Modeling McKibben artificial muscles through differential geometry

Motivations and background

McKibben artificial muscles are soft actuators made of an elastomeric chamber surrounded by a cylindrical braided mesh of helical fibers. Their robust and reproducible mechanical response makes them particularly suitable for applications in Soft Robotics. However, their high non-linearity poses modeling challenges. Historically, their modeling has followed two opposite approaches:

- **Minimal geometric models**: by approximating the mesh as a cylinder, we can derive a simple formula for the pressure-force relationship of a McKibben actuator by applying the Principle of Virtual Work (PVW). This approach is computationally low-cost and potentially insightful, but too simplistic to reproduce many real-life scenarios.
- **Complex Finite Elements Method (FEM) simulations**: fibers are solved for as individual beams linked together through boundary conditions. These simulations are accurate, yet computationally expensive, and do not reveal the inner working principles of these structures.

Courtesy of Claudia Mariagiulia De Chirico and Debora Zrinšćak

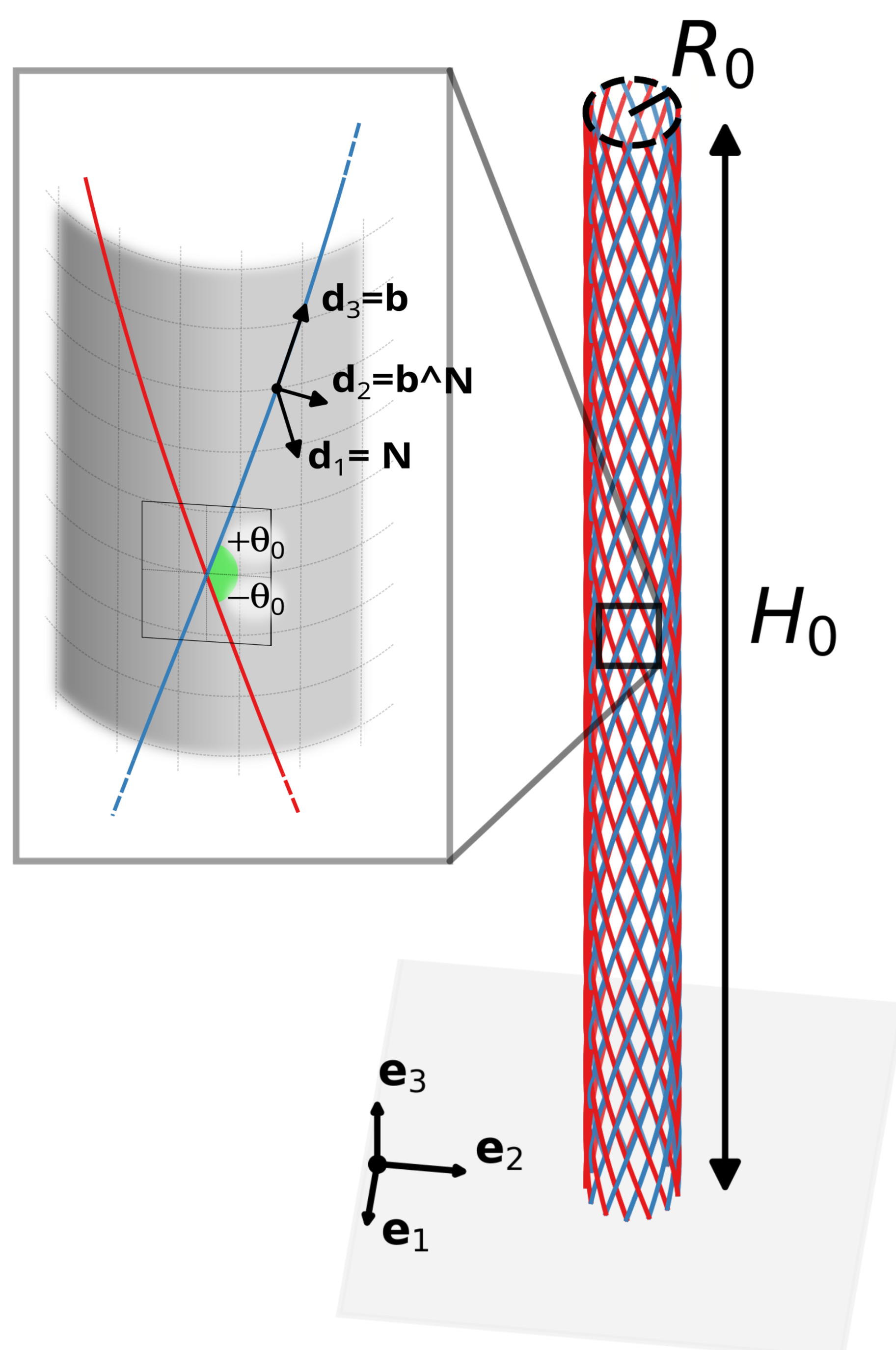
A novel coarse-grained model

In [1], we develop a coarse-grained model based on differential geometry and the "**Darboux hypothesis**": the directors' frame of the fibers always matches the corresponding Darboux frame on the envelope surface, i.e., the surface of revolution where center-lines of all fibers lie.

Moreover, we assume that the inner surface of the balloon coincides with the envelope surface of the braided mesh. Thus, the kinematics of the envelope surface resolves the one of the whole structure, both in its initial and deformed configurations.

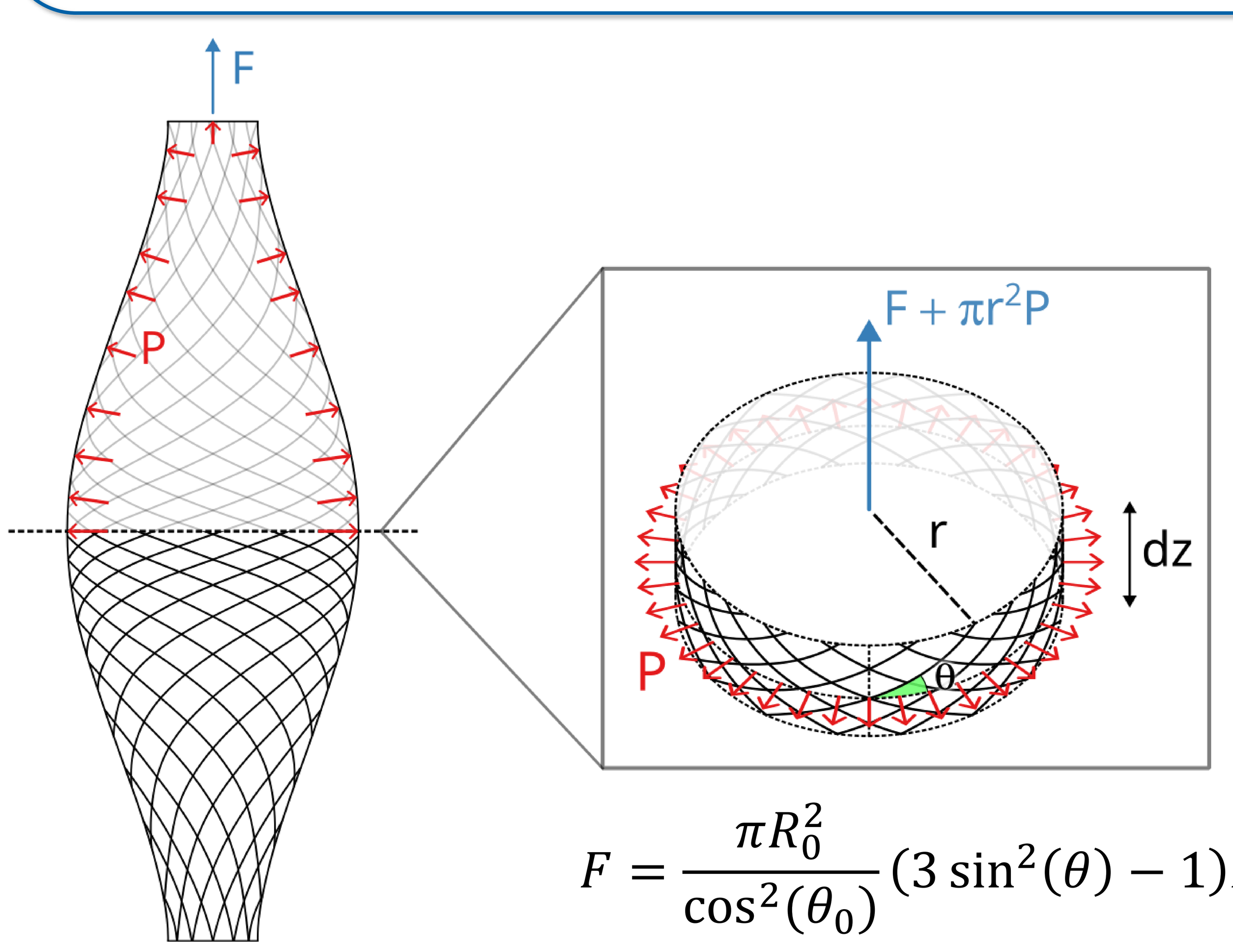
Since the kinematics of the envelope surface is expressed in terms of the current braiding angle $\theta(v)$, the problem boils down to solving for a single scalar field.

We implemented our model in MATLAB and validated it against results from Hassan et al. [2], who compared FEM-based simulations of a McKibben actuator with experimental results.



Assumptions and modeling choices

- Axisymmetric deformations;
- Braided mesh and inner chamber are mechanically in parallel;
- Fibers are modeled as Kirchhoff rods, with directors $\{\boldsymbol{d}_1, \boldsymbol{d}_2, \boldsymbol{d}_3\}$, and their center-lines are helices with braiding angle θ_0 in the initial configuration;
- The inner chamber is described as a rubber balloon, with elastic energy following an incompressible Mooney-Rivlin model.



$$F = \frac{\pi R_0^2}{\cos^2(\theta_0)}\left(3\sin^2(\theta) - 1\right)P$$

Materials and methods

Four cases were investigated:

- **Quasi-static isometric tests** A pre-stretched (case A) or pre-compressed (case B) McKibben actuator is pressurized, and the axial load is recorded;
- **Quasi-static isobaric relaxation** A pre-stretched (case C) or pre-compressed (case D) McKibben actuator is pressurized; the top base is then let free to relax until the axial load vanishes, and the corresponding displacement is recorded.

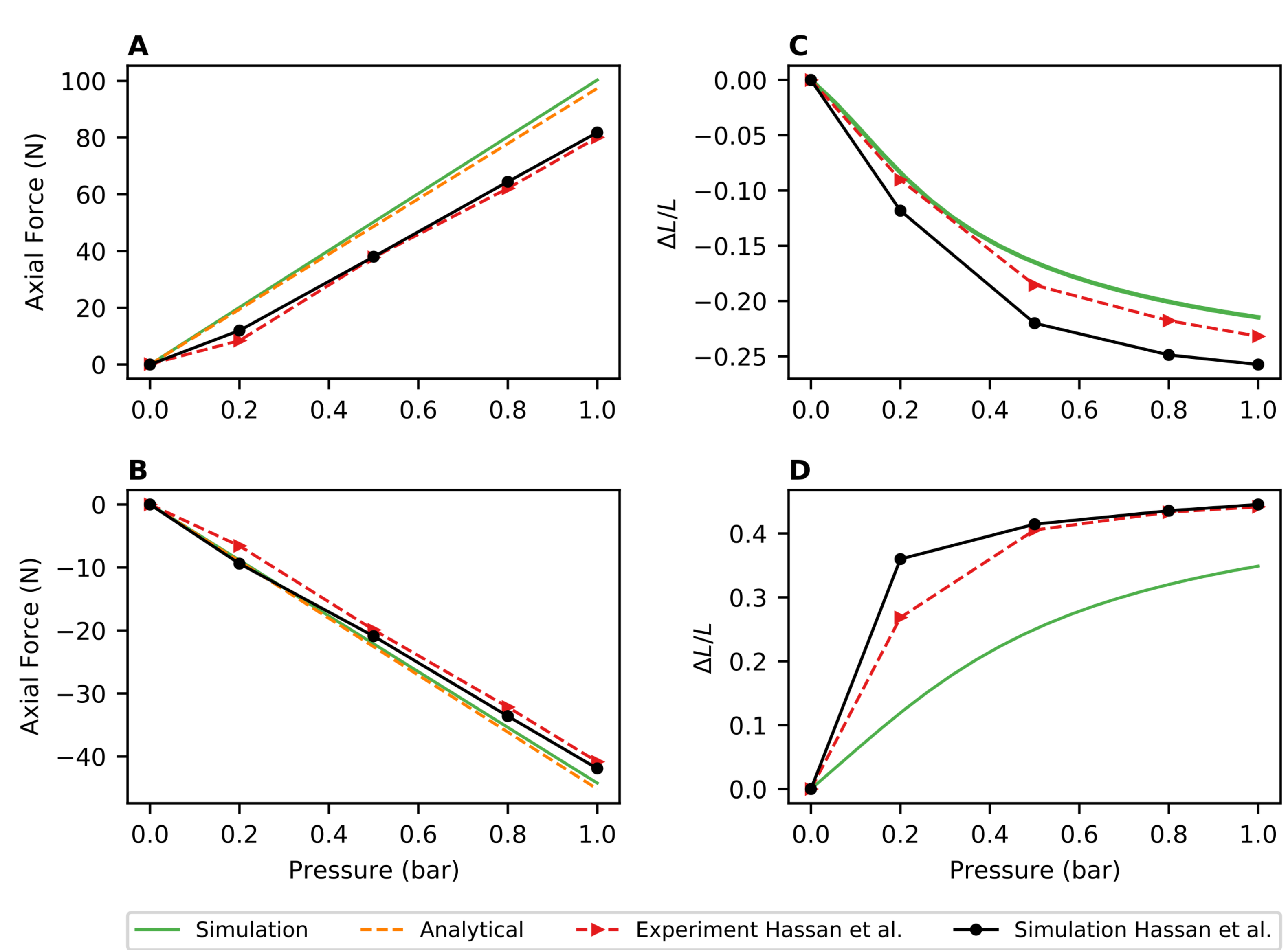
Analytical profiles obtained by applying the PVW in cases A,B are a good approximation if

1. the mesh stores no elastic energy;
2. loads on the structure are sustained entirely by the mesh;
3. upon pressurization, the angle θ at the center of the mesh does not change.

Results and discussion

Our results show small discrepancies with those from [2] in cases B,C and more pronounced ones in cases A,D. These can be attributed to the differences with respect to both their FE model and experimental prototype.

Our model matches well the analytical approximations for cases A,B, as simulations highlight a separation of roles: loads are sustained mostly by the braided mesh, whereas the inner chamber stores most of the external work as elastic energy. This phenomenon, together with the fact that θ does not change much upon pressurization, explains why simplified PVW-based formulas lead – in this case – to good approximation. However, our model succeeds where this approximation fails.



Outlook

Overall, our model shows a good matching with more complex FEM-based ones, at a fraction of their computational cost: our model uses only 43 DoFs, compared to the $\sim 10^5$ in [2]. This makes our model a physically insightful, yet computationally efficient tool for design optimization of McKibben artificial muscles.

In future work, we plan to extend our formulation to capture effects not yet included in our model such as friction, sliding, and interweaving of fibers.

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References

1. Quaglierini et al. "Mechanics of tubular meshes made of helical fibers and application to modeling McKibben artificial muscles." (2023)
2. Hassan et al. "Finite-element modeling and design of a pneumatic braided muscle actuator with multifunctional capabilities." (2018)