Evolution of depth estimation techniques by using potential field data

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SUMMARY

Depth estimation methods should be selected according to the data quality and the nature of one's particular geologic problem. Magnetic depth estimation is a cost-effective and useful tool of quantitative interpretation and helps reduce exploration risk. To produce a reliable depth solution, the experience of an interpreter is important and other independent controls are necessary. This follows from an understanding of what the indicated depths might represent. Also, as a gradient calculation is usually involved, noise creates a spread of depth solutions.
The new Cauchy derivatives by integration are stabilized and more precise. This stems from exploiting potential field holomorphic properties, which have largely been ignored in exploration geophysics. This leads to enhanced noise suppression and a more stable version of the `signal` in complex number form. Falcon airborne gravity gradiometer surveys may benefit from some of this work as well.
Padé Approximation in conjunction with Cauchy methods leads to a coherent downward continuation of the magnetic field.
Depth to basement, is now nuanced to include depth to weathering, depth to top/bottom of discreate magnetic sources.

Key words: potential field depths, vertical gradients, tensor gradients, random dipoles.

INTRODUCTION

A complete quantitative interpretation of potential field data aims to estimate three types of information about sources of geological interest: depth, dimensions, and the distribution of relevant physical properties. In many applications depth estimates are of great interest. The confusing aspect is "depth to what?".

Historically, different methods have been developed to estimate depths to magnetic sources including Euler/Werner deconvolution, Naudy, Source Parameter Imaging- improved (iSPI), and spectral analysis. All these methods have benefits and issues. All methods rely on calculated derivatives of the measured magnetic field. As Moore's law has delivered effectively unlimited computing capacity, today more correct methods can be implemented. In attempting to explain the typical results obtained, the habit of calling the results the "magnetic basement", as opposed to the "seismic basement", or "density contrast" basement has crept into the lexicon. This is an approximation at best.

None of this mathematics is new, being more than 100 years old. Existing approaches cut corners which leads to numerical instability and compromised results.

The Cauchy method provides high fidelity gradient estimation and detection and reduced ambiguity.

In this paper, the performance of the Padé Approximation in conjunction with Cauchy methods is demonstrated using a range of synthetic models and "borehole calibrated" real-life examples. The popular Tikhonov inversion pipeline also comes under scrutiny. These data sets have been used to check the older methods named above, providing a comparison of methods. To investigate the robustness of the method, noise is added to the synthetic dataset and the results are considered with the other methods.

DIFFERENT METHODS FOR ESTIMATING MAGNETIC DEPTHS to 2005

(Li, 2003) laid out a clear summary of historic *popular* magnetic depth estimation algorithms. This is neither magnetic inversion nor a "black box." Most importantly, an accurate inversion requires many constraints.

Depth estimation methods work for simplified source geometries and are independent of the susceptibility contrast. The automatic methods are also independent of magnetization direction. These depth estimate methods can be briefly assessed as follows:

1) The Naudy method (Naudy, 1971) is a curve-matching technique with the estimate obtained via a look-up table technique. The accuracy is closely associated with these tables.
2) The virtue of Werner deconvolution is its transformation of a complex, nonlinear magnetic inversion (for depth, dip, and susceptibility) into a simple linear inversion.
3) Euler deconvolution (Reid, 1990) uses Euler's homogeneity equation to construct a system of linear equations and solve it in least-squares sense for the single putative source of a given type. Since Euler's homogeneity equation holds not only for the magnetic field itself, but also for its derivative and a combination of derivatives, others have developed Euler deconvolution of the analytical signal or of the first- or second-order vertical derivative of the magnetic field to determine the structure index (SI) and/or to improve the resolution. Two advantages of Euler deconvolution over many other methods are its easy generalization from 2D (profile analysis) to 3D (grid analysis) and its direct application to observations with variable altitudes.
4) SPI (Thurston, 1997) is a complex analytical signal technique, using not only the magnitude of the analytical signal but also the phase.
5) Mathematically, continuous wavelet transform (CWT) is just the upward continuation of the analytical signal amplitude multiplied by the continuation distance.

UNIFICATION OF ESTIMATING MAGNETIC DEPTHS

The extended Euler deconvolution algorithm (Nabighian & Hansen, 2001) is shown to be a generalization and unification of 2-D Euler de-convolution and Werner deconvolution. After recasting the extended Euler algorithm in a way that suggests a natural generalization to three dimensions, they show the 3-D extension can be realized using generalized Hilbert transforms. The resulting algorithm is both a generalization of extended Euler deconvolution to three dimensions and a 3-D extension of Werner deconvolution. An implementation of this extended algorithm plus extensions to also include FTG measured gradients were made by FitzGerald et al 2004.

At a practical level, the new algorithm helps stabilize the Euler algorithm by providing at each point three equations rather than one. What was not realised was that the Hilbert transform also acted to stabilise the signal, as now understood following the Cauchy insights. The advantage of the extended algorithm is the ability to also estimate the SI as well as the depths. See figure 1. The extended method can also now estimate the SI as well as HOT-SPOT depths. This then allows the interpreter to use SI as a discriminator and to plot only those solutions with SI > 1. This achieves a result where the four kimberlites are identified (with a tight clusters of solutions) Note, a kimberlite, is approximated by a vertical pipe, and therefore have an SI of 2.

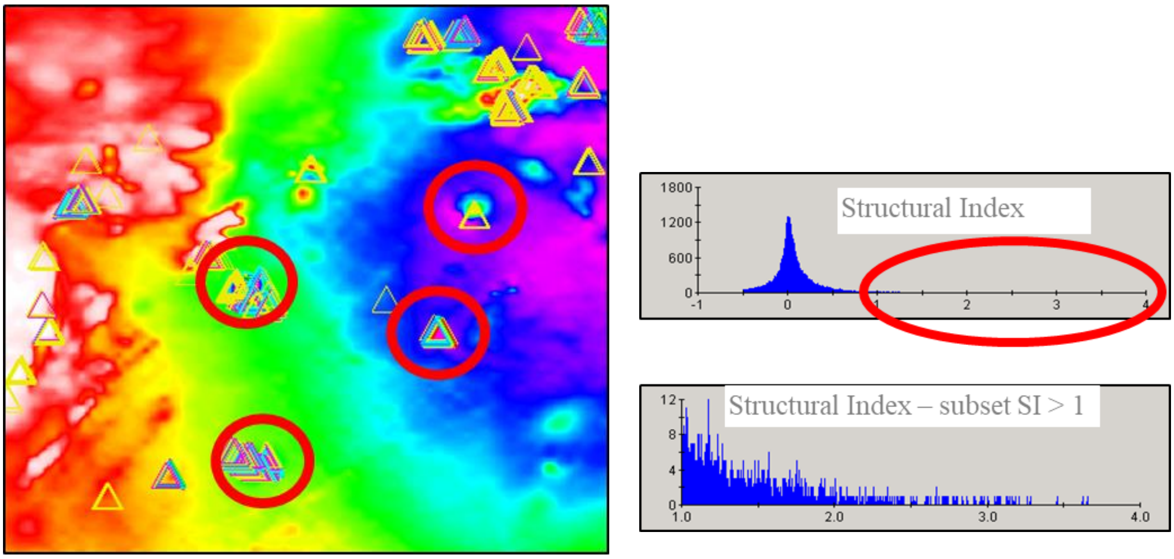

Figure 1. Extended Euler advantages: Looking for Kimberlite Targets on magnetic data, reduced to the pole at Sheoak Hill, South Australia. The two plots to the right show histograms of the sources and their SI. Examination of the computed Structural Index shows that only a small percentage of solutions have a high Structural Index (SI) value. Most HOT-SPOT solutions are associated with contacts.

ESTIMATING MAGNETIC DEPTHS USING SPECTRAL METHODS

A method of finding magnetic depths (Clifton, 2015) simulates the heterogeneity of magnetic bodies by assuming that the source of signal is one or more layers of randomly located dipoles. A layer of dipoles contributes shape characteristic of the depth, across a range of wavelengths on the power spectrum. Often a second or even a third layer contributes the shape characteristic of their depths to other parts of the spectrum. Inversion to depths results in a depth profile, and a series of depth profiles in a transect resembles a noisy seismic section. The method works well when the magnetic bodies are wide, have at most a gentle gradient and the geology above is magnetically quiet. The Energy Spectral Analysis methods developed by Archimedes, Kivor et al, 2016, has similarities to this approach.

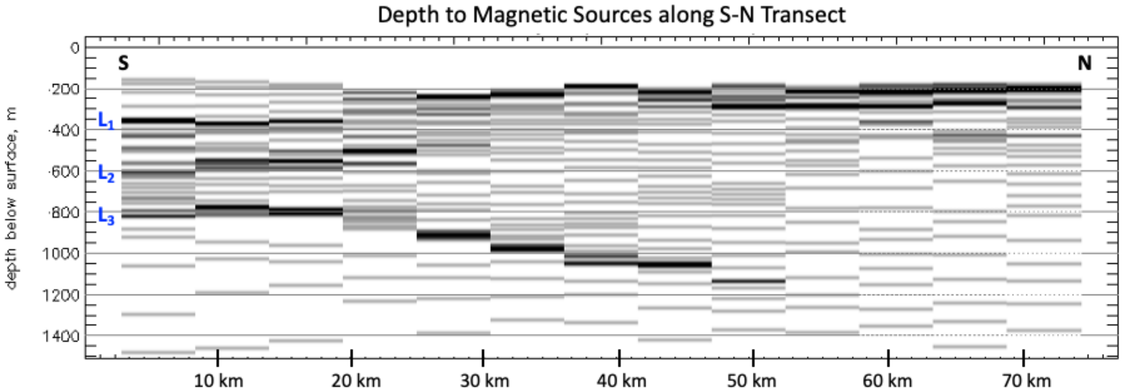

Figure 2. Transect of depth estimates along a north-south profile in the Barkley Basin, NT. The tops of three layers (L_1, L_2, L_3) can be distinguished in the southmost three depth profiles. A deepening feature assumed to be basement (L_3) can be identified across the middle of the figure.

RECENT INNOVATIONS – CAUCHY INTEGRATION

Numerical differentiation can be undertaken in the Fourier domain with linear filters, or in the space-domain with convolution filters; both instances are Finite Impulse Response Filters. This technique is applicable to virtually any digital input and is widely used in several signal processing settings. However, when applied to potential-field data, high-order differentiation is ill-conditioned; rectifying this requires severe filtering of the short wavelength portion of the spectrum. Severe filtering can generate quasi-sinusoidal gradients lacking a rich dynamic range and limited useable signal. This leads to the resigned position that only first and maybe second order differentiation, can be calculated for potential fields. The historic reasons for cutting corners with the physics has its origins back to the start of digital filtering for telephony and limited capacity to do anything else due to hardware limitations.

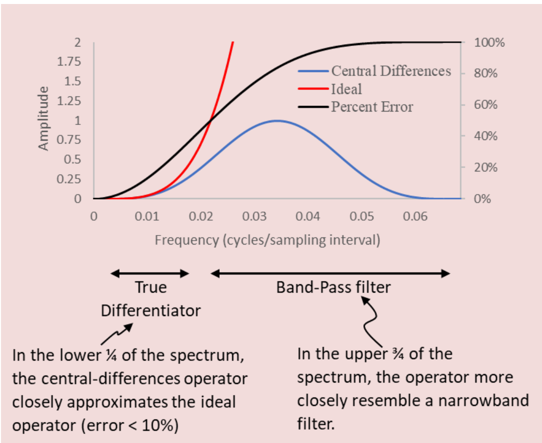

Figure 3. High-order FIR differentiators behave somewhat like band-pass filters. This is illustrated by the amplitude spectrum of a 4th-order central differences operator. Only the lower 25% of this spectrum is actually a “true differentiator”; the upper 75% more closely resembles a band-pass filter.

Cauchy’s integral formula recasts this unstable process to so-called differentiation by integration. Adapting this method to the processing of gravity and magnetic data allows setting aside the troublesome spectral operators (and their attendant high-frequency distortions) in favour of a scheme that relies on potential-field (rather than signal-processing) theory. Importantly, when using Cauchy's formula, there is no need to apply a filter only capable of isolating one specific band in a largely inseparable spectrum. Instead, differentiation is replaced by integration, which, unlike differentiation, is a stable and well-conditioned undertaking. Holomorphic functions comprise real parts and their harmonic conjugates. For a scalar field (e.g., total magnetic intensity) the Hilbert transform is used to compute the imaginary signal. While feasible, this process is not without its pitfalls. This leads to 1D using complex and 3D using Quaternion notation arithmetic. Viable third order vertical gradients immediately show more promise in further detail interpretation applications.

DOWNWARD CONTINUATION

While Cauchy gradients are useful, we have also found use of these quantities for extended downward continuation. This method is a counterpart of Taylor series methods, albeit with higher order terms and a rational approximating function. Various attempts are mentioned in the literature, Cooper, 2004, Fedi and Florio, 2002 and Zhang, Ravat, and Hu, 2013. However, in this case the method is augmented with:

1. Convergence acceleration
2. Gradients calculated using Cauchy's integral theorem - differentiation by integration
3. Expansion by complex (rather than purely real) Taylor series approximation

These enhancements improve both accuracy and stability, allowing for continuation to distances that are much greater than has conventionally been possible.

The image in Figure 4 shows synthetic data generated for thin sheets (white lines) and continued downward. The leftmost body is a vertical source with a finite depth extent. Note the polarity flips first as the data are continued to the source level, and once again below the body. The sources to the right have infinite depth extent and exhibit a single change in polarity near the sources' tops. Further, the asymmetric response of the dipping sheet (rightmost anomaly) is owing to the source dip (note the dip of this sheet is skewed because of the vertical exaggeration.

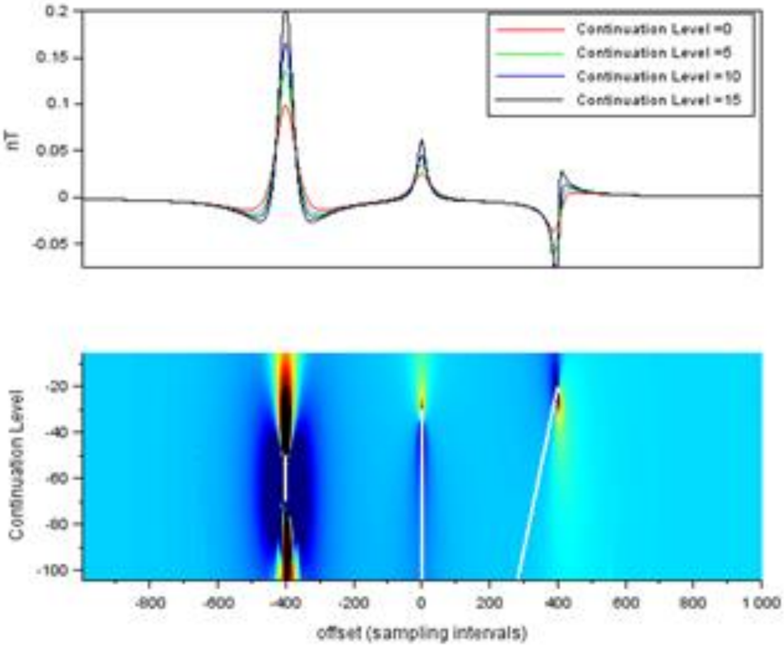

Figure 4. Synthetic data generated for thin sheets (white lines) and continued downward. Horizontal and vertical scales are in sampling interval, so the vertical exaggeration is about 10x.

The image in Figure 5 shows TMI data for a flight line over the Ring of Fire in the Hudson Bay Lowlands of Northern Ontario. The sources primarily bear the signature of sub-vertical sheets with deeply rooted bases. That is, they are largely symmetrical with a lone change in polarity). Distances on the vertical axis are with respect to the sensor height, which was flown at a mean terrain clearance of 100 m. Thus, source depths (i.e., abrupt changes from red to blue) range between 50 and 100 m below ground.

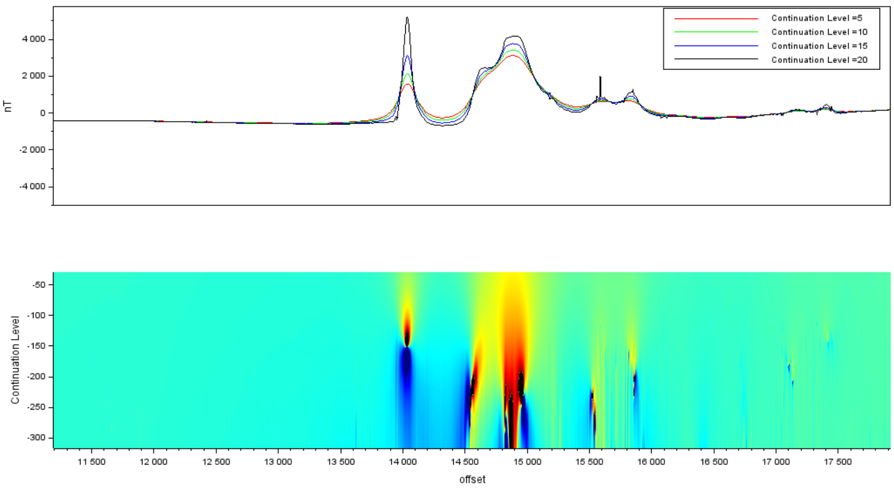

Figure 5. TMI data for a flight line over the Ring of Fire in the Hudson Bay Lowlands of Northern Ontario

EXTENSIONS TO GRADIOMETRY

Measured gradients have been used with Euler, Worming, and Naudy methods for many years now. As these algorithms already require gradients to function, it is a relatively simple exercise to substitute and upgrade the tools. Generally, a measured gradient produces a much superior estimate of depths compared to a method requiring a gradient to be computed.

All potential field gradient surveys (Falcon (AGG), FTG, magnetic tensor gradients), can benefit from the uplift offered by new approaches. Turning specifically to Cauchy methods, Falcon is well suited as the actual measurements are a Hilbert pair (the A and B measures). BHP's original processing stream makes use of what was termed BRUTE methods, involving packing the Hilbert pair into a complex number. Unlike magnetics, the inherent lower signal to noise ratios in gravity gradiometry are an ever-present obstacle. Despite noise floor numbers given by the survey providers, noise levels need to be assessed on the as collected survey data, and where the gradients fall below the signal threshold, they should be ignored or set to zero.

The Noise can be estimated by conventional L(1) and L(2) methods. However, methods that treat the tensor as a whole as the signal require non-scalar methods. Fisher statistics can be employed on the Eigenvectors of each gradient measure. This leads to summary statistics of survey tensor signal. The tensor data signal floor can then be estimated by histograms of the cube root of the determinant, which exhibits a notch (or gradient deficit) at the minimum resolvable gradient magnitude. The notch represents the missing gradient measures, as the instrument system was not able to record the gradients below that threshold. Because the data acquisition systems have improved industry wide, recently acquired data can be expected to provide better results than older surveys.

To date, we have observed better results by downwards continuing after pre-processing with Full Tensor Noise Reduction strategies, and then undertaking a finer than normal altitude levelling stage, followed by a process to zero out the dubious measures, and using what is left. The aim is to produce a more classic Gaussian noise signature, indicating the gradients in play have reduced noise. Full Tensor gridding also benefits by pre-processing to ignore the noisier low gradients.

CASE STUDIES

We show an example using aeromagnetic data acquired over the Gawler Craton in central South Australia, under the auspices of the PACE Copper project. The top left panel of Figure 6 below shows the TMI response over the Mt Hawker target, a gold and copper prospect in the Prominent Hill mining district. Oz Minerals drilled this ~400nT response in 2011 and 2012. Their explorationists interpreted the structure to be a fold hinge, at about 400 m depth, with steeply dipping flanks.

Figure 6 shows a depth section generated directly from the magnetic data gridded at a 40m interval. At the deeper 400m level, the single peak, observed on the TMI data, is resolved into a sharp low flanked by highs on either side. This is also apparent on the cross-section. We propose the high-low pair at the apex of the response images a singularity. In the shadow of the singularity, the continued data is likely unreliable. At lower levels, and adjacent to the singularity, the continued data is a valid representation of the magnetic response at that level. An isolated feature that has been drill tested, was chosen as a calibration test for this new technology.

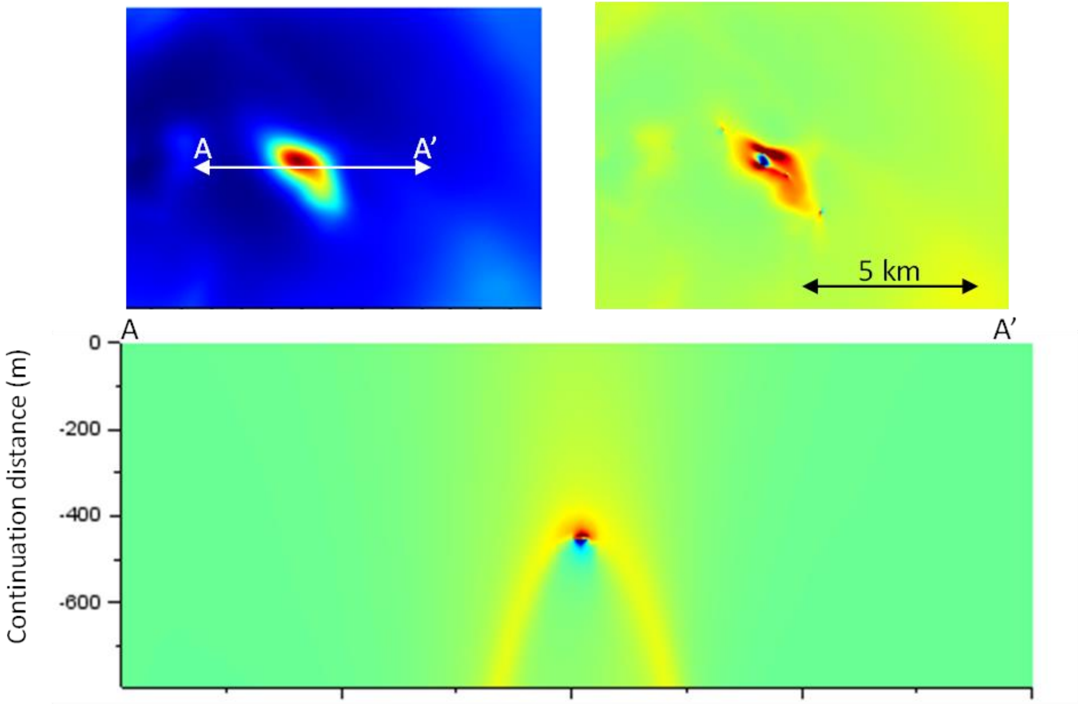

Figure 6. TMI anomaly, over the Mt Hawker, Gawler Craton, South Australia.
(Top left is the TMI showing this discrete feature and an East-West section line. The top right panel shows the aeromagnetic data continued downwards 400 m, roughly to the level of the top of the interpreted source.
The bottom image is the downwards continued East-West cross-section, depth registered.)

VICTORIAN GOLDFIELDS

Older Victorian government aeromagnetic data acquired over the Horsham area, has been reprocessed, had next generation levelling and gridding, and then subjected to this new downwards continuation process. Unlike the Euler methods, this approach continues all the measured signal downwards, without attempting to highlight the HOT-SPOT magnetic sources. HOT-SPOTs are edges and or centroids of compact bodies. The results obtained (Figure 7) start to echo the sort of results seen with 1D inversion methods for AEM datasets. In this case, typically, the depth to water table, or weathering depth, when nothing much else is present magnetically. None of the other algorithms for depths from magnetic data show this style of result.

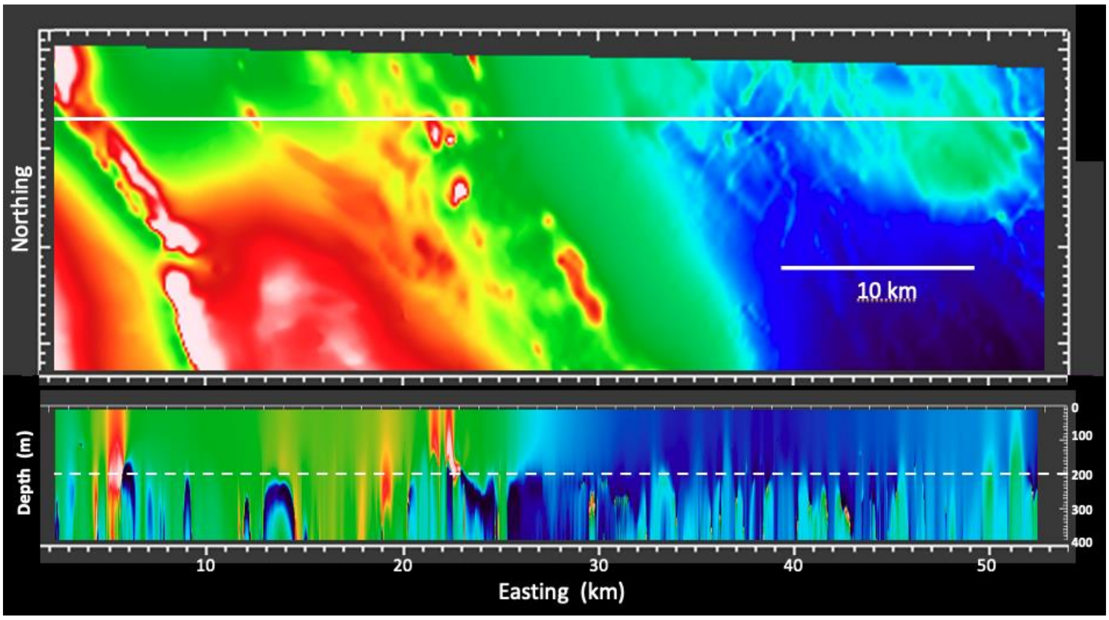

Figure 7. Top: TMI aeromagnetic data in Western Victoria. Bottom: Vertical cross section along the East-West traverse of the downward continued TMI data. Around 200m below surface (dashed line), what looks to be the weathered layer and its variable base has emerged. In a magnetically quite zone, this is the main signal content.

CONCLUSIONS

Conventional FIR differentiation is incapable of producing reliable high-order gradients. However, differentiation based on holomorphic function theory is a promising alternative. Since gradients are a key ingredient for many potential-field depth estimation processes, this approach presents several opportunities for improved algorithms. Despite the improved ability to calculate gradients, demonstrated in this paper, the case for commissioning gradiometry surveys to achieve better resolution geophysical mapping, is not compromised.

The early work on applying Cauchy integral differentiation to the extended Euler algorithm already shows some of this promise.

Here we have shown one such possibility; inclusion of higher-order terms (i.e., higher-order gradients) in the complex series expansion increases the region of convergence. This allows continuation to levels currently unattainable by conventional methods. These methods do not require a depth weighting function, as used by common potential field inversion schemes.

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