Is the Aki and Richards approximation really practical to estimate AVO responses?: Case study of the Browse Basin

ISHIKAWA Satoshi
INPEX Corporation
satoshi.ishikawa@inpex.co.jp

SUMMARY

AVO analysis is an important basic technique in exploration geophysics for lithology and fluid estimation. Currently, the AVO equation of Aki and Richards (ARE), which is approximation of the Zoeppritz equation (ZE) is widely used for AVO analysis and AVO inversion. However, ARE assumes small an elastic property contrast between media. This study investigated a condition where the approximated AVO relation can be deviated from the actual equation, by using the regional rock physics model in the Browse Basin and calculating angle-dependent reflectivity. The intercept, gradient (and curvature) term originally defined by ARE are calculated on ZE as if ZE equation is approximated with these two and three terms. The calculation is performed between two media, seal as the upper medium, and sandstone and shale as the lower medium. These approximated two and three terms are compared with the original parameters of ARE. The study revealed that the 3-term ARE has more aligned with ZE derived angle-dependent reflectivity than that of the 2-term. Deviation of the angle-dependent reflectivity from ZE occurs around 20 degree. Comparison of two approximation methods (2-term vs 3-term) indicates, the 3-term equation has smaller gradient errors with ARE than the 2-term equation at target depth. In the 2-term approximation, gradient errors get larger with the larger angle utilized for the approximation. This implies that, for the Browse Basin, the 3-term ARE approximation is preferable to correctly estimate angle-dependent reflectivity. For use of the 2-term approximation, narrower angle range (around 20 degrees) is suggested otherwise error correction is required (at far angle range). These results emphasize the requirement of careful investigation of seismic quality specifically at far angle, and analysis of angle-dependent reflectivity of expected lithology for AVO analysis.

Key words: AVO, Zoeppritz equation, Aki and Richards approximation, Gradient difference, Curvature difference.

INTRODUCTION

2019 is the 100th year after the Zoeppritz equations (Zoeppritz, 1919) emerged which has most remarkable impact on seismic industry. Analysis of angle variation with offset (AVO) is an important basic technique in exploration geophysics for lithology and fluid estimation (e.g. Avseth *et al*., 2005, Chopra, 2014). Currently, the AVO equation of PP reflection suggested by Aki and Richards (Aki and Richards, 2002), which is approximation of the Zoeppritz equation is widely used for AVO analysis and AVO inversion as the approximation provides linearized forms suitable for inversion problems.

Although AVO analysis is a basic exploration technique, there are several issues impacting AVO relations. Anisotropy, attenuation, thin layer assumption and spherical waves are not considered both in the Zoeppritz equation and the Aki and Richards equations. The most fundamental problem in the Aki and Richards equations is that the equations assume a small elastic-property contrast between media. This implies that the approximated AVO relations can be deviated from the actual AVO relations in a certain condition where large property contrast between media such as sandstone and shale combination as indicated by Haase *et al*. (2004).

To overcome this issue, other approximations are suggested (Wang, 1999, Riede *et al*., 2005). Causse *et al*. (2007a) and Causse *et al*. (2007b) suggested compensation methodologies of the Aki and Richards equation. Although such an obvious issue of the assumption is well known and investigated (e.g. Li, *et al*., 2007), there are few studies indicating an impact led by this assumption and workable solution are yet to well understood.

The Browse Basin is one of the main oil and gas field in the North Weste Shelf in Australia. Although there are several production fields in the Browse Basin, still exploration potential and fields in an appraisal phase remained. AVO analysis to estimate lithology and/or fluid is still an important method to decrease uncertainty of such remained potentials.

To effectively and quickly estimate lithology and/or fluid and its uncertainty, it is important to understand degrees of the AVO errors induced by the Aki and Richard approximation with a simple analysis in the Browse Basin. This study investigated a condition when the AVO relation can be distorted and suggested its solution, by using the regional rock physics model in the Browse Basin. The previous study focusing rock physics is also well integrated in the current study.

ROCK PHYSICS MODEL

We utilized the published rock physics model focusing the Browse Basin suggested by Desaki *et al*. (2019). This model focuses the Upper Vulcan Formation, the one of the main reservoirs in the Browse Basin, that is suitable for our objective. Comparison between the Zoeppritz equation and the Aki and Richards equation is conducted by Monte Carlo simulation. Single interface separated by two media are assumed for reflectivity calculation. We referred simulation parameters based on the Desaki's work. Eight different but possible scenario were analysed which are summarised in following Table 1. Applied simulation parameters are explained in Table 2. Gaussian Distribution is used to populate of random parameters for volume of clay (Vclay), density (ρ), P-wave velocity (Vp) and S-wave velocity (Vs).

Case	***Upper medium (assuming seal)***	***Lower medium (assuming reservoir)***
1	End-member shale	End-member sandstone
2	End-member shale	Non-end-member sandstone
3	Non-end-member shale	End-member sandstone
4	Non-end-member shale	Non-end-member sandstone
5	End-member shale	End-member shale
6	End-member shale	Non-end-member shale
7	Non-end-member shale	End-member shale
8	Non-end-member shale	Non-end-member shale

Table 1. Cases summarizing upper medium and lower medium for the AVO stochastic simulation.

Parameters	***Remarks***
Number of Simulations	50000
Depth range (m)	3700 to 4100 with uniform distribution

Table 2. Main parameters for our Monte Carlo simulation

COMPARISON METHOD

To compare AVO responses in the domain of AVO parameters, the similar methods utilized by Causse *et al*. (2007) are used in this analysis. Angle-dependent reflectivity by the Zoeppritz equation is approximated by the AVO parameters, the intercept, gradient and curvature as if the Zoeppritz equation is expressed by these parameters as follows.

$$R_{ZE2} = I_{ZE2} + G_{ZE2} \sin^2 \theta \quad \text{(Equation 1)}$$
$$R_{ZE3} = I_{ZE3} + G_{ZE3} \sin^2 \theta + C_{ZE3}(\tan^2 \theta - \sin^2 \theta) \quad \text{(Equation 2)}$$
$$R_{AR} = I_{AR} + G_{AR} \sin^2 \theta + C_{AR}(\tan^2 \theta - \sin^2 \theta) \quad \text{(Equation 3)}$$

Where, R_{ZE2} is angle-dependent reflectivity by the approximated pseudo-2-term intercept I_{ZE2} and pseudo-2-term gradient G_{ZE2} based on the Zoeppritz equation. R_{ZE3} is angle-dependent reflectivity by the approximated pseudo-3-term intercept I_{ZE3}, pseudo-3-term gradient G_{ZE3}, and pseudo curvature C_{ZE3} based on the Zoeppritz equation. R_{AR}, I_{AR}, G_{AR} are the angle dependent reflectivity, intercept and gradient respectively defined by Aki and Richards (2002). The approximated intercept, gradient and curvature from the Zoeppritz equation are estimated by Nash variant of Marquardt nonlinear least squares solution.

Difference of both 2-term approximation and 3-term approximation of the Zoeppritz equation against Aki Richards approximations were analysed in this study. Following angle ranges, 0-20, 0-25, 0-30, 0-35, 0-40, 0-45 and 0-50 degrees were used for the calculations. Difference between the equations is calculated by

$$P_{d2} = \frac{P_{ZE2} - P_{AR}}{P_{ZE2}} \quad \text{(Equation 4)}$$
$$P_{d3} = \frac{P_{ZE3} - P_{AR}}{P_{ZE3}} \quad \text{(Equation 5)}$$

where, P_d is the difference indicator between the Zoeppritz equation and the Aki and Richards equation, such as reflectivity, the intercept, the gradient and the curvature, and subscript of 2 and 3 indicates the utilized terms for the approximation of the Zoeppritz equation, the 2 term means the intercept and gradient, and the 3 term means the intercept, gradient and curvature.

Since the Aki and Richards equation assumes a small contrast between media, elastic property contrast (C_{EP}) is also compared with the AVO parameters defined by the following equation,

$$C_{EP} = \frac{1}{3}\sqrt{\left(\frac{\rho_1}{\rho_2}\right)^2 + \left(\frac{Vp_1}{Vp_2}\right)^2 + \left(\frac{Vs_1}{Vs_2}\right)^2} \qquad \text{(Equation 6)}$$

where, its subscript 1 and 2 stands for the upper media and the lower media, respectively.

RESULTS AND DISCUSSIONS

Figure 1(a) to (d) shows an example of angle-dependent reflectivity of Case 1. As shown, reflectivity difference is observed between the Zoeppritz equation and the Aki and Richards equation at the angle above 20 degrees. Thus, it is clear that reflectivity deviation from the actual value at far angle is expected in the Browse Basin when the Aki and Richards approximation is applied. Also, even between the 2-term and the 3-term Aki and Richards equation, we can observe reflectivity difference from the angle of 20 degrees onward.

Figure 2(a) to (d) show an example of crossplots of Case 1 between the intercept (I) and the gradient (G). Summarized results of the I and G plots are shown in Figure 3. In the 2-term approximation by the Zoeppritz equation, the larger deviated results of the intercept and gradient from the Aki and Richards equation are obtained with the larger angles utilized for the approximation. Considering the reflectivity deviation between the equations above the angle of 20 degrees, this implies that curvature effect emerges from the angle of 20 degrees in the case of (a) and (b) in Figure 3. In the 3-term approximation by the Zoeppritz equation, similar intercept and gradient values between the equations are obtained if the far side is restricted within 20 to 40 degrees. Large deviation of intercept and gradient are obtained if the utilized angle range is above 40 degrees.

Figure 4 to 6 show examples of difference of the gradient and curvature of Case 1 between the equations. Summarized results of the difference are shown in Figure 7. Although, in some cases, the elastic property contrast has a relationship with the gradient difference, it is difficult to say there is a clear relationship between the gradient difference and the elastic property contrast. Meanwhile, weak relationship between the elastic property contrast and the curvature difference is observed.

There is large variation of the difference of the gradient, from 1 to 100% between the Cases. In Case 1, the end-member shale overlying the end-member sandstone, the difference between the Zoeppritz equation and the Aki and Richards equation is around 10 % in the 2-term approximation, and 3% in the 3-term approximation. This value of the 3% shows that the 3-term approximation provide better fit than 2-term approximation and the 3-term approximation appear reasonable.

Meanwhile, most of the curvature difference is more than 10%. Also, no clear relationships are observed between the curvature difference and the utilized angle range. This implies that, curvature estimation using seismic data need detail angle optimisation which may be far complex than current industry practice. Moreover, one should consider the errors of the curvature carefully since the difference of the curvature is always high.

From the point of view of obtained errors of the gradient between the Zoeppritz equation and the Aki and Richards equation, it is recommended to use the 3-term approximation or to use the 2-term approximation with smaller angle range such as 0 to 20 degrees to minimize the errors of the gradient. While, if using the 3-term approximation, seismic quality of far stack data should be a critical issue. Possible practical solution to effectively estimate the gradient term is that 1. application of the 2-term approximation for AVO analysis and/or to AVO seismic inversion with smaller angle range around 0 to 20 degrees and applying gradient error functions of the estimated lithologies, and 2. application of the 3-term approximation with appropriate angle range, 25-40 degrees. It depends on both seismic quality and seismic responses with expected lithology, which solution to be used. Utilization of the curvature is way forward.

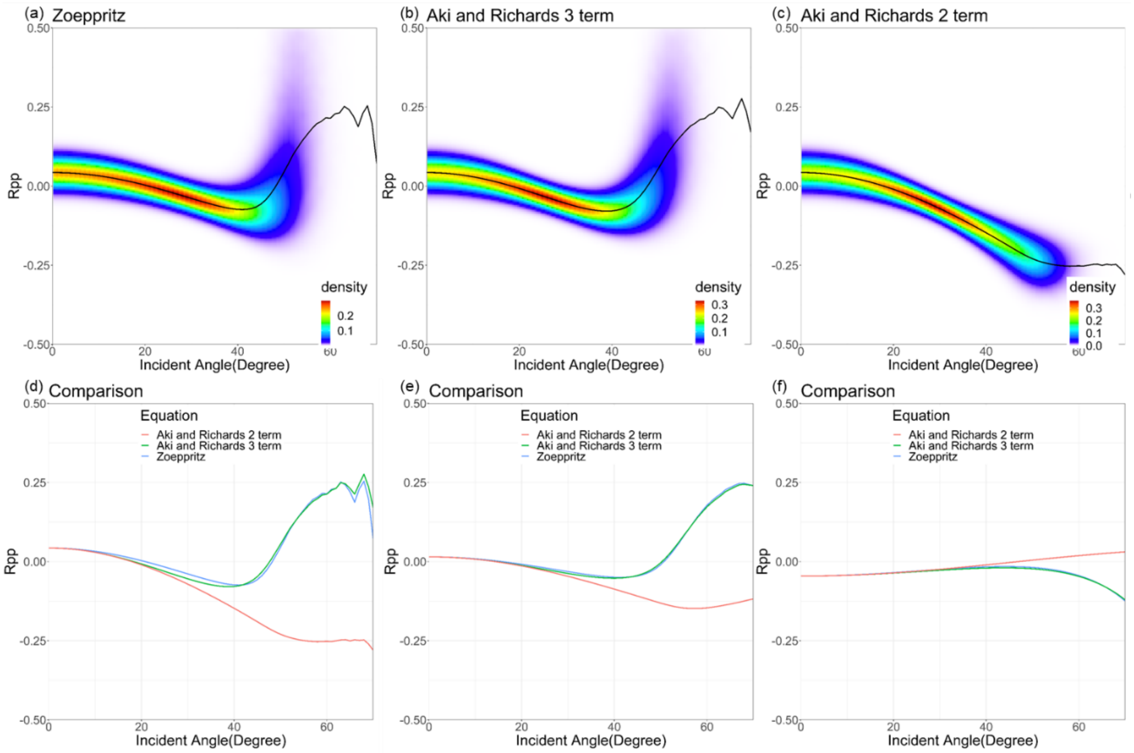

Figure 1. An example of the simulation results of angle-dependent reflectivity with Case 1 between (a) the Zoeppritz equation, (b) 2-term Aki and Richards equation, (c) 3-term Aki and Richards equation, and (d) comparison of (a), (b) and (c). (a), (b) and (c) are 2D histogram. (e) and (f) are comparison of angle-dependent reflectivity between these 3 equations of Case 4 and Case 7. The colour and the solid line of (a), (b) and (c) indicates the data density and mean reflectivity in each angle.

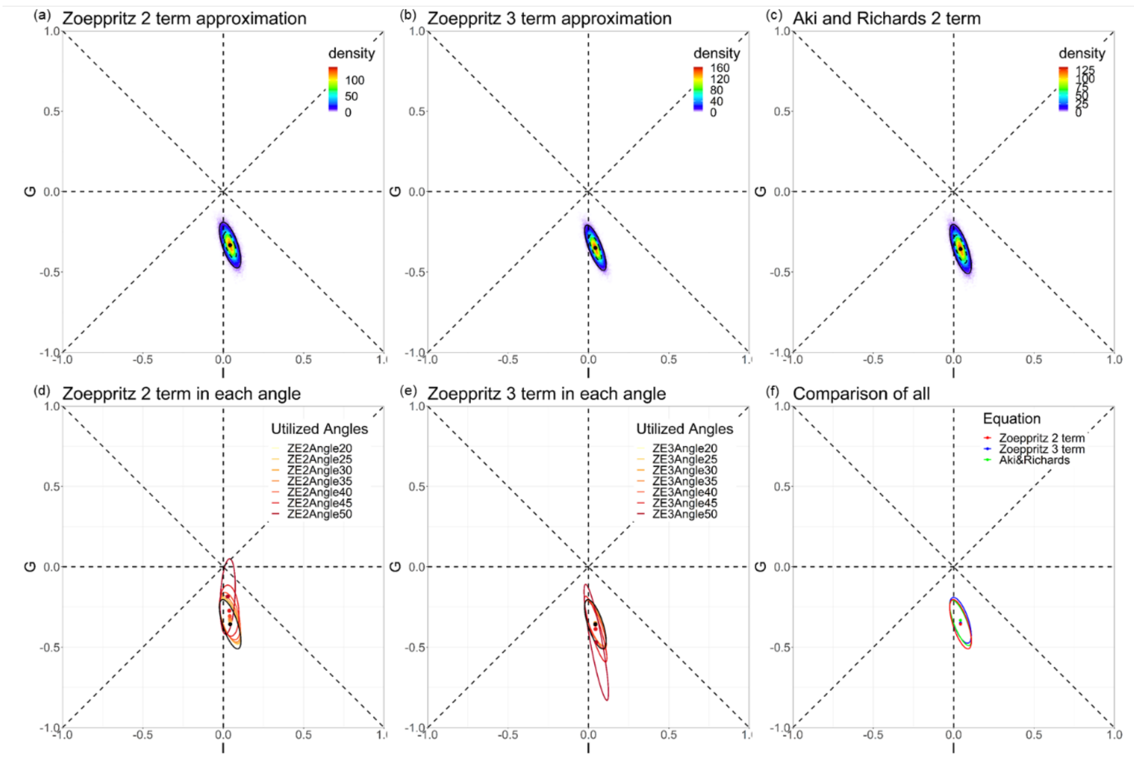

Figure 2. An example of the simulation results of 2D histogram between intercept and gradient with Case 1 superimposed with 1-sigma (dashed line) and 2-sigma ellipse (solid line), and mean value shown by the black circle. (a): Results of the 2-term approximation with 0 to 30 angle range by the Zoeppritz equation. (b) Results of the 3-term approximation with 0 to 30 angle range by the Zoeppritz equation. (c): Results of the Aki and Richards equation. The colour of (a), (b) and (c) shows the data density. (d): Angle-range comparison of the 2-

sigma ellipses (solid lines) and the mean values (the circles) for the 2-term approximation by the Zoeppritz equation, 0 to 20, 0 to 25, 0 to 30, 0 to 35, 0 to 40, 0 to 45 and 0 to 50 degrees. The black line of the 2-sigma ellipse and circle of the mean value are from the Aki and Richards equation. (e): Angle-range comparison for the 3-term approximation by the Zoeppritz equation. The legend is same as (d). (f): Comparison of the 2-sigma ellipses and the mean values between (a), (b) and (c).

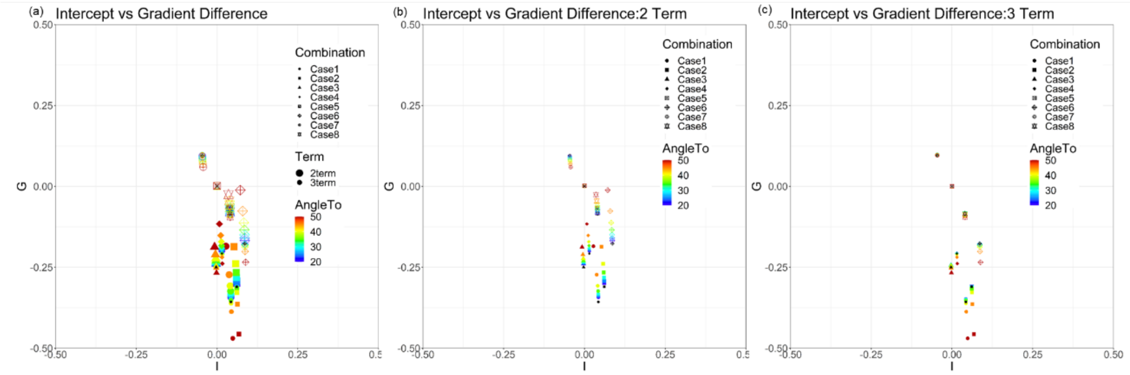

Figure 3. Comparison of the simulated average intercept and gradient of (a) both the 2-term and 3-term approximation, (b) the 2-term and (c) the 3-term.

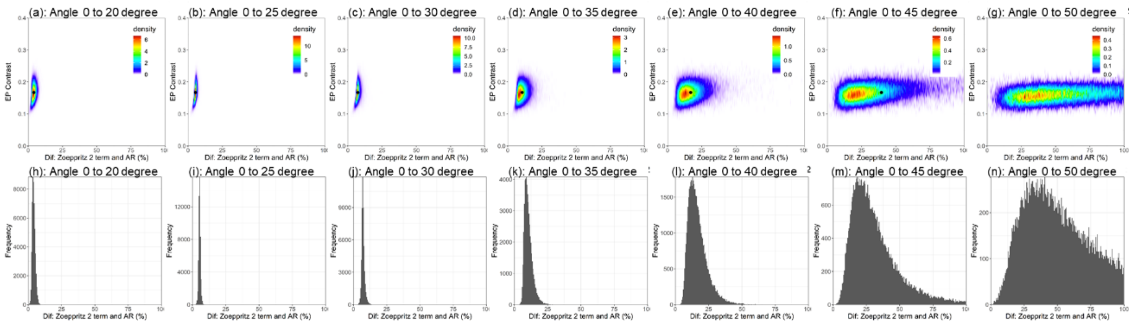

Figure 4. 2D histograms (a to g) between the elastic property contrast and the difference between the simulated gradient of the Aki and Richards equation and the 2-term approximation by the Zoeppritz equation with Case 1, and histograms (h to n) of the difference comparing between angle range used for the approximation between (a)(h) 0 to 20 degrees, (b)(i) 0 to 25 degrees, (c)(j) 0 to 30 degrees, (d)(k) 0 to 35 degrees, (e)(l) 0 to 40 degrees, (f)(m) 0 to 45 degrees, and (g)(n) 0 to 50 degrees. The black circle in each figure shows the mean value.

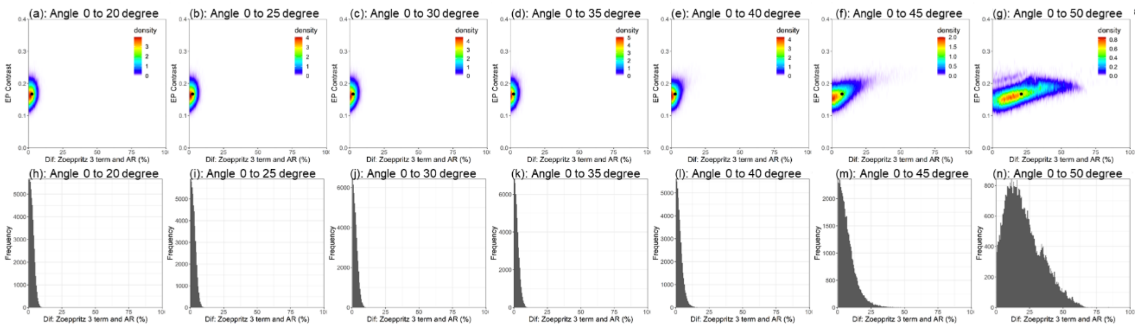

Figure 5. 2D histograms (a to g) between the elastic property contrast and the difference between the simulated gradient of the Aki and Richards equation and the 3-term approximation by the Zoeppritz equation with Case 1, and histograms (h to n) of the difference comparing between angle range used for the approximation between (a)(h) 0 to 20 degrees, (b)(i) 0 to 25 degrees, (c)(j) 0 to 30 degrees, (d)(k) 0 to 35 degrees, (e)(l) 0 to 40 degrees, (f)(m) 0 to 45 degrees, and (g)(n) 0 to 50 degrees. The black circle in each figure shows the mean value.

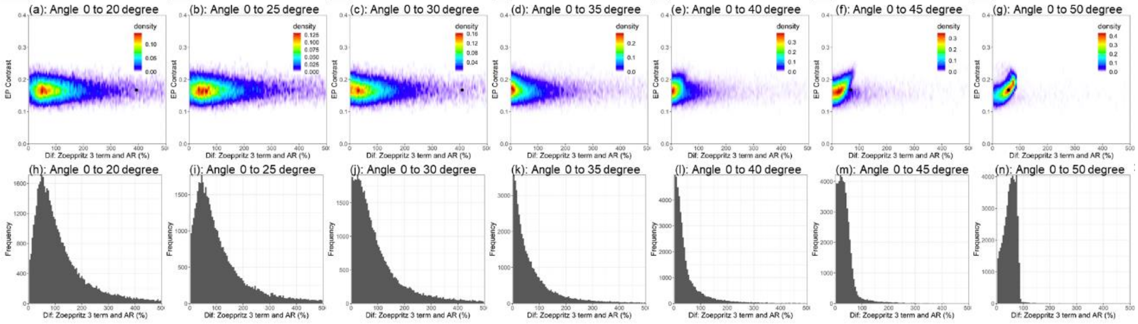

Figure 6. 2D histograms (a to g) between the elastic property contrast and the difference between the simulated curvature of the Aki and Richards equation and the 3-term approximation by the Zoeppritz equation with Case 1, and histograms (h to n) of the difference comparing between angle range used for the approximation between (a)(h) 0 to 20 degrees, (b)(i) 0 to 25 degrees, (c)(j) 0 to 30 degrees, (d)(k) 0 to 35 degrees, (e)(l) 0 to 40 degrees, (f)(m) 0 to 45 degrees, and (g)(n) 0 to 50 degrees. The black circle in each figure shows the mean value.

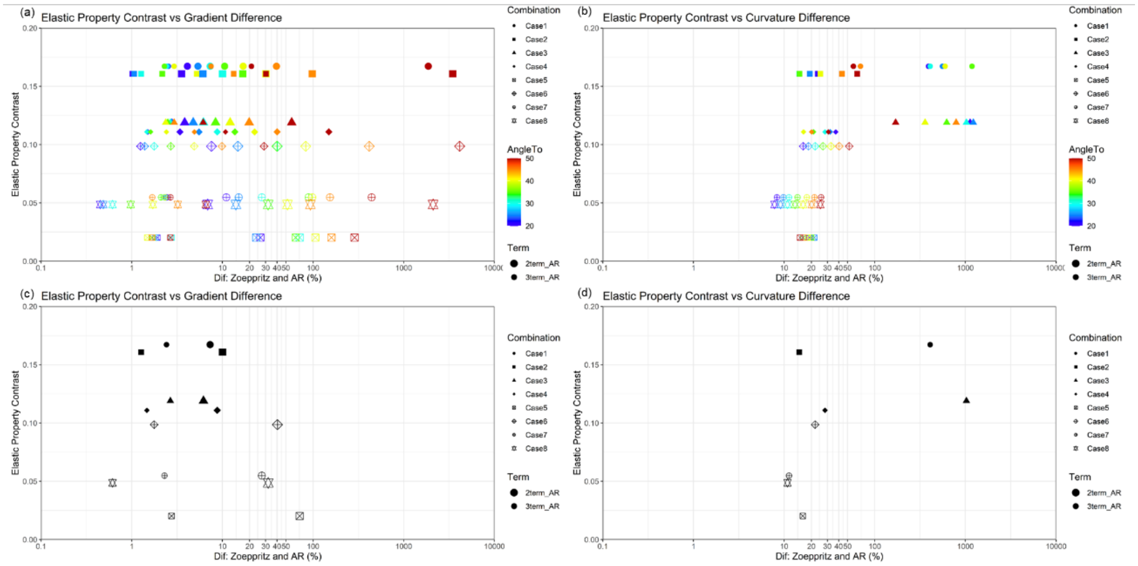

Figure 7. Comparison of the simulated average (a) gradient and (b) curvature difference of all angles, and (c) gradient and curvature difference of the angle range of 30 degrees against the elastic property contrast.

CONCLUSIONS

In-depth analysis of an AVO approximation, their impact of reflectivity, gradient and curvature (for 3-term only) conducted in Browse Basin using the publicly available rock physics model. The study indicates smaller deviation of angle dependent reflectivity is observed when 3-term approximation is used than that of 2-term. Smaller gradient errors are obtained with using 2 term approximation only when the angle range is limited within 20 degrees. In regard to the error in the gradient induced due to Aki Richard approximation, it is preferred to use either 3-term approximation or use of 2-term approximation with smaller angle range around 20 degrees. Error correction or compensation can also be considered to obtain true angle reflectivity from AVO approximations.

ACKNOWLEDGMENTS

I would like to be grateful for INPEX Corporation to permit publishment of this study. Also, I would like to thank Mr. Shuichi Desaki, Dr. Yuki Kobayashi and Iftikhar Arsalan Syed for fruitful discussion and contribution.

REFERENCES

Aki, K., and Richards, P.G., 1980, Quantitative seismology: Second Edition, University Science Books.

Avseth, P., Mukerji, T., & Mavko, G., 2005, Quantitative seismic interpretation: Applying rock physics tools to reduce interpretation risk. Cambridge university press.

Chopra, S., and Castagna, J., P., 2014, AVO. Investigatiosin Geophysics. No. 16. SEG.

Causse, E., Riede, M., van Wijngaarden, A. J., Buland, A., Dutzer, J. F., Fillon, R., 2007a, Amplitude analysis with an optimal model-based linear AVO approximation: Part I — Theory. Geophysics, 72(3), C59-C69.

Causse, E., Riede, M., van Wijngaarden, A. J., Buland, A., Dutzer, J. F., Fillon, R., 2007b, Amplitude analysis with an optimal model-based linear AVO approximation: Part II — Field data example. Geophysics, 72(3), C71-C79.

Desaki, S., Kobayashi, Y., and Miklavs, P., 2019, Statistical Rock Physics Analysis and Modelling in The Browse Basin. AEGC Annual Conference.

Hasse, A., B., 2004, Modelling of linearized Zoeppritz approximations. CREWES Research Report 16.

Li, Y., Downton, J., and Xu, Y., 2007, Practical aspects of AVO modeling, The Leading Edge, 26(3), 249-384.

Riede, M., Causse, E., van Wijngaarden, A.J., Buland, A., J.F. Dutzer, R. Fillon, 2005, Optimized AVO analysis by using an optimal linear approximation.

Wang, Y., 1999, Approximations to the Zoeppritz equations and their use in AVO analysis. Geophysics, 64(6), 1920-1927

Zoeppritz, K., B., 1919, VIIb. Über Reflexion und Durchgang seismischer Wellen durch Unstetigkeitsflächen.