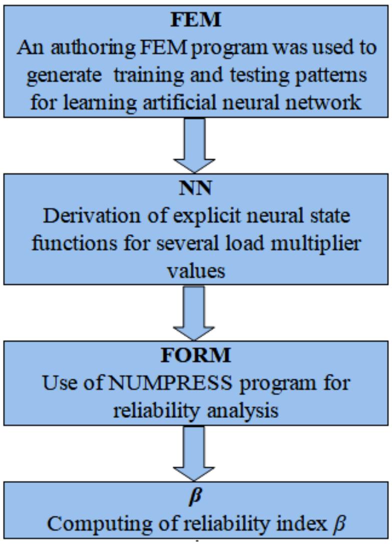

Fig. 2. Scheme of Applied Approach 2 to Reliability Analysis.

A detailed description of this method is presented in the paper [11].

C. Approach 3 – Hybrid Monte Carlo

The Classical Monte Carlo (CMC) method is often used to assess the reliability of engineering structures. This method also belongs to the group of fully probabilistic methods of reliability analysis. In the CMC method, probability failure estimation is performed in a simulation manner. The main limitation in using this method is the need to generate a very large number of samples for simulation. When considering the reliability of engineering structures, samples used for simulations are usually calculated using FEM. The probability of failure is usually related to the limit load value of the structural system. At the same time, a large number of design parameters (contained in vector **X**) should be taken into account. In the general case, the measure of structure reliability are the probability of reliability q and the probability of failure p_f in relation defined by the formula:

$$p_f = \text{Prob}\left[G(\mathbf{X}) \le 0\right] = \int \cdots \int_{G(\mathbf{X}) \le 0} f_{\mathbf{X}}(\mathbf{x})\, \mathrm{d}x \quad , \qquad (1)$$

where: $G(\mathbf{X})$ - limit state function; R - resistance of structure; S - actions (loads) applied to the structure. In the Monte Carlo simulation method, after conducting the N_s times experiments, the probability of total failure can be calculated from the formula:

$$p_f \cong n[G(\hat{\mathbf{x}}_i) \le 0]/N_s \quad , \qquad (2)$$

where: $n[G(\hat{\mathbf{x}}_i) \le 0]$ – number of simulations for which the structure has failed, N_s – the total number of samples in the simulation. In the reliability analysis of engineering structures, the reliability indicator β is more often used, which, in approximation methods, can be determined in several ways [18]. The relation between the reliability indicator and the probability of failure is as follows:

$$\beta = -\Phi^{-1}\left(p_f\right), \qquad (3)$$

where: Φ^{-1} – inverse function to the cumulative distribution function of a standardized random variable. Correct estimation of the probability of structure failure requires the generation of a large number of random samples (min. 10^8). This is a big limitation in using the classic Monte Carlo method. Sometimes the problem considered is very complicated and the time needed to evaluate a single drawn set of parameters is too long. To partially solve this problem, it can apply:

- numerical modifications of the classic MC method, based on the reduction of variance [19]–[24], or
- hybrid Monte Carlo method, connecting FEM and artificial neural networks (NN), [25]–[27].

In this approach, the limit value of the structure load parameter was computed using NN for the each *i*-th randomly selected vector $\mathbf{X}_i$

$$X_{N\times1} = \{x_1, \ldots, x_N\} \xrightarrow{NN} y = \mu_{ult} , \qquad (4)$$

where: x_i - random variable corresponding to the load, geometric values or material properties, μ_{ult} - ultimate load multiplier. Training and testing patterns of ANN are computed by means of authoring FEM program implemented in Matlab environment [28]. In this case, the single load parameter is considered, $P = \mu_{ult} P^*$, where P^* – the reference load vector. The ultimate load parameter μ_{ult} corresponds to the global loss of stability. The course of action in the applied this approach presents block diagram (Fig. 3).

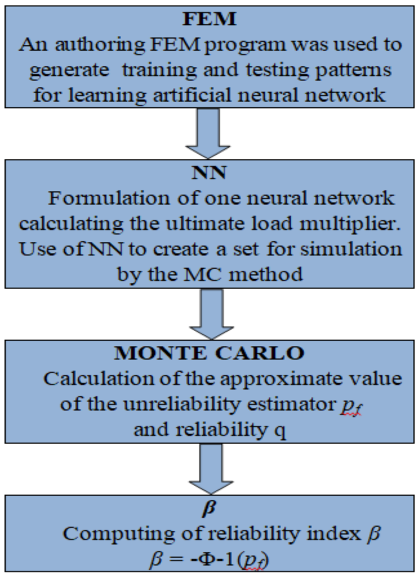

Fig. 3. Scheme of Applied Approach 3 to Reliability Analysis.

III. NUMERICAL EXAMPLE

The structure of the spatial truss susceptible to loss of stability through node snapping was analyzed.

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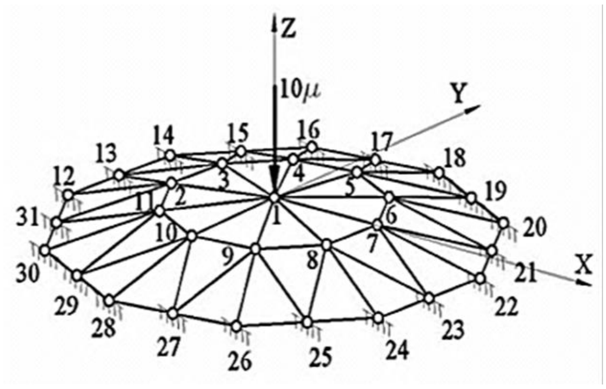

Fig. 4. Scheme of Space Truss Structure.

All elements of structure as tubular sections RO 135x5 was designed and were made of S275NH steel. Material characteristics for this kind of steel are: the yield point f_y=275 MPa and elastic modulus E=210 GPa. Conditions support defined as a simply supported at nodes 12-31. Scheme of analysed structure on Figure 4 was presented. For a single element, load bearing capacity was determined due to buckling $N_{b,Rd}$=278.72 kN. Buckling carrying capacity condition was checked. The buckling carrying capacity condition due to buckling was met, therefore it can be concluded that the stability loss occurs through the node snapping. The first stage of analysis was determination of the equilibrium path, and consequently, coordinates of the limit point. These coordinates q=7.20 cm (q – vertical displacement) and μ=6.65 (μ – load multiplier) to define the limit function were used. This function refers to the ultimate limit state. Condition of the non-exceeding of the admissible vertical load multiplier of node 1 was formulated as:

$$G_1 = 1 - \frac{\mu(\mathbf{X})}{6.652}, \qquad (5)$$

where

μ(**X**) – function of random variables while approaching to the limit point,

X={P, EA, Z} – vector of random variables.

In reliability analysis three random variables were used: load of node (P), longitudinal stiffness (EA), coordinate of node 1 (Z). The correlation of variables and time was not included in the analysis. Description of random variables is shown in Table- I.

Table- I: Distribution Parameters of Random Variables

Random variables	Variable parameters			
	Probability density function	Mean value	Standard deviation	Coefficient of variation
P	Normal	10 kN	1 kN	10 %
EA	Normal	428610 kN	42861 kN	10 %
Z	Normal	0.524 m	0.05 m	9.54 %

IV. RESULT AND DISCUSSION

Using the three proposed methods of safety structure assessment reliability index values were determined. In this paper Approach 1 was reference method. Approach 2 and 3 are attempts to look for an accurate method that does not require high calculation costs. Values for selected points on the equilibrium path around the limit point were estimated.

Figure 5 shows changes in reliability index values while following the non-linear geometric solution path, when all parameters (EA, P and Z) are random variables. Each of the methods is fraught with some errors, they mainly result from approximation performed several times. Despite this, the results should be considered correct. The largest error in the reliability ratio estimate compared to the reference method does not exceed 12%.

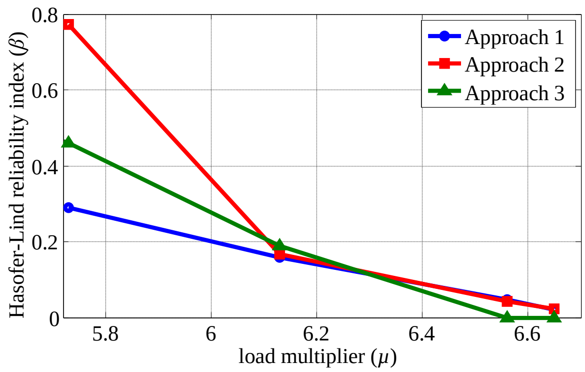

Fig. 5. Values of Reliability Index.

Figure 6 shows values of relative error between the reference method (Approach 1) and others. We observed that differences between the reliability index values decrease as the limit point approaches.

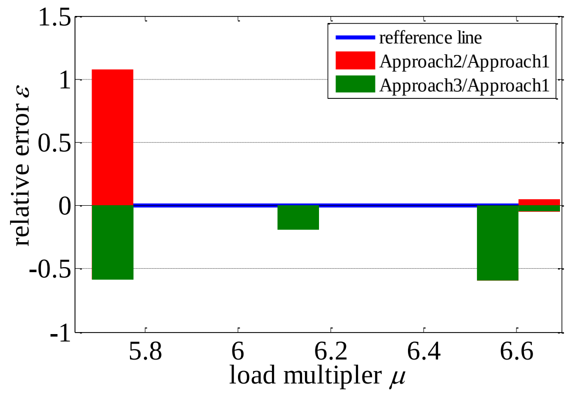

Fig. 6. Values of Relative Error.

On the basis of the performed calculations, the advantages and disadvantages of the methods used can be formulated. This information is presented in Table- II.

Table- II: Advantages and disadvantages of each Approach

Type of analysis	Advantages	Disadvantages
Approach 1	▪ The FORM method allows computing the sensitivity of the reliability index to random variables. ▪ Simple method.	▪ The need to integrate the MES program with a reliability program. ▪ Time-consuming. ▪ Not applicable to strongly nonlinear boundary functions.

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Type of analysis	Advantages	Disadvantages
Approach 2	▪ Possibility of formulating explicit state functions depending on many random variables. ▪ Shortening the calculation time in the reliability program compared to Approach 1.	▪ The need to develop several NNs. ▪ The accuracy of the results depends indirectly on the quality of the network. ▪ High computational costs.
Approach 3	▪ Significant reduction of computation time compared to CMC. ▪ Development of only one neural network. ▪ All algorithm elements are implemented in a common computing environment.	▪ The results depend on the quality of the random number generator.

V. CONCLUSION

The search for an effective and accurate method of reliability analysis for real constructions is a matter of a lot of space in contemporary literature. We conclude, based on the results, that way of analysis of structural reliability in safety assessment plays a secondary role. We obtained similar reliability index values, especially for coordinates around the limit point.

Therefore decisive criterion when choosing how to assess reliability is the complexity of the analyzed structure because in engineering practice, it is a very difficult to use implicit form of limit state function. Besides the calculation time increases with the number of random variables and the complexity of the structure so it is also important the time we can spend on calculations. From the point of view of the accuracy of the calculations made, the hybrid Monte Carlo method is the most appropriate.

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