

Application of Elmer Software Package in Nonlinear Dynamic Analysis to Develop Poincaré Map based Damage Detection Techniques

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1. Step-by-step procedure for dynamic analysis in Elmer

1.1 Modeling and analysis of natural frequency of beam using Elmer

The steps involved to find the natural frequency of the nonlinear undamped beam are described with figures.

- At first, 'FreeCAD' model is meshed in 'Gmsh' using 32 tetrahedron elements and saved as 'beam.msh'. To perform meshing the following general steps are followed: Open Gmsh> File > Open model > Expand 'Mesh' > Expand 'define'> Expand 'Transfinite' > select Curve > select edges of the model to be discretised with required number of elements > 'Recombine' with '3D'. After generating the 3D mesh the model with mesh is exported and saved with '.msh' extension. The whole process is shown in Fig. 1.
- The 'beam.msh' is opened up directly in the graphical user interface (GUI) of 'Elmer' as shown in Fig. 2.
- The material property, load, and output requests can be assigned to the member in two ways. The new properties are added directly from the 'Model' section of graphical user interface as shown in Fig.1. Another way is to define those properties in the environment of '.sif'. The 'sif' (solver input file) tab can be accessed with the 'Edit' option (Fig. 3).

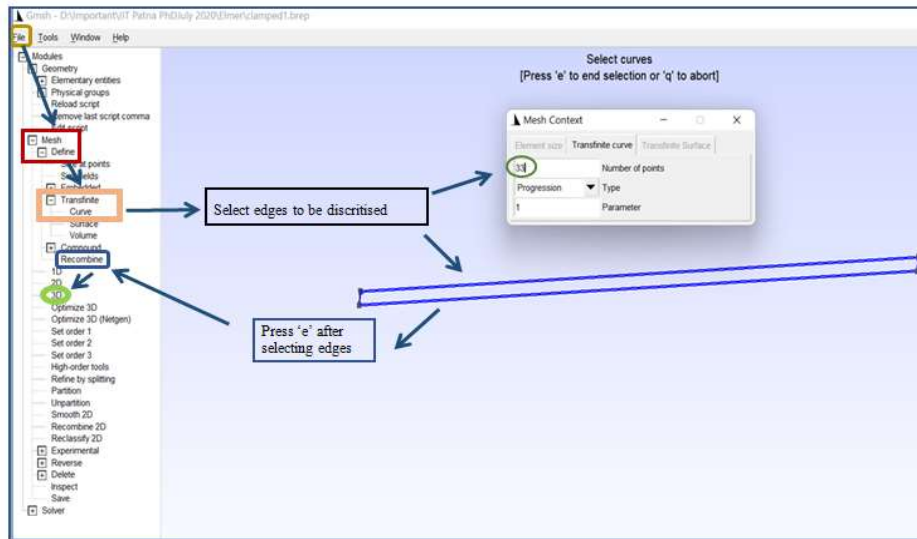


Fig. 1. Flow of work to mesh a model using Gmsh

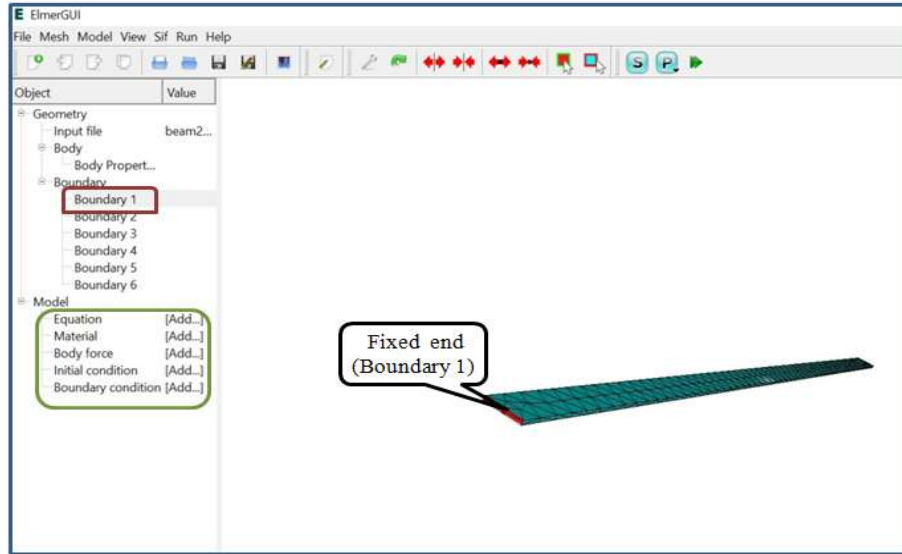


Fig. 2. Graphical user interface of Elmer

- d) The nonlinear undamped beam is modeled with the property of aluminium which are Young's modulus=70 GPa, mass density=2778 kg/m³, and Poisson's ratio=0.3. In case of damped analysis, Rayleigh coefficients are added as shown in (Fig. 4). In the .sif file, the assigned properties are checked and saved.
- e) In a cantilever beam, all degrees of freedom of the end boundary are restricted to zero. It is modeled by assigning the displacement value of the first node of element 1 to zero (Fig. 5).

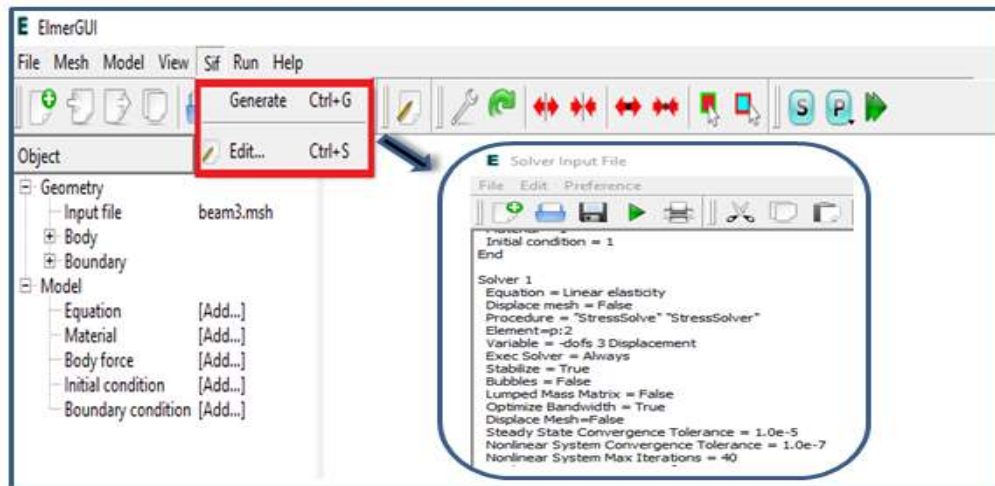


Fig. 3. Tab to enter .sif environment

```

E Solver Input File
File Edit Preference
Initial condition = 1
End

Solver 1
Equation = Nonlinear elasticity
Displace mesh = False
Procedure = "ElasticSolve" "ElasticSolver"
Element=p:2
Variable = -dofs 3 Displacement
Exec Solver = Always
Stabilize = True
Bubbles = False
Lumped Mass Matrix = False
Optimize Bandwidth = True
Displace Mesh=False
Steady State Convergence Tolerance = 1.0e-5
Nonlinear System Convergence Tolerance = 1.0e-7
Nonlinear System Max Iterations = 40
Nonlinear System Newton After Iterations = 3
Nonlinear System Newton After Tolerance = 1.0e-3
Nonlinear System Relaxation Factor = 0.7
Linear System Solver = Direct Iterative
Linear System Direct Method = Umfpack
Linear System Iterative Method = BICGStab
Linear System Max Iterations = 10000
Linear System Convergence Tolerance = 1.0e-7
BICGStab polynomial degree = 2
Linear System Preconditioning = ILU0
Linear System ILUT Tolerance = 1.0e-3
Linear System Abort Not Converged = False
Linear System Residual Output = 10
Linear System Precondition Recompute = 1
End

Equation 1
Name = "Equation 1"
Active Solvers(1) = 1
End

Material 1
Name = "Material 1"
Density = 2778
Youngs modulus = 70.0e9
Poisson ratio = 0.3
Rayleigh Damping=Logical True
Rayleigh Damping Beta=Real .0001
End

Initial Condition 1
Name = "InitialCondition 1"
Displacement 1 = 0
Displacement 2 = 0
Displacement 3 = 0
End

```

Fig. 4. Material section in .sif environment for both damped (green) and undamped member (red)

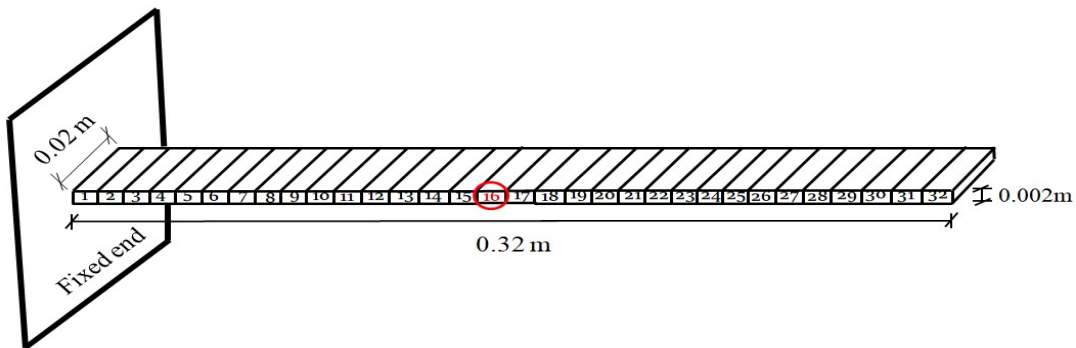


Fig. 5. Physical description of a model of undamaged cantilever beam

- f) In the .sif file, the boundary of the model is identified by the 'Target boundaries' number as in Fig. 6 of graphical interface of 'Elmer'. The associated number with 'Displacement' in Fig. 4 and 6 denotes the corresponding direction of the axis where '1', '2' and '3' represents 'x', 'y' and 'z' axis respectively. To modify the value of boundary condition, default values can be directly edited in the coding. The 'Target Boundaries' in Fig. 6 indicates the boundary number to be operated on.

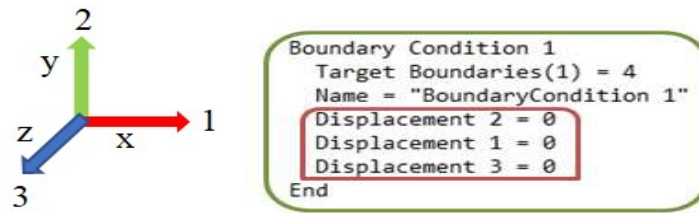


Fig. 6: Boundary condition in .sif environment

- g) The axes direction for the corresponding model is obtained from the 'compass' which opens up with 'view' when right clicked on the GUI as shown in Fig. 7.
- h) To find the natural frequency, 'Stress solver' with 'Linear Elasticity' (Fig. 8) equation is used and required number of modal values is set in 'Eigen System Values'. For nonlinear analysis, 'Linear Elasticity' in 'equation' is replaced by 'Nonlinear elasticity'.

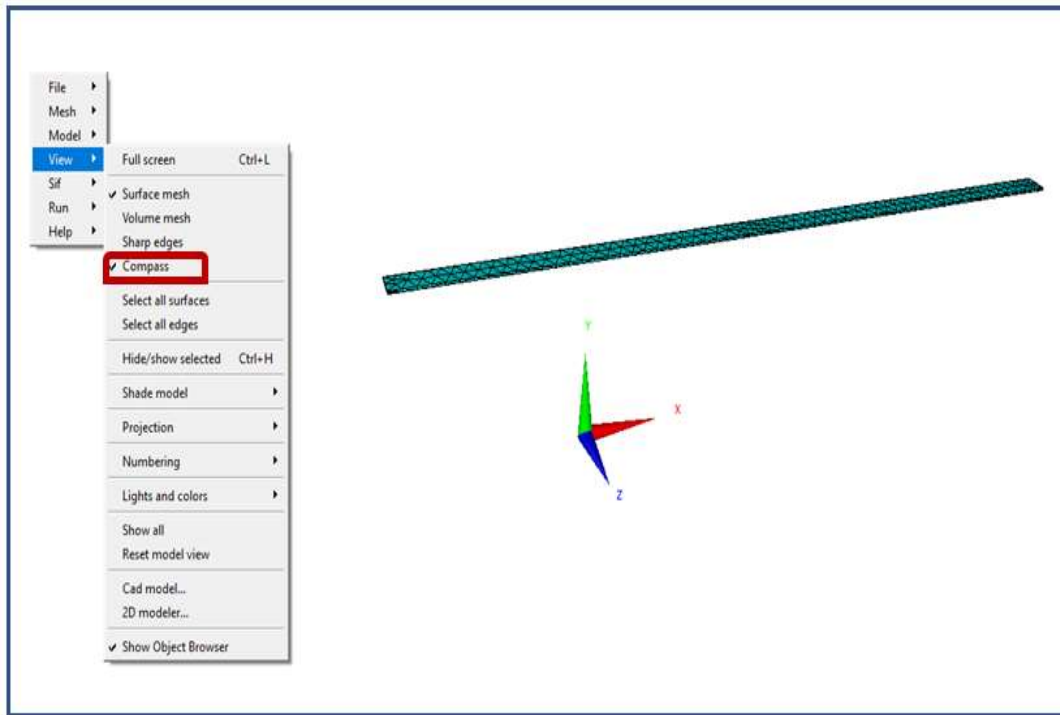


Fig. 7. Description of axes directions of the model

```

Solver 1
Equation = Linear elasticity
Variable = -dofs 3 Displacement
Geometric Stiffness = True
Eigen Analysis = True
Eigen System Select = Smallest magnitude
Eigen System Values = 5
Procedure = "StressSolve" "StressSolver"
Element-p:2
Exec Solver = Always
Stabilize = True
Bubbles = False
Lumped Mass Matrix = False
Optimize Bandwidth = True
Steady State Convergence Tolerance = 1.0e-5
Nonlinear System Convergence Tolerance = 1.0e-7
Nonlinear System Max Iterations = 20
Nonlinear System Newton After Iterations = 3
Nonlinear System Newton After Tolerance = 1.0e-3
Nonlinear System Relaxation Factor = 1
Linear System Solver = Direct
Linear System Direct Method = Umfpack
End

```

Fig. 8. Solver section in '.sif' environment

- i) The simulation is performed under 'steady-state' solver in case of any dynamic analysis. It is also required to define the input file name so that whenever the file is referred to after saving last modified coding, will appear in '.sif' file for the corresponding member. The output file name can also be modified and will be saved in '.vtu' format for post-processing of result. The solver will ask for the intervals that need to be stored in the output format as per the requirement (Fig. 9).

```

Simulation
Max Output Level = 5
Coordinate System = Cartesian
Coordinate Mapping(3) = 1 2 3
Simulation Type = Steady state
Steady State Max Iterations = 1
Output Intervals = 1
Timestepping Method = BDF
BDF Order = 1
Solver Input File = case.sif
Post File = case.vtu
End

```

Fig. 9. Section to define simulation type in '.sif' environment

- j) After modifications, the file is run for the analysis, and output is stored in '.vtu' file. The natural frequency is obtained from the square root of the eigenvalues.

2. Forced vibration analysis of beam using Elmer

The same .sif file can be used for the dynamic analysis of the member with some modifications in the simulation method. The geometric stiffness of the member is considered automatically by the software package during the dynamic analysis when the elastic solver is applied. On the other hand, linear analysis is performed by renaming it to 'StressSolver'. The following changes are also made for the damped vibration of beam subjected to external harmonic loading of frequency 41.88 rad/s.

- a) To add damping in the beam, the Rayleigh damping coefficient is defined in the material section of '.sif' file as shown in Fig. 10(a). The loading environment is modified with harmonic loading and

target boundaries are fixed by respective boundary numbers as per modeling shown in Fig.10(b). Here ‘MATC’ refers to the library of solutions for time-varying excitation function.

```
Material 1
  Name = "Material 1"
  Density = 2778
  Youngs modulus = 70.0e9
  Poisson ratio = 0.3
  Rayleigh Damping=Logical True
  Rayleigh Beta Damping=Real .0001
End
```

```
Boundary Condition 2
  Target Boundaries(3) = 5 11 17
  Name = "BoundaryCondition 2"
  Force 2 = Variable time
  Real MATC "75*cos(41.88*tx(0))"
End
```

(a)
(b)

Fig. 10. Sections to define (a) damping parameter and (b) excitation force in the .sif file

- b) The analysis is intended to perform for 10s with 0.005s intervals. Accordingly, the time step interval with total array size of $10/0.005=2000$ is added in the simulation part. Any dynamic analysis in ‘ELMER’ is performed by ‘Transient’ type of simulation instead of ‘Steady state’ used in case of natural frequency determination (Fig. 11).

```
Simulation
  Max Output Level = 1
  Coordinate System = Cartesian
  Coordinate Mapping(3) = 1 2 3
  Simulation Type = Transient
  Steady State Max Iterations = 1
  Output Intervals = 1
  Timestepping Method = BDF
  BDF Order = 1
  Timestep Intervals =2000
  Timestep Sizes = 0.005
  Solver Input File = case.sif
  Post File = case.vtu
End
```

Fig. 11. Section to define analysis time-interval

The post-processing of the output result is performed in ‘Paraview’ instead of its own environment for the detailed response history of the member with better graphical interface.

Same process is applied for the analysis of the axial bar where harmonic load is applied along the longitudinal direction of the member at the free end. The output history of displacement is finally extracted for both beam and axial members to review the performance of different damage indicators.

3. Application of Elmer

3.1 Development of Poincaré map

3.1.1 Theoretical Background of Poincaré map

The efficiency of any damage indicator is dependent on the appropriate timeframe associated with the motion if time-history response is used as fault signature. Poincaré map is one of the solutions to this time-dependent variation of state of the system. A Poincaré section is a lower-dimensional subspace of a continuous dynamic system, where a periodic orbit intersects the section. When the motion is purely periodic in nature and the

system behaves linearly as well, it creates a single dot on Poincaré section after each returning period. The reason behind this is explained through Fig. 12. The flow of a continuous dynamic system is presented here by the trajectory of x_n . The advantage of a Poincaré map is that it converts a continuous dynamic system into discrete-time system by the mapping of the flow. The discrete form of the flow preserves the information about the periodicity of the orbit along with the information about the stability of the system. At time t , the trajectory of ' x_n ' is located on the space ' Σ '. After one sampling period, if a cross-section is placed transversely to the flow and the process is continued for a number of time periods it will create a series of dots

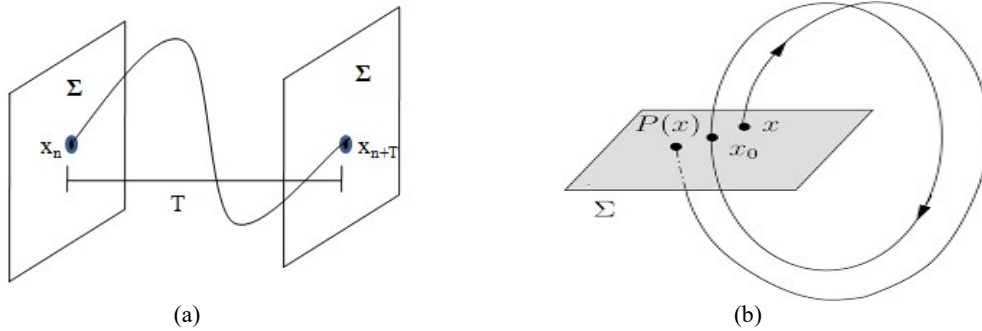


Fig. 12. (a) Trajectory of an orbit in state space, and (b) Poincaré section transverse to the flow

on the cross-section. Thus, if any periodic orbit is sampled at a fixed time period and intersection points are mapped on a suitable cross-section transverse to the flow, the series of dots will generate Poincaré map of the dynamic system. The sparse dispersion of dots in phase space or Poincaré section holds the periodic, quasiperiodic, or aperiodic information of the system. To mitigate the effect of time-dependent function on the indicator performances, Poincaré map is hence utilized in some work to detect the discrepancy in the signal between the damaged state and the reference state of the signal. The Poincaré map considered in the study of time-domain based damage detection technique is constructed

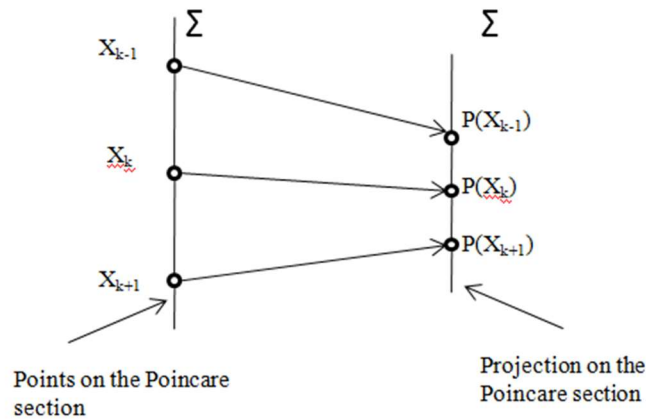


Fig. 13. Mapping of trajectory to Poincaré section

by phase-space projection of the state variables at a fixed time interval. As the Poincaré section captures each returning point of the trajectory at a fixed time period, a nearly periodic orbit generates a set of closely spaced

dots. For any change in the property of the material, the phase space plot of the same member changes from its reference state. This compact form of time-series response and graphical description of the state of the member leads to the development of some indicators for the detection of damage in the member. The Poincaré map can also be described as the map of Σ to Σ as shown in Fig.13. It describes the stability as well as periodicity of the dynamic system. Mathematically this mapping is explained as $f: \Sigma \rightarrow \Sigma$. In the Fig. 13, $P(x_k)$ is the Poincaré projection of the trajectory x_k . The converging behavior of the points on Poincaré section explains the stability of the limit cycle. In case if $P(x_i)$ diverges, it is considered as unstable system. Also in the figure mapping of the point x_k on the Poincaré section is $P(x_k)$ and $x_k = P(x_k)$. The point x_k is hence called the fixed point and the flow is called limit cycle of the flow. The stability of x_{k+1} or x_{k-1} is analyzed by the limit cycle or fixed point. A stable fixed point or limit cycle attracts the surrounding trajectory towards itself. The trajectory of the flow diverges in case of an unstable fixed point of the Poincaré map. Thus the Poincaré of $P(x_{k+h})$ of a point on Poincaré section close to fixed point x_k can be defined with respect to x_k as,

$$P(x_{k+h}) = x_k + h \cdot DP(x_k) + \Theta(\|h\|^2) \quad (1)$$

Eq. (1); where D is the first differentiative operator of the fixed point and Θ is the truncated error with the square of small perturbation h . The magnitude value of $DP(x_k)$ determines the stability of the dynamic system. In this article, the performance of any Poincaré map-based damage indicator is evaluated from the time-history response taken at each time period and plotted for nonlinear structures. The displacement and velocity coordinate of Poincaré map are then used to derive different indices. For a damaged beam element, the reduction in local stiffness value affects the global stiffness of the system. Consequentially, the trajectory of state variables changes from that of the healthy state of the member. This change even on a small scale can be captured from the compact form of the Poincaré map. In case of damped vibration, the steady-state displacement and velocity are generally considered instead of total response to eliminate the transient part of the motion. For an unknown system of motion at first, the period of motion is calculated from the response signal. The response signal in time-domain is transformed into the frequency domain to extract the frequency and thus the natural period or fundamental motion of the system. The state variables considered for the Poincaré map are then plotted after each period of time. The coordinate of each intersection point on the Poincaré section defines the behavior of the system. The development of Poincaré map using Elmer based analysis result of a nonlinear damped beam is explained with figures in the next section.

3.1.2 Poincaré Map from Elmer result

It is developed with integrated MATLAB code ‘ElmerMAT.m’. The developed code can be directly used for the nonlinear system identification and extended to the purpose of structural health monitoring.