Approximated Convex-lifting-based Robust Control for a Heat Exchanger

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Abstract

In this work we investigate the effect of the implementation of convex-lifting-based robust control with approximated control law on a laboratory heat exchanger. The analyzed setup is the reference tracking problem of the heat exchanger output temperature. To analyze the efficiency of the approximated convex-lifting-based robust control, the tunable convex-lifting-based robust control, and the robust model predictive control were also implemented. The control performance was evaluated using the energy consumption and the corresponding carbon footprint emissions of the heat exchanger. The real-time computational complexity was evaluated based on the volume of the robust positive invariant set and the average time necessary to compute the value of the manipulated variable. The expected advantage of the robust control method is increased control performance and decreased computational complexity compared to the other designed robust controllers.

Problem Statement

Robust convex-lifting-based control is an optimisation-based method. The robust controller design is performed in two phases.

The offline phase – the convex lifting, robust controller and linear control law are constructed.

The online phase – the optimal control input is computed either by solving linear programming or by linear control law.

The offline stage produces, in general multiple, in this case, two tunable robust positively invariant (RPI) sets denoted by $\mathbb{Z}_1$ and $\mathbb{Z}_2$, with corresponding gain matrix K_1 and K_2.

By turning of the weighting matrices we enforce $\mathbb{Z}_1$ to be *large-damped* RPI set and $\mathbb{Z}_2$ to be *small-aggressive* RPI set.

Obviously, considering multiple RPI sets and control laws, sudden switching between associated controller gains occurs. The sudden switching may lead to a decreased control performance, which is the problem discussed in this work.

Control Method

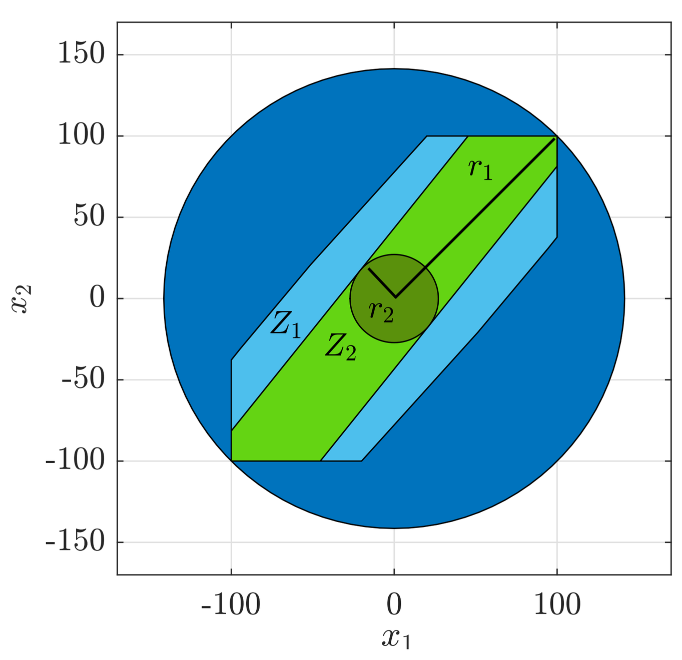

Figure 1: Illustrative figure of the RPI sets: RPI set $\mathbb{Z}_1$ (bright blue), RPI set $\mathbb{Z}_2$ (bright green), the outer approximation of RPI set $\mathbb{Z}_1$ (dark blue), the inner approximation of RPI set $\mathbb{Z}_2$ (dark green).

In this approach, a linear interpolation is considered, if the system states are present in the set difference of the *large-damped* RPI set and *small-aggressive* RPI set, i.e., $x \in \mathbb{Z}_1 \wedge x \notin \mathbb{Z}_2$. The approximation is based on the Euclidean distance of the states from the origin $d(k)$ during the real-time control. The maximal admissible distance $d_{\max}$ is determined by the radius r_1 of the minimum-volume *outer approximation* of RPI set $\mathbb{Z}_1$, see Figure 1. The value of the distance $d_{\min}$ from origin presented in *large-damped*. RPI set $\mathbb{Z}_1$ is determined by the radius r_2 of the ball with maximal volume *inner approximation*. The interpolated values are the proportional and integral parts of the gain matrix $K_{\text{approx}}(k) = [K_{\text{P,approx}}(k), K_{\text{I,approx}}(k)]$. The interpolation is determined as follows:

$$K_{\text{P,approx}}(k) = K_{\text{P},2} + (d(k) - r_2)\frac{K_{\text{P},1} - K_{\text{P},2}}{r_1 - r_2},$$

$$K_{\text{I,approx}}(k) = K_{\text{I},2} + (d(k) - r_2)\frac{K_{\text{I},1} - K_{\text{I},2}}{r_1 - r_2},$$

where $K_{\text{P},1}$, $K_{\text{P},2}$ and $K_{\text{I},1}$, $K_{\text{I},2}$ stand for the proportional and integral part of the controllers K_1 and K_2, respectively.

Case Study: Heat Exchanger

The considered plant was a liquid-liquid plate heat exchanger is shown in Figure 2, device I. The manipulated variable was the flow rate of the hot medium. The goal was to control the output temperature of the cold medium, which was the control variable.

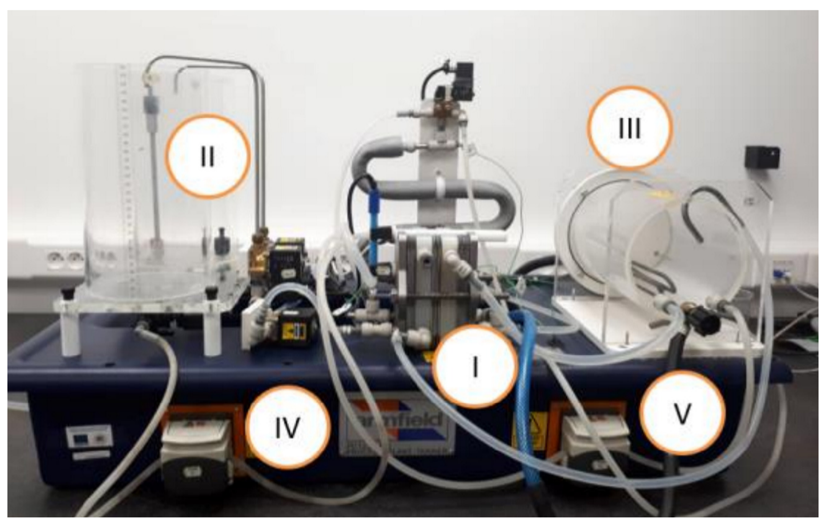

Figure 2: Plate heat exchanger (device I), retention tanks for cold medium (device II), preheating tank (device III), peristaltic pump for cold medium (device, IV), peristaltic pump for hot medium (actuator) (device V).

The control perfromance of the heat exchanger in Figure 3, was judged w.r.t. the evaluated criteria particularly, settling time t_{set}, maximal overshoot $\sigma_{\max}$, the energy consumption E, and carbon footprint m_{CO_2}. The computational complexity was analyzed w.r.t. the average time to compute the optimal value of the control input during the real-time control t_{RT}, and the values of $V_{\text{RPI}}^{\text{I}}$ and $V_{\text{RPI}}^{\text{II}}$ represent the volume of the RPI sets. The implementation of the convex-lifting-based methods reduced almost all of the evaluated control performance criteria, except for the overshoot, see Table 1 and Table 2. From the environmental viewpoint, the convex-lifting-based method outperformed RMPC also in the energy consumption E and the corresponding carbon footprint m_{CO_2} were reduced by 19 %. The convex-lifting-based method with approximated control law also outperformed the method, where the sudden switching of control law occurs, in each investigated control performance criterion.

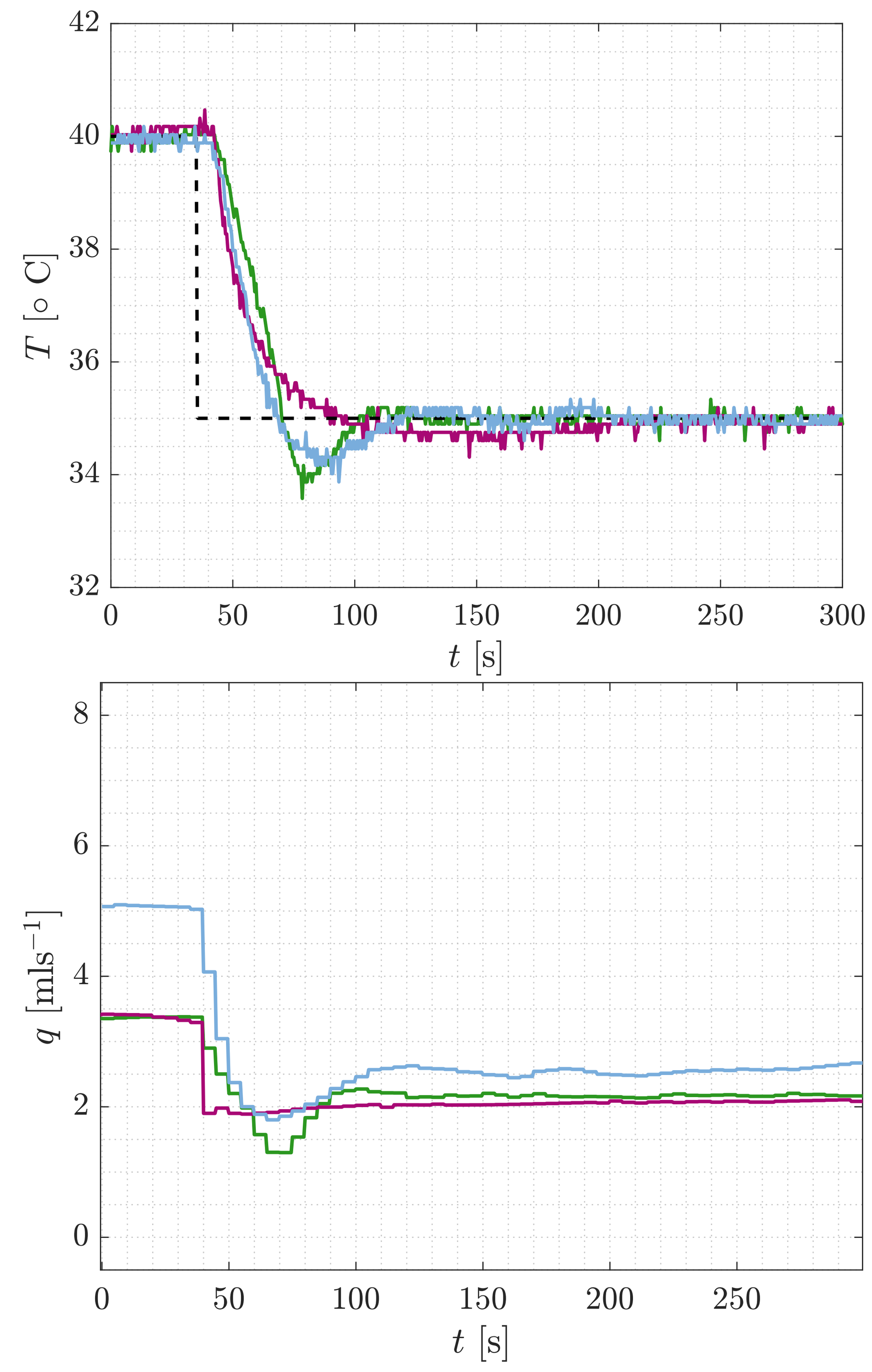

Figure 3: Closed-loop performance of controlled plant using convex-lifting-based robust control (green), convex-lifting-based robust control with approximated control law (purple), robust MPC (blue), and reference (dashed black).

Table 1: Control performance criteria.

Method	t_{set}[s]	$\sigma_{\max}$[%]	E[kJ]	m_{CO_2}[g]
RMPC	105	3.22	273	0.35
Original	94	4.81	233	0.30
Proposed	79	0.69	222	0.28

Table 2: Computational complexity criteria.

Method	$V_{\text{RPI}}^{\text{I}}$ [$\times 10^3$]	$V_{\text{RPI}}^{\text{II}}$ [$\times 10^3$]	t_{RT} [ms]
RMPC	31.4	1.9	160
Original	7.8	6.8	1
Proposed	7.8	6.8	50

Stability Analysis

In this paper, we propose a more straightforward way to directly enforce robust stability. Following the quadratic Lyapunov function candidate $V(\tilde{x})$, suppose that $V(\tilde{x})$ satisfies all Lyapunov conditions. Then any value of K_{approx} computed as a convex combination of K_1 and K_2 assures robust stability, if the following holds:

$$X_1^{-1} - \Omega_{\text{approx}}(k)^{(v)\top} X_1^{-1} \Omega_{\text{approx}}^{(v)}(k) \succ 0,$$

where $\Omega_{\text{approx}}^{(v)}(k) = (A^{(v)} + B^{(v)} K_{\text{approx}}(k))$, $v = 1 \ldots n_{\text{v}}$, where v is the number of vertices, X_1 is the weighted inverted Lyapunov matrix. If the condition is not satisfied, then the last valid gain matrix K_{approx} is implemented.

Acknowledgments

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