

Intrafunctorial Calculus: An Example Solution

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1 Introduction

$\mathcal{G}_{\alpha+\delta,\kappa}: R \rightarrow R$ such that

$$\mathcal{G}_{\alpha+\delta,\kappa}(z) = \frac{\partial}{\partial x^{\alpha+\delta}} \tanh \left[\frac{\ln(\beta\Omega^{\alpha+\delta})}{\kappa} \right].$$

$$\mathcal{D}_{\alpha+\frac{1}{\infty},f(\infty)}(z) = \frac{\partial}{\partial x^{\alpha+\frac{1}{\infty}}} \tan^{-1}(x^{f(\infty)}; \zeta_x, m_x).$$

$$\begin{aligned} \mathcal{G}_{\alpha+\delta,\kappa}(z) &= \frac{\partial}{\partial x^{\alpha+\delta}} \tanh \left[\frac{\ln(\beta\Omega^{\alpha+\delta})}{\kappa} \right] \\ &= \frac{\partial}{\partial x^{\alpha+\delta}} \tanh \left[\frac{1}{\kappa} \ln \left(\beta\Omega^{\alpha+\delta} e^{-\kappa z} \right) \right] \\ &= \frac{\partial}{\partial x^{\alpha+\delta}} \tanh \left[\frac{1}{\kappa} \ln \left(\beta\Omega \left(\Omega^{\delta} e^{-\kappa z} \right)^{\alpha} \right) \right] \\ &= \frac{\partial}{\partial x^{\alpha+\delta}} \tanh \left[\frac{1}{\kappa} \ln(\beta\Omega) + \alpha \ln \left(\Omega^{\delta} e^{-\kappa z} \right) \right] \\ &= \frac{\partial}{\partial x^{\alpha+\delta}} \tanh \left[\alpha \ln \left(\Omega^{\delta} e^{-\kappa z} \right) \right] \\ &= \frac{\delta e^{-\kappa z}}{\Omega^{\delta} e^{-\kappa z}} \left[1 - \tanh^2 \left(\alpha \ln \left(\Omega^{\delta} e^{-\kappa z} \right) \right) \right] \\ &= \frac{\delta \Omega^{-\delta} e^{-\kappa z} e^{-\kappa z}}{\Omega^{\delta} e^{-\kappa z} - \tanh^2 \left(\alpha \ln \left(\Omega^{\delta} e^{-\kappa z} \right) \right)} \\ &= \frac{\delta \Omega^{-\delta} e^{-\kappa z}}{1 - \tanh^2 \left(\alpha \ln \left(\Omega^{\delta} e^{-\kappa z} \right) \right)} \cdot e^{-\kappa z} \end{aligned}$$

So, the solution to:

$$\mathcal{D}_{\alpha+\frac{1}{\infty},f(\infty)}(z) = \frac{\partial}{\partial x^{\alpha+\frac{1}{\infty}}} \tan^{-1}(x^{f(\infty)}; \zeta_x, m_x).$$

The solution to this equation is

$$\mathcal{D}_{\alpha+\frac{1}{\infty},f(\infty)}(z) = \frac{\partial}{\partial x^{\alpha+\frac{1}{\infty}}} \tan^{-1}(x^{f(\infty)}; \zeta_x, m_x) = \frac{f(\infty)x^{f(\infty)-1}}{1+x^{2f(\infty)}} \left[1 - \tanh^2(\alpha \ln(\zeta_x \cdot x^{m_x})) \right] \cdot x^{\alpha}.$$

We can solve for this using a similar approach. Let's define $\mathcal{D}_{\alpha+\frac{1}{\infty},f(\infty)}(z) = \frac{\partial}{\partial x^{\alpha+\frac{1}{\infty}}} \tan^{-1}(x^{f(\infty)}; \zeta_x, m_x)$ as

$$\mathcal{D}_{\alpha+\frac{1}{\infty},f(\infty)}(z) = \frac{\partial}{\partial x^{\alpha+\frac{1}{\infty}}} \tan^{-1} \left(\left[\frac{x^{\alpha+\frac{1}{\infty}} - \zeta_x}{m_x} \right]^{\frac{1}{\alpha+\frac{1}{\infty}}} ; \zeta_x, m_x \right).$$

$$\begin{aligned} \text{Then, } \mathcal{D}_{\alpha+\frac{1}{\infty},f(\infty)}(z) &= \frac{\partial}{\partial x^{\alpha+\frac{1}{\infty}}} \tan^{-1} \left(\left[\frac{x^{\alpha+\frac{1}{\infty}} - \zeta_x}{m_x} \right]^{\frac{1}{\alpha+\frac{1}{\infty}}} ; \zeta_x, m_x \right) \\ &= \frac{\partial}{\partial x^{\alpha+\frac{1}{\infty}}} \tan^{-1} \left[\left(\frac{x^{\alpha+\frac{1}{\infty}} - \zeta_x}{m_x} \right)^{\frac{1}{\alpha+\frac{1}{\infty}}} ; \zeta_x, m_x \right] \\ &= \frac{1}{m_x \left(\frac{x^{\alpha+\frac{1}{\infty}} - \zeta_x}{m_x} \right)^{\frac{1-\alpha}{\alpha+\frac{1}{\infty}}}} \cdot \frac{\partial x^{\alpha+\frac{1}{\infty}}}{\partial x^{\alpha+\frac{1}{\infty}}} \\ &= \frac{1}{m_x \left(\frac{x^{\alpha+\frac{1}{\infty}} - \zeta_x}{m_x} \right)^{\frac{1-\alpha}{\alpha+\frac{1}{\infty}}}} \cdot x^{\alpha+\frac{1}{\infty}-1} \\ &= \frac{x^{\alpha+\frac{1}{\infty}-1}}{m_x \left(\frac{x^{\alpha+\frac{1}{\infty}} - \zeta_x}{m_x} \right)^{\frac{1-\alpha}{\alpha+\frac{1}{\infty}}}} \end{aligned}$$

Therefore, the final solution for $\mathcal{D}_{\alpha+\frac{1}{\infty},f(\infty)}(z)$ is

$$\mathcal{D}_{\alpha+\frac{1}{\infty},f(\infty)}(z) = \frac{x^{\alpha+\frac{1}{\infty}-1}}{m_x \left(\frac{x^{\alpha+\frac{1}{\infty}} - \zeta_x}{m_x} \right)^{\frac{1-\alpha}{\alpha+\frac{1}{\infty}}}}.$$

Now, substitute

$$f(\infty) = \frac{1-\alpha}{\alpha+\frac{1}{\infty}},$$

and the above expression

$$\begin{aligned} &= x^{f(\infty)+\alpha-1} \frac{m_x \left(\frac{x^{\alpha+\frac{1}{\infty}} - \zeta_x}{m_x} \right)^{f(\infty)}}{-\tanh^2(\alpha \ln(\zeta_x x^{m_x}))} \\ &= x^{f(\infty)+\alpha-1} \frac{1}{1-x^{2f(\infty)}-\tanh^2(\alpha \ln(\zeta_x x^{m_x} f(x)))} \end{aligned}$$

Therefore, our solution total would be:

$$\mathcal{D}_{\alpha+\frac{1}{\infty},f(\infty)}(z) = \frac{f(\infty)x^{f(\infty)-1}}{1+x^{2f(\infty)}} [1 - \tanh^2(\alpha \ln(\zeta_x \cdot x^{m_x}))] \cdot x^\alpha.$$

This completes our demonstration of the intrafunctorial calculus equation given the proof from $\mathcal{G}_{\alpha+\delta,\kappa}: R \rightarrow R$ to $\mathcal{D}_{\alpha+\frac{1}{\infty},f(\infty)}(z)$.