Modelling of Cosmic-ray Transport in Galaxies

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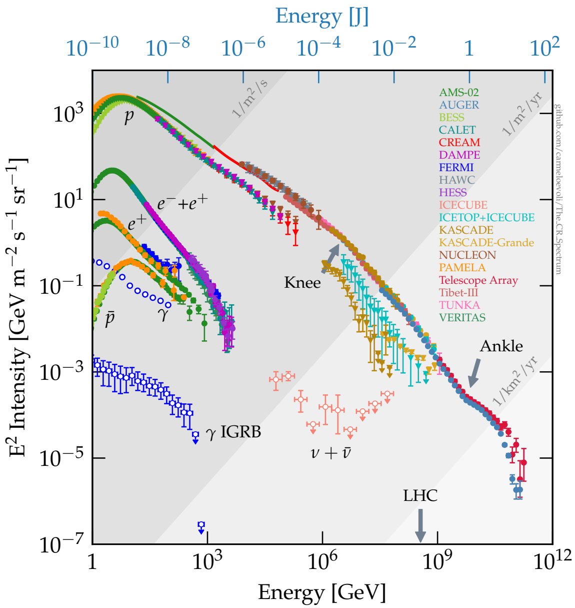
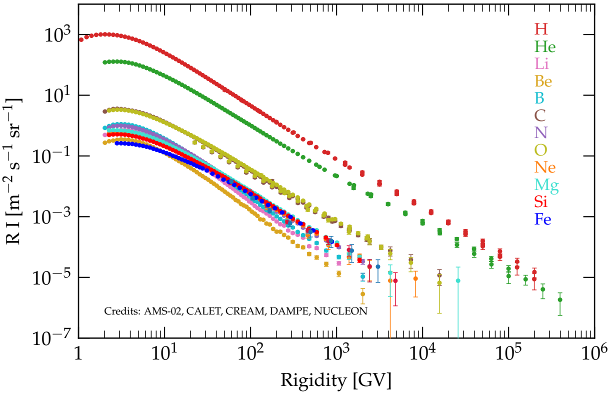
The cosmic-ray spectrum in 2023

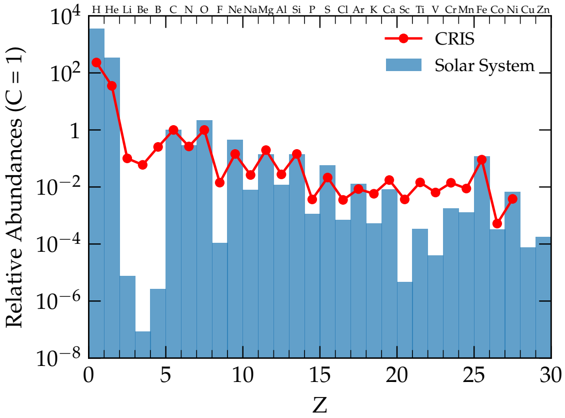


Galactic Cosmic Rays: unprecedented measurements



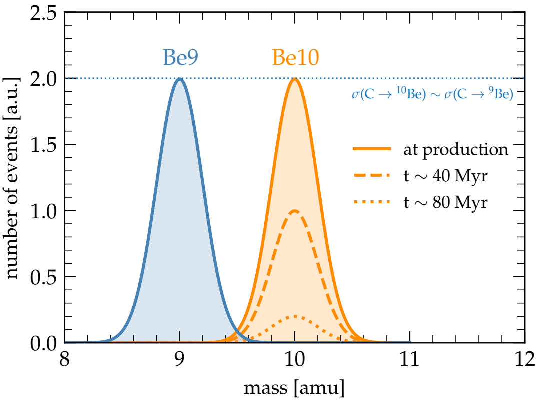
- Amazing new data: The spectrum of each isotope includes contributions from many different parents (both in terms of fragmentation and decays) giving to each observed isotope a potentially very complex history
- In these lectures we will mainly focus on our Galaxy as we take advantage of far more information than any other environment, bearing in mind that the very same physical picture can be straightforwardly applied to the vast majority of astrophysical territories.

Basic indicators of diffusive transport: stable elements



- ▷ Thermal particles in the average interstellar medium are somehow accelerated to relativistic energies becoming CRs ⟶ primary CRs
- ▷ It must exist also a second population which is produced during propagation by primary fragmentation ⟶ secondary CRs

Basic indicators of diffusive transport: unstable elements



- ^{10}Be is a β^- unstable isotope decaying in ^{10}B with an half-life of ~ 1.5 Myr
- Similar production rates than other (stable) isotopes $\sigma_{\rm Be9} \sim \sigma_{\rm Be10}$
- Traditionally ^{9}Be/^{10}Be has been used as CR clock pointing to a residence time of $\mathcal{O}(100)$ Myr

Basic definitions: The grammage pillar

- The grammage X is the amount of material that the particle go trough along propagation (a sort of column density):

$$X = \int dl \rho(l)$$

- I assume a simple system with one primary species n_p and one secondary n_s only.
- The evolution of primary and secondary along the grammage trajectory is given respectively by:

$$\begin{aligned} \frac{dn_p}{dX} &= -\frac{n_p}{\lambda_p} \\ \frac{dn_s}{dX} &= -\frac{n_s}{\lambda_s} + P_{p \to s} \frac{n_p}{\lambda_p} \end{aligned}$$

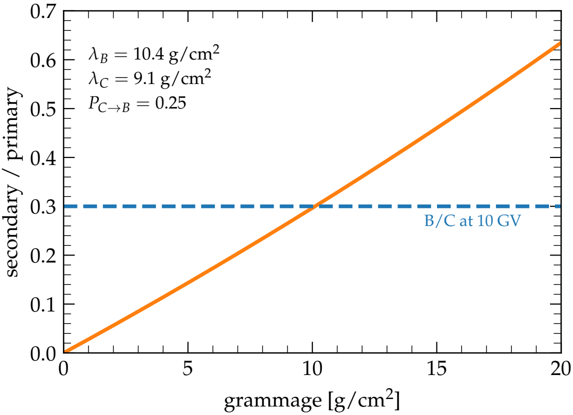
  with initial conditions $n_p(X = 0) = n_0$ and $n_s(X = 0) = 0$, where λ_i are some kind of interaction lenght (probability) and P is the fraction resulting in that specific channel.

- Solving this, I can get n_s/n_p in terms of X, λ_s and λ_p only:

$$\frac{n_s}{n_p} = P_{p \to s} \frac{\lambda_s}{\lambda_s - \lambda_p} \left[\exp\left(-\frac{X}{\lambda_s} + \frac{X}{\lambda_p} \right) - 1 \right]$$

$\longrightarrow$ I quantify the transport process, whatever it is, in something that can be either directly measured in CRs n_s/n_p or provided by a nuclear physics experiment (λ's, P's).

Basic definitions: The grammage pillar



$$\text{B/C} \sim 0.3 \longrightarrow \chi = 10\,\text{g/cm}^2$$

Basic definitions: The grammage pillar

- Let me assume that the grammage is accumulated in the gas disc of our Galaxy
- At each crossing of the disc $n_{\rm gas} \sim 1~{\rm cm}^{-3}$, $h \sim 200$ pc:

$$\chi_d \sim m_p n_{\rm gas} h_d \sim 10^{-3}~{\rm g/cm}^2 \ll \chi_{\rm B/C}$$

- The grammage accumulated in one crossing is clearly inconsistent with the grammage we estimate from CR measurements $\rightarrow$ the particles have to cross the disk many times
- The time spent in the gas region before escaping the Galaxy must be not less than:

$$t_{\rm esc,min} \sim \frac{\chi_{\rm B/C}}{\chi_d}\frac{h}{v} \sim 7 \times 10^6~{\rm years} \gg \frac{\rm kpc}{c}$$

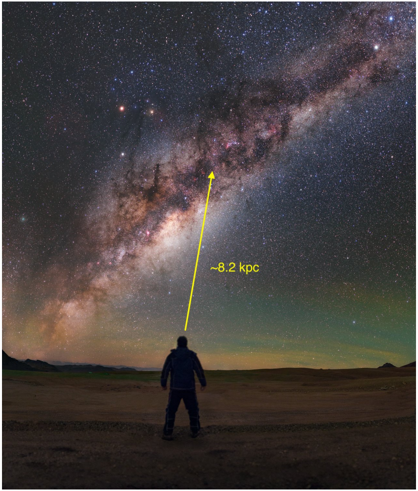
which exceeds by order of magnitudes any possible ballistic timescale in the MW $\sim \mathcal{O}(\frac{\rm kpc}{c})$

- We deduce that CRs follow something more similar to a Brownian motion in the Galaxy

Open question

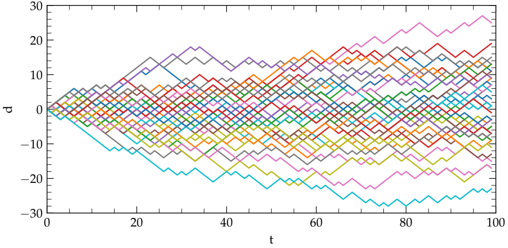
What is the origin of confinement of these particles in the Galaxy?

Basic definitions: The grammage pillar



Wanderer above the Sea of Stars

Basic definitions: random walk and diffusion coefficient



- After N steps $\vec{\lambda}_i$ of the same size $\|\lambda_i\| = \lambda$ and random direction a particle has reached a distance:

$$\vec{d} = \sum_{i=1}^{N} \vec{\lambda}_i$$

- The scalar product of d with itself is

$$\vec{d} \cdot \vec{d} = \sum_{i=1}^{N} \sum_{j=1}^{N} \vec{\lambda}_i \cdot \vec{\lambda}_j \longrightarrow d^2 = N\lambda^2 + 2\lambda^2 \sum_{i=1}^{N} \sum_{j<1}^{N} \cos\theta_{ij} \sim N\lambda^2$$

as we assumed that the angles θ_{ij} are chosen randomly and thus the off-diagonal terms are uncorrelated.

Basic definitions: random walk and diffusion coefficient

- The continuity equation for the number density n and its current $\vec{j}$ reads

$$\frac{\partial n}{\partial t} + \nabla \vec{j} = q$$

  assuming q to be the sum of all sources or losses.

- Combined together with Fick's law for an isotropic flux $\vec{j} = -D \nabla n$ leads to the diffusion equation:

$$\frac{\partial n}{\partial t} - \nabla (D \nabla n) = q$$

- The propagator (Green function) of the 1D diffusion equation with constant D is

$$G(d) = \frac{1}{(4\pi D t)^{1/2}} \mathrm{e}^{-\frac{d^2}{4Dt}}$$

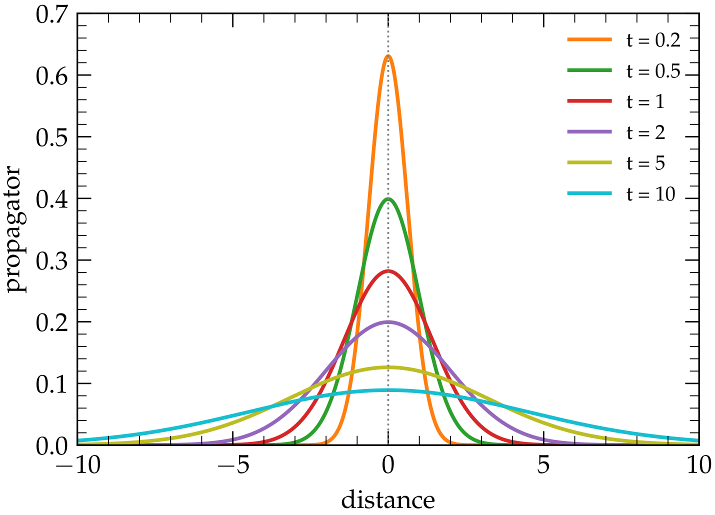
  thus the mean distance traveled outward is $\propto \sqrt{Dt}$

- Connecting the two pictures we obtain that D is the product of particle velocity v and mean free path λ:

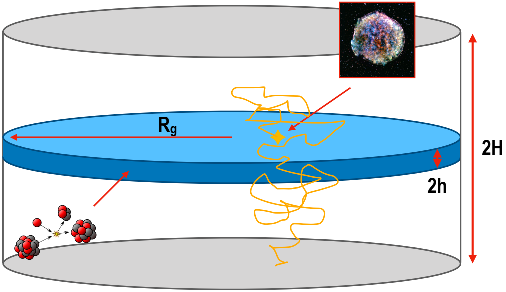
$$\boxed{D \sim \frac{N\lambda^2}{t} \sim \frac{v\lambda}{3}}$$

  where the numerical factor is obtained in 3D with a more accurate derivation.

Basic definitions: random walk and diffusion coefficient

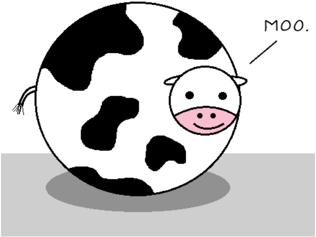


A toy model for protons in our Galaxy: main assumptions



- In the standard model for the origin of Galactic CRs, these particles are accelerated in the disc $h \sim 100$ pc with an injected spectrum $q_p \propto p^{-\gamma}$ where $\gamma \gtrsim 4$
- after injection, CRs propagate diffusively throughout the Galactic halo $H \sim \mathcal{O}(\text{kpc})$ with a diffusion coefficient $D \propto p^{\delta}$ where $\delta \sim 1/3 - 1/2$ and free escape at the boundaries
- $R_{\rm g} \gg H$ is the radius of the Galactic disc $\to$ 1D problem
- Secondary production, e.g. LiBeB, takes place predominantly in the disc h where all the gas is confined.

A toy model for protons in our Galaxy: main assumptions



$$\frac{\partial}{\partial z}\left(j_{\text{diff}} + j_{\text{adv}} + \dots\right) = \boxed{\text{sources}} - \boxed{\text{losses}}$$

e.g., SNRs, fragmentation, decay, ionization...

A toy model for protons in our Galaxy

- The simplest transport equation for protons, assuming relativistic particles $p \simeq E$:

$$-\frac{\partial}{\partial z}\left[D(E)\frac{\partial n_p}{\partial z}\right] = Q(E,z) = \frac{\xi E_{\rm SN}\mathcal{R}_{\rm SN}}{\pi R_d^2} q_0(E)\delta(z)$$

where $n_p(E)$ is the cosmic ray density, $E_{\rm SN} \simeq 10^{51}$ erg is the SN kinetic energy converted to proton with efficiency ξ, and $\mathcal{R}_{\rm SN} \simeq 1/100\ {\rm yr}^{-1}$ is the SN galactic rate.

- For $z \neq 0$, and using the boundary condition $n_p(z = \pm H, E) = 0$:

$$j_{\rm diff} = D\frac{\partial n_p}{\partial z} = \text{Constant} \longrightarrow n_p(z) = n_0\left(1 - \frac{z}{H}\right)$$

- Since the diffusive flux is constant in z, in particular at the disc $z = 0$:

$$D\frac{\partial n_p}{\partial z}\bigg|_{z=0+} = -D\frac{n_{p,0}}{H}$$

- We now integrate the diffusion equation around $z = 0$

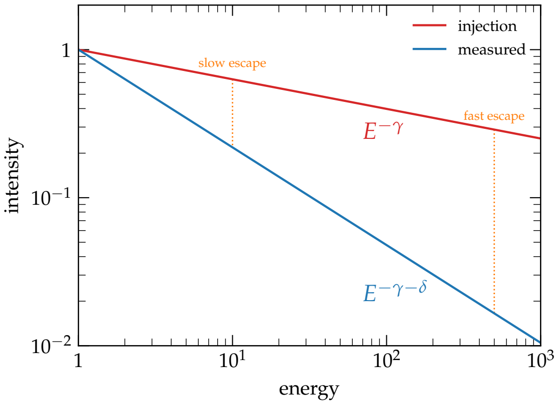
$$\lim_{\epsilon\to 0}\int_{\epsilon^-}^{\epsilon^+} dz \left\{-\frac{\partial}{\partial z}\left[D\frac{\partial n_p}{\partial z}\right] = Q(E,z)\right\} \longrightarrow -2D\frac{\partial n_p}{\partial z}\bigg|_{z=0+} = \frac{E_{\rm SN}\mathcal{R}_{\rm SN}}{\pi R_d^2} q_0(E)$$

- and using the equation for the flux:

injection rate per unit volume | escape rate

$$n_p(E) = \frac{E_{\rm SN}\mathcal{R}_{\rm SN} q_0(E)}{2\pi R_d^2}\frac{H}{D(E)} = \boxed{\frac{E_{\rm SN}\mathcal{R}_{\rm SN} q_0(E)}{2\pi R_d^2 H}}\boxed{\frac{H^2}{D(E)}} \propto E^{-\gamma-\delta}$$

A toy model for protons in our Galaxy

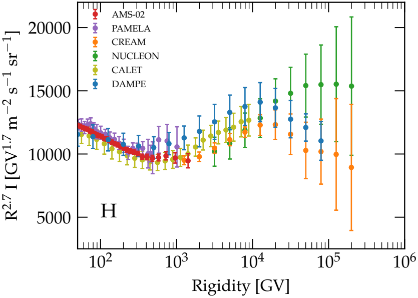


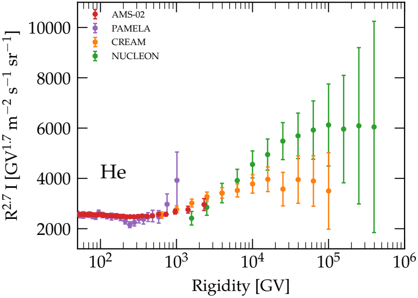
Open question

Having evidence of a feature in the proton spectrum, how to distinguish if due to injection or propagation?

Galactic Cosmic Rays: novel features

PAMELA Coll., Science 2011; AMS-02 Coll., PRL 2015; CREAM Coll., ApJ 2017; NUCLEON Coll., JETP 2018; DAMPE Coll., Science 2019





- Spectra of protons and helium are not a single power law below the knee $\rightarrow$ some physics kicking in?
- The hardening at $R = p/Z \sim 300 - 400$ GV is well established since first observation by PAMELA
- AMS-02 confirmed the same break for almost all nuclei
- The softening at $R = p/Z \sim 10$ TV is observed by different experiments, first strong evidence in DAMPE

The cosmic ray density

- Cosmic rays come from all directions in outer space over large energy intervals.
- The number of particles in volume element d^3r about $\vec{r}$ and in the momentum interval d^3p about $\vec{p}$ is given by

$$dn = F(\vec{r}, \vec{p}, t)\, d^3r\, d^3p$$

  with F the distribution function.
- Expanding in spherical coordinates d^3p:

$$dn = F(\vec{r}, p, t)\, d^3r\, p^2\, dp d\Omega$$

- Typically we are not able to measure F but only averages over momentum space, thereby we conveniently introduce the phase-space distribution function as:

$$f(\vec{r}, p, t) = \frac{1}{4\pi} \int_\Omega F(\vec{r}, p, t) d\Omega$$

- Correspondingly, the number of particles dN in $d^3\, r$ and in $(p, p + dp)$ (independent of direction of $\vec{p}$) is:

$$dN = \int_\Omega d\Omega\, F(\vec{r}, p, t)\, d^3r\, p^2\, dp = 4\pi p^2 f(\vec{r}, p, t)\, d^3r\, dp$$

Description of transport of nuclei

- For nuclei of mass A, it is customary to introduce the **intensity** (number of particles per unit surface time solid angle and energy) as a function of the **kinetic energy per nucleon T**:

$$I_\alpha(T)dT = p^2 f_\alpha(p) v(p) dp \longrightarrow I_\alpha(T) = Ap^2 f_\alpha(p)$$

> **!** fragmentation preserves the energy per nucleon

- I is the quantity to be directly compared with measurements

- We explicit $q_{\rm SN} = 2h_d \delta(z) q_\alpha(p)$ and $(\tau_\alpha^{\rm in})^{-1} = 2h_d\delta(z) n_d v \sigma_\alpha$, thereby the transport equation becomes:

diffusion — spallation of nuclei α

$$-\frac{\partial}{\partial z}\left[D_\alpha \frac{\partial I_\alpha(T)}{\partial z}\right] + 2h_d n_d v(T) \sigma_\alpha \delta(z) I_\alpha(T) =$$

injection of nuclei α — contribution to nuclei α from spallation of $\alpha' > \alpha$

$$= 2Ap^2 h_d q_{0,\alpha}(p)\delta(z) + \sum_{\alpha' > \alpha} 2h_d n_d v(T) \sigma_{\alpha' \to \alpha} \delta(z) I_{\alpha'}(T)$$

The transport equation for primary Nuclei

- Formally similar to the equation for protons but with spallation taken into account:

$$-\frac{\partial}{\partial z}\left[D_\alpha \frac{\partial I_\alpha(T)}{\partial z}\right] + 2h_d n_d v(T)\sigma_\alpha \delta(z) I_\alpha(T) = 2Ap^2 h_d q_{0,\alpha}(p)\delta(z)$$

- The equation is solved in the same way:
  - first we consider the solution for $z \neq 0$ ($z > 0$ or $z < 0$)
  - then integrate around $z = 0$ between 0^- and 0^+
- It follows:

$$D_\alpha \frac{\partial I_\alpha}{\partial z} = \text{constant} \longrightarrow I_\alpha = I_{0,\alpha}\left(1 - \frac{z}{H}\right)$$

which we use to derive

$$-D_\alpha \frac{\partial I_\alpha}{\partial z}\bigg|_{z=0} = -h_d n_d v(T)\sigma_\alpha I_{0,\alpha} + Ap^2 h_d q_{0,\alpha}(p)$$

The transport equation for primary Nuclei

- The intensity of a primary nucleus of type α is

$$I_{0,\alpha}(T) = \frac{\frac{Ap^2 h_d q_{0,\alpha}(p)}{H} \frac{H^2}{D_\alpha}}{1 + \frac{X_\alpha(T)}{\hat{X}_\alpha}} = \frac{\frac{Ap^2 q_{0,\alpha}(p)}{n_d m_p v} X_\alpha(T)}{1 + \frac{X_\alpha(T)}{\hat{X}_\alpha}}$$

- Where the grammage traversed by nuclei of type α:

$$X_\alpha(T) = n_d \left(\frac{h}{H}\right) m_p v \frac{H^2}{D_\alpha} = \bar{n} m_p v \tau_{\rm esc}(T)$$

- and the critical grammage (energy independent) is:

$$\hat{X}_\alpha = \frac{m_p}{\sigma_\alpha}$$

- Relevant limits:

diffusion dominated: for $X \ll \hat{X}$ the equilibrium spectrum is $I_0 \propto T^{-\gamma-\delta}$

spallation dominated: for $X \gg \hat{X}$ the equilibrium spectrum is $I_0 \propto T^{-\gamma}$

The transport equation for secondary Nuclei: secondary/primary ratio

- Let's work out a simple case with only Carbon as primary species $\alpha' = \mathrm{C}$, and Boron as secondary $\alpha = \mathrm{B}$:

destruction of B — production of B from C spallation

$$-\frac{\partial}{\partial z}\left[D_{\mathrm{B}}\frac{\partial I_{\mathrm{B}}(T)}{\partial z}\right] + 2h_d n_d v \sigma_{\mathrm{B}} \delta(z) I_{\mathrm{B}}(T) = 2h_d n_d v \sigma_{\mathrm{C}\to\mathrm{B}} \delta(z) I_{\mathrm{C}}(T)$$

- following the same approach as before (and assuming $X_{\mathrm{B}} \simeq X_{\mathrm{C}} \equiv X$):

$$I_{\mathrm{B},0}(T) = I_{\mathrm{C},0}(T)\frac{X(T)}{\hat{X}_{\mathrm{C}\to\mathrm{B}}}\left(1 + \frac{X(T)}{\hat{X}_{\mathrm{B}}}\right)^{-1}$$

- which reflects in the following B/C ratio:

$$\frac{\mathrm{B}}{\mathrm{C}} = \frac{\frac{X(T)}{\hat{X}_{\mathrm{C}\to\mathrm{B}}}}{1 + \frac{X(T)}{\hat{X}_{\mathrm{B}}}}$$

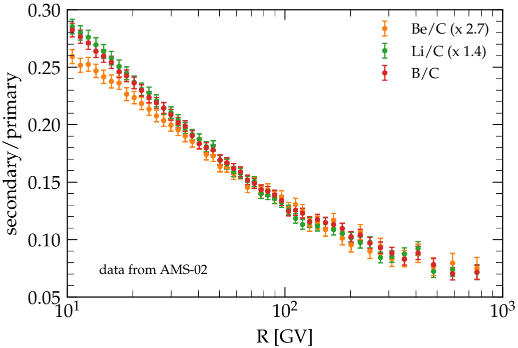
- Relevant limits:

diffusion dominated: for $X \ll \hat{X}$ the ratio is B/C $\propto X(T) \propto 1/D(T)$

spallation dominated: for $X \gg \hat{X}$ the ratio is B/C $\sim$ constant

The transport equation for secondary Nuclei: the diffusion coefficient slope

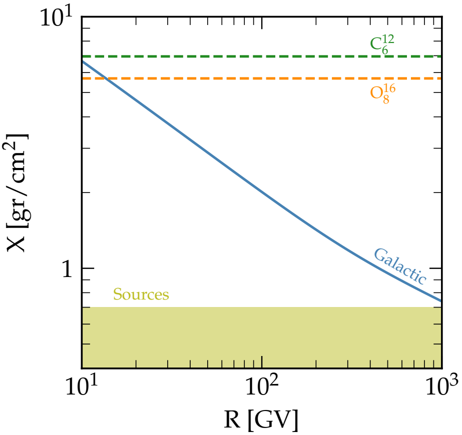
AMS-02 Coll., PRL 120 (2018)



- ▷ Evidence of rigidity dependent grammage $\longrightarrow$ high-energy particles spend less time in our Galaxy than low-energy ones (advection may play a role only at low energies)
- ▷ At $T \gtrsim 50$ GeV/n the B/C ratio scales as the grammage $\longrightarrow$ we can measure the slope of $D(E)$ from the energy dependence of B/C.
- ▷ Notice however that B/C is sensitive only to the H/D ratio, remember $\chi = n_d m_p v h \frac{H}{D_\alpha}$!

The transport equation for primary Nuclei

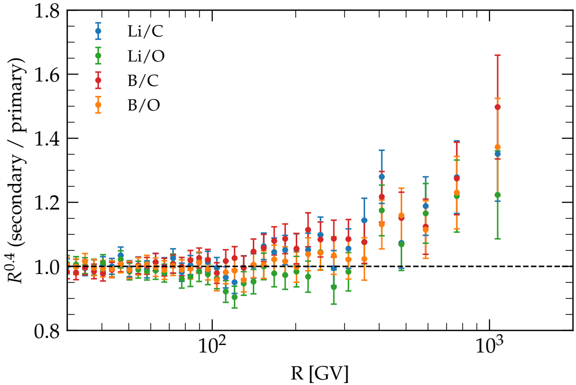
Evoli et al., PRD 101 (2020)



- Since $\hat{X}_{\rm B,C}$ are independent on energy and $X(E)$ is a decreasing function of energy we can distinguish the regime where spallation dominates $\lesssim 10$ GeV/n and the regime where diffusion dominates $\gtrsim 10$ GeV/n.
- Reference value $\sigma^{\rm in} \sim 45 A^{0.7}$ mb

The transport equation for secondary Nuclei: the origin of the spectral feature

AMS-02 Coll., PRL 120 (2018)



the same feature detected in the primary spectra is observed in the secondary/primary ratio which depends only on the grammage $\longrightarrow$ propagation effect

The transport equation

Parker, Planet. Space Sci. (1965); Ginzburg & Syrovatskii (1964); Berezinskii et al. (1980)

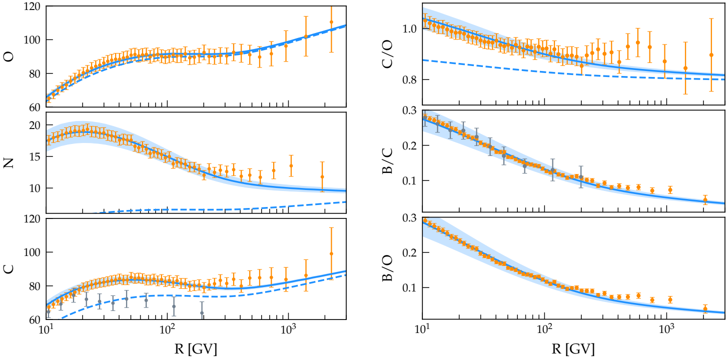
The transport of a CR species $\alpha = \mathrm{H}^1, \mathrm{He}^4, \mathrm{C}^{12}, ..., \mathrm{Fe}^{56}$ is well described by an advection-diffusion equation with losses for :

$$\cancelto{0}{\frac{\partial f_\alpha}{\partial t}} - \frac{\partial}{\partial z}\left(D\frac{\partial f_\alpha}{\partial z}\right) + u\frac{\partial f_\alpha}{\partial z} - \frac{du}{dz}\frac{p}{3}\frac{\partial f_\alpha}{\partial p} = q_{\mathrm{SN}}\delta(z) - \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2\dot{p}f_\alpha\right] - \frac{f_\alpha}{\tau_\alpha^{\mathrm{in}}} + \sum_{\alpha'>\alpha} b_{\alpha'\alpha}\frac{f_{\alpha'}}{\tau_{\alpha'}^{\mathrm{in}}}$$

- Stationarity is ensured by proper boundary conditions $f_\alpha(z = \pm H) = 0$
- Spatial diffusion: $\vec{\nabla} \cdot \vec{J}$
- Advection by Galactic winds/outflows: $u = u_w + v_A \sim v_A$
- Source term proportional to Galactic SN rate $\mathcal{R}$: $q_{\mathrm{SN}} \propto \frac{E_{\mathrm{SN}}\mathcal{R}}{\pi R_g^2} q_\alpha(p)$
- Energy losses: ionization, Coulomb losses, Inverse Compton, Synchrotron, ...
- Production/destruction of nuclei due to inelastic scattering (or decay) $\rightarrow \sigma_\alpha^{\mathrm{in}}$ is the inelastic cross-section , $b_{\alpha'\alpha}$ is the fragmentation branching ratio

Quick look at the data: The CNO element

Evoli et al., PRD 101 (2020), Weinrich et al., A&A 639 (2020), De La Torre Luque et al., JCAP 03 (2021), Schroer et al., PRD 103 (2021)

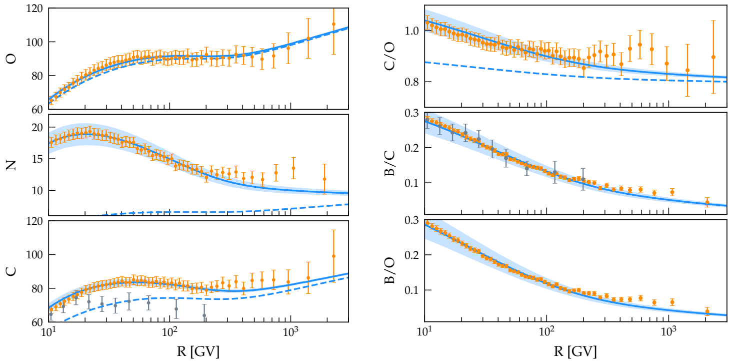


▷ We assume a phenomenological motivated $D(R)$ as a smoothly-broken power-law:

$$D(R) = \boxed{2v_A H} + \frac{\boxed{\beta D_0 (R/\text{GV})^{\delta}}}{\boxed{[1 + (R/R_b)^{\Delta\delta/s}]^s}}$$

Quick look at the data: The CNO element

Evoli et al., PRD 101 (2020), Weinrich et al., A&A 639 (2020), De La Torre Luque et al., JCAP 03 (2021), Schroer et al., PRD 103 (2021)



- by fitting primary and secondary/primary measurements we found: $\delta \sim 0.54$, $D_0/H \sim 0.5 \times 10^{28}$ cm/s^2/kpc, $\Delta\delta \sim 0.2$, $v_A \sim 5$ km/s
- All nuclei injected with $\gamma \sim 4.3$
- Shaded areas: uncertainty from cross sections (small for pure primary species as Oxygen).

Decay of unstable isotopes

B/C only gives the grammage $\propto H/D \rightarrow$ how to break the degeneracy?

- We now look at the ratio of unstable and stable species, as the lifetime introduces a clock breaking the degeneracy
- ^{10}Be is β^- unstable with an half-life $\tau_{1/2} \sim 1.39 \times 10^6$ years $\longrightarrow$ ^{10}B
- The transport equation for ^{10}Be is the first case we discuss where the source or loss term is not in the form of a δ-function in z:

destruction of Be — Be decay — production of Be from C spallation

$$-\frac{\partial}{\partial z}\left[D_{\rm Be}\frac{\partial I_{\rm Be}(T)}{\partial z}\right] + \frac{\mu v \sigma_{\rm Be}}{m}\delta(z) I_{\rm Be}(T) + \frac{I_{\rm Be}(T)}{\gamma \tau_d} = \frac{\mu v \sigma_{\rm C \to Be}}{m}\delta(z) I_{\rm C}(T)$$

where $\mu = 2 h_d n_d m \sim 10^{-3}$ g/cm^2 is the disk surface density.

- ^{10}Be decays on a time scale $\gamma \tau_d$ that at some high-E becomes longer than $\tau_{\rm esc} \longrightarrow$ stable
- ^{10}Be decays mainly into ^{10}B so that it changes the abundance of stable elements.

Decay of unstable isotopes

- Outside the disk $z \neq 0$ the transport equation becomes

$$-\frac{\partial}{\partial z}\left[D_{\mathrm{Be}}\frac{\partial I_{\mathrm{Be}}(T)}{\partial z}\right]+\frac{I_{\mathrm{Be}}(T)}{\hat{\tau}_d}=0$$

- the solution is in the form

$$I = A\mathrm{e}^{-\alpha z} + B\mathrm{e}^{\alpha z}$$

which implies $\alpha^{-1} \equiv \sqrt{D\hat{\tau}_d}$

- after imposing the proper boundary conditions we obtain (introducing $y \equiv \mathrm{e}^{\alpha H}$):

$$\frac{I_{\mathrm{Be}}(z)}{I_{\mathrm{Be},0}} = -\frac{y^2}{1-y^2}\mathrm{e}^{-\alpha z} + \frac{1}{1-y^2}\mathrm{e}^{\alpha z}$$

- the value of the distribution function at $z = 0$ can be obtained by the usual integration above/below disc:

$$-2D_{\mathrm{Be}}\frac{\partial I_{\mathrm{Be}}(T)}{\partial z}\bigg|_{0^+} + \frac{\mu v \sigma_{\mathrm{Be}}}{m} I_{\mathrm{Be},0}(T) = \frac{\mu v \sigma_{\mathrm{C}\to\mathrm{Be}}}{m} I_{\mathrm{C},0}(T)$$

$$\longrightarrow I_{\mathrm{Be},0}(T)\left[\frac{\sigma_{\mathrm{Be}}}{m} - \frac{2D_{\mathrm{Be}}}{\mu v H}\alpha H \frac{1+y^2}{1-y^2}\right] = \frac{\sigma_{\mathrm{C}\to\mathrm{Be}}}{m} I_{\mathrm{C},0}(T)$$

Decay of unstable isotopes

- The transport equation in terms of X's becomes:

$$\frac{I_{\text{Be},0}}{I_{\text{C},0}}(T) = \frac{1}{\hat{X}_{\text{C}\to\text{Be}}}\left[\frac{1}{\hat{X}_{\text{Be}}} + \frac{1}{X'_{\text{Be}}(T)}\right]^{-1}$$

- At high energy: $\frac{H^2}{D_{\text{Be}}} \ll \hat{\tau}_d \longrightarrow \alpha H \to 0$

$$X'_{\text{Be}}(T) \longrightarrow X_{\text{Be}}(T)$$

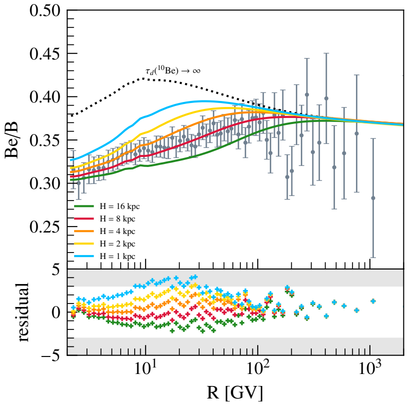
- At low energy: $\frac{H^2}{D_{\text{Be}}} \gg \hat{\tau}_d \longrightarrow \alpha H \to \infty$

$$X'_{\text{Be}}(T) \longrightarrow \frac{\mu v}{2}\sqrt{\frac{\hat{\tau}_d}{D_{\text{Be}}}} = \frac{\mu v}{2H}\sqrt{\hat{\tau}_d \tau_{\text{esc}}}$$

- It is crucial to consider the additional contribution to B production by Be decay (! homework!).

Quick look at the data: The Beryllium-over-Boron ratio and the Halo size

Evoli et al., PRD 101 (2020), Weinrich et al., A&A 639 (2020), Korsmeier & Cuoco, PRD 105 (2022), Maurin et al., arXiv:2203.07265



- Preference for large halos $H \gtrsim 5$ kpc
- Notice that H and $\tau_{\rm esc}$ are mutual corresponding

$$\tau_{\rm esc}(10\,\mathrm{GV}) \sim \frac{H^2}{2D} \sim 20\,\mathrm{Myr}\left(\frac{H}{\mathrm{kpc}}\right)\left(\frac{0.25\times 10^{28}\,\mathrm{cm}^2/\mathrm{s}/\mathrm{kpc}}{D_0/H}\right)$$

Potential sources of galactic CRs

D. Ter Haar, Reviews of Modern Physics, 1950; Ginzburg & Syrovatskii, 1963

- The grammage is also a crucial piece of information to identify galactic CR sources.
- The galactic CR luminosity is:

from B/C

$$L_{\rm CR} \sim \frac{\epsilon_{\rm CR} V_{\rm MW}}{\tau_{\rm esc}} \sim \pi \epsilon_{\rm CR} R_d^2 \boxed{\frac{H}{D}} \sim 10^{41} \text{ erg/s}$$

where

- $\epsilon_{\rm CR} \sim 1$ eV/cm^3 is the local CR energy density
- $V_{\rm MW} = \pi R_d^2 2H$ is the Milky Way Volume (for CRs)
- $\tau_{\rm esc} \sim H^2/D$ is the escape time

- This is also the luminosity required (on a timescale of $\sim \tau_{\rm esc}$) to sustain the CR population.
- The SNe energy rate in our Galaxy:

$$L_{\rm SN} = E_{\rm SN} R_{\rm SN} \sim 10^{42} \text{ erg/s} \sim 10 \times L_{\rm CR}$$

- Galactic SNe provide the right energetics if $\sim 10\%$ efficiency in CR acceleration is achieved $\longrightarrow$ a mechanism able to transfer such an energy was discovered in the 70's (DSA).

Cosmic ray transport for the poor physicists

- Generic rule of thumb:

$$\text{Intensity} \sim \text{Injection Rate} \times \frac{\text{Relevant lifetime}}{\text{Relevant volume}}$$

- Primary species equilibrium spectrum:

$$I_p(T) \propto Q(T) \frac{\tau_{\rm esc}(T)}{H}$$

- Secondary stable species equilibrium spectrum:

$$I_s(T) \propto I_p(T) \sigma v n_{\rm d} h_d \frac{\tau_{\rm esc}(T)}{H}$$

- Secondary unstable(*) species equilibrium spectrum:

$$I_s^*(T) \propto I_p(T) \sigma v n_{\rm d} h_d \frac{\tau_{\rm d}(T)}{\sqrt{\tau_{\rm d}(T) D(T)}}$$

- Stable secondary over primary ratio:

$$\frac{I_s(T)}{I_p(T)} \propto \chi(T) \propto \frac{H}{D(T)}$$

- Unstable secondary over stable secondary ratio:

$$\frac{I_s^*(T)}{I_s(T)} \propto \frac{\sqrt{D(T)}}{H^2} \quad \leftarrow \text{break the degeneracy!}$$

An old friend: the leaky-box model

Seo & Ptuskin, ApJ 431 (1994)

- An extremely simplified approximation of the diffusion model is the so called leaky-box model.
- Widely popular to infer on the nail relevant properties of the galactic transport from the data but watch at the caveats!
- In this case we ignore any spatial dependence and express the diffusion term as a leakage rate:

$$\nabla(D\nabla I_\alpha) \longrightarrow -\frac{I_\alpha}{\tau_\alpha^{\text{esc}}}$$

where τ_{esc} is the usual H^2/D.

- The steady-state transport equation then reads

$$0 = q_\alpha - \frac{I_\alpha}{\tau_\alpha^{\text{esc}}} - \left(\frac{1}{\tau_\alpha^{\text{in}}} + \frac{1}{\tau_\alpha^{\text{d}}}\right) I_\alpha + \sum_{\alpha'>\alpha} \left(\frac{1}{\tau_{\alpha'\to\alpha}^{\text{in}}} + \frac{1}{\tau_{\alpha'}^{\text{d}}}\right) I_{\alpha'}$$

where $\tau_\alpha^{\text{in}} = (v\bar{n}\sigma_\alpha)^{-1}$

The leaky-box model: secondary-over-primary ratio

- Considering only stable species $\tau^{\rm d} \to \infty$ the solution becomes

$$I_\alpha = \left(q_\alpha + \frac{I_{\alpha'}}{\tau^{\rm in}_{\alpha' \to \alpha}} \right) \left(\frac{1}{\tau^{\rm esc}_\alpha} + \frac{1}{\tau^{\rm in}_\alpha} \right)^{-1}$$

- In terms of the grammage:

$$I_\alpha = \left(\frac{q_\alpha}{\bar{n} m_p v} + \frac{I_{\alpha'}}{\hat{\chi}_{\alpha' \to \alpha}} \right) \frac{\chi_\alpha}{1 + \frac{\chi_\alpha}{\hat{\chi}_\alpha}}$$

- The solution we just derived is precisely the same as the one we obtained using the full approach if the source term is taken as:

$$q_\alpha = A p^2 q_{0,\alpha}(p) \frac{h}{H}$$

- which reflects in the following B/C ratio:

$$\frac{\rm B}{\rm C} = \frac{\frac{\chi(T)}{\hat{\chi}_{\rm C \to B}}}{1 + \frac{\chi(T)}{\hat{\chi}_{\rm B}}}$$

The leaky-box model: unstable-over-stable ratio

- For a secondary unstable species the solution becomes

$$I_\alpha = \left(\frac{I_{\alpha'}}{\tau^{\text{in}}_{\alpha' \to \alpha}} \right) \left(\frac{1}{\tau^{\text{esc}}_\alpha} + \frac{1}{\tau^{\text{in}}_\alpha} + \frac{1}{\tau^{\text{d}}_\alpha} \right)^{-1}$$

- assuming $\sigma_{10} \simeq \sigma_9$, the ratio:

$$\frac{\text{Be}^{10}}{\text{Be}^9} = \frac{(\tau^{\text{esc}})^{-1} + (\tau^{\text{in}})^{-1}}{(\tau^{\text{esc}})^{-1} + (\tau^{\text{in}})^{-1} + (\tau^{\text{d}}_{10})^{-1}} \xrightarrow{\tau^{\text{esc}} \ll \tau^{\text{in}}} \frac{1}{1 + \frac{\tau^{\text{esc}}}{\tau^{\text{d}}_{10}}}$$

- To be compared against the solution we derived for the thin disk case

$$\frac{\text{Be}^{10}}{\text{Be}^9} \simeq \sqrt{\frac{\tau^{\text{d}}_{10}}{\tau^{\text{esc}}}}$$

- At $\gamma \sim 1$, measurements points to a ratio $\frac{\text{Be}^{10}}{\text{Be}^9} \sim 0.3$ which would corresponds to:

$$\tau_{\text{esc}} = \begin{cases} \sim 6 \text{ Myr} & \text{in the Leaky-box approximation} \\ \sim 20 \text{ Myr} & \text{in the thin disc model} \end{cases}$$

this equation should not be used for simple estimates of the escape time of CRs or the halo height.

Some implications on microphysics

Charged particles in a ordered B-field

- In general the equation of motion of a charged particle in a electromagnetic fields

$$\frac{d\vec{p}}{dt} = q\left[\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right]$$

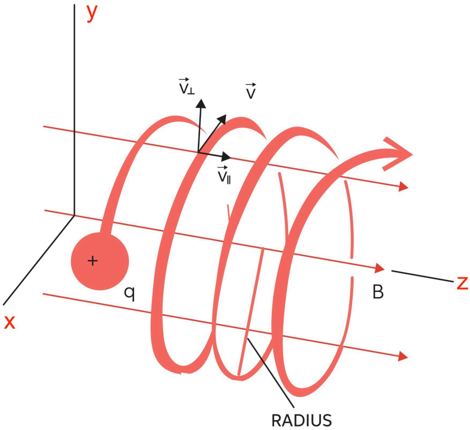
- Given the absence of regular electric fields $\vec{E} \to 0$ we will limit ourselves to the case where only $\vec{B}$ is present $\longrightarrow$ as a consequence the particle energy γ cannot change
- We specify the eq. of motion for a regular magnetic field B_0 oriented along the z-axis

$$\begin{aligned} m\gamma\frac{dv_x}{dt} &= q\frac{v_y}{c}\mathsf{B}_0 \\ m\gamma\frac{dv_y}{dt} &= -q\frac{v_x}{c}\mathsf{B}_0 \\ \frac{dv_z}{dt} &= 0 \longrightarrow v_z \equiv v_{||} = \text{constant} \end{aligned}$$

- Combining the first two:

$$m\gamma\frac{d^2 v_{x,y}}{dt^2} = -\left(\frac{q\mathsf{B}_0}{mc\gamma}\right)^2 v_{x,y} \equiv -\Omega^2 v_{x,y}$$

Charged particles in a ordered B-field



$$\Omega \equiv \frac{qB_0}{mc\gamma} = \frac{v}{r_L}$$ Gyration frequency, $$\mu \equiv \cos\theta = \frac{v_\parallel}{v}$$ Pitch angle

Charged particles in a ordered B-field

- The solution can be written as

$$
\begin{aligned}
v_x(t) &= A\cos(\Omega t) + B\sin(\Omega t) \\
v_y(t) &= -A\sin(\Omega t) + B\cos(\Omega t)
\end{aligned}
$$

- where A and B satisfy the initial conditions that

$$
\begin{aligned}
v_x(t=0) &= A \equiv v_\perp \cos(\phi) \\
v_y(t=0) &= B \equiv v_\perp \sin(\phi)
\end{aligned}
$$

- hence

$$
\begin{aligned}
v_x(t) &= v_\perp \left[\cos(\phi)\cos(\Omega t) + \sin(\phi)\sin(\Omega t)\right] = v_\perp \cos(\phi - \Omega t) \\
v_y(t) &= v_\perp \left[-\cos(\phi)\sin(\Omega t) + \sin(\phi)\cos(\Omega t)\right] = v_\perp \sin(\phi - \Omega t)
\end{aligned}
$$

- The unperturbed motion of the particle is periodic in the XY plane and rectilinear uniform in the z direction where

$$
v_{||} = v\mu = \text{constant} \rightarrow \mu = \text{constant}
$$

and the equation of motion along z is simply

$$
z = v\mu t
$$

Charged particles in a ordered + random B-field

- For simplicity let's consider the case of a perturbation that only propagates along the ordered magnetic field $\vec{B} = \mathrm{B}_0 \hat{z}$ and only having components along x and y axes: $\delta\mathrm{B}_x\,,\ \delta\mathrm{B}_y$
- Is it still true that we can neglect induced electric fields?
- The equation of motion of the particle is

$$m\gamma \frac{d\vec{v}}{dt} = \frac{q}{c} \begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_x & v_y & v_z \\ \delta\mathrm{B}_x & \delta\mathrm{B}_y & \mathrm{B}_0 \end{pmatrix} \overset{\delta\mathrm{B} \ll \mathrm{B}_0}{\simeq} \frac{q}{c} \begin{pmatrix} v_y \mathrm{B}_0 \\ -v_x \mathrm{B}_0 \\ v_x \delta\mathrm{B}_y - v_y \delta\mathrm{B}_x \end{pmatrix}$$

- It follows that the perturbed motion along z becomes

$$m\gamma \frac{dv_z}{dt} = \frac{q}{c} \left[v_x(t)\delta\mathrm{B}_y - v_y(t)\delta\mathrm{B}_x \right]$$

as a consequence now the pitch angle changes with time

$$\rightarrow m\gamma v \frac{d\mu}{dt} = \frac{q}{c} v_\perp \left[\cos(\phi - \Omega t)\delta\mathrm{B}_y - \sin(\phi - \Omega t)\delta\mathrm{B}_x \right]$$

Charged particles in a ordered + random B-field

▷ Let's assume that the perturbed field is circularly polarized (and for simplicity we ignore the phase):

$$\begin{aligned} \delta \mathrm{B}_y &= \delta \mathrm{B} \exp\left[i(kz - \omega t)\right] \\ \delta \mathrm{B}_x &= \pm i \delta \mathrm{B}_y \end{aligned}$$

▷ Taking the real part gives

$$\begin{aligned} \delta \mathrm{B}_y &= \delta \mathrm{B} \cos(kz - \omega t) \\ \delta \mathrm{B}_x &= \mp \delta \mathrm{B} \sin(kz - \omega t) \end{aligned}$$

▷ thereby

$$m\gamma v \frac{d\mu}{dt} = \frac{q}{c} v_\perp \delta \mathrm{B} \left[\cos(\phi - \Omega t)\cos(kz - \omega t) \pm \sin(\phi - \Omega t)\sin(kz - \omega t)\right]$$

or

$$m\gamma v \frac{d\mu}{dt} = \frac{q}{c} v_\perp \delta \mathrm{B} \cos(\phi - \Omega t \mp kz \pm \omega t)$$

Charged particles in a ordered + random B-field

- For Alfvén waves the dispersion relation holds $\omega = k v_\mathrm{A}$ where v_A is the Alfvén velocity, as a consequence

$$\frac{kz}{\omega t} \simeq \frac{kv\mu t}{kv_\mathrm{A} t} \sim \frac{v\mu}{v_\mathrm{A}} \gg 1$$

  unless $\mu \ll v_\mathrm{A}/v$.

- We are allowed to neglect the term ωt with respect to kz. More formally, this is equivalent to choose the reference system in which waves are stationary. This implies that in this frame there is no electric field associated with the waves.

- Finally, using again that for unperturbed orbit $z = v\mu t$

$$\frac{d\mu}{dt} = \frac{q\mathrm{B}_0}{mc\gamma} \frac{v_\perp}{v} \frac{\delta \mathrm{B}}{\mathrm{B}_0} \cos\left[\phi + (\Omega \pm kv\mu)t\right]$$

  or

$$\boxed{\frac{d\mu}{dt} = \frac{q\mathrm{B}_0}{mc\gamma}(1-\mu^2)^{\frac{1}{2}} \frac{\delta \mathrm{B}}{\mathrm{B}_0} \cos\left[\phi + (\Omega \pm kv\mu)t\right]}$$

Charged particles in a ordered + random B-field

- Once averaged over a long period of time, the mean value of the displacement in the cosine of the pitch angle must vanish

$$\langle \Delta\mu \rangle = \int_0^{\Delta t} dt \frac{d\mu}{dt} \to 0$$

- However, its variance does not

$$\langle \Delta\mu\Delta\mu \rangle = \int_0^{2\pi} \frac{d\phi}{2\pi} \int_0^{\Delta t} dt \frac{d\mu}{dt}(t) \int_0^{\Delta t} dt' \frac{d\mu}{dt}(t')$$

- Notice however that

$$\int_0^{\Delta t} dt \int_0^{\Delta t} dt' \cos\left[(\Omega \pm kv\mu)t\right] \cos\left[(\Omega \pm kv\mu)t'\right]$$

$$\overset{\Delta t \gg t, t'}{\simeq} \frac{1}{2} \int_0^{\Delta t} dt \cos\left[(\Omega \pm kv\mu)t\right] \int_{-\infty}^{\infty} dt' \cos\left[(\Omega \pm kv\mu)t'\right]$$

$$= \pi \int_0^{\Delta t} dt \cos\left[(\Omega \pm kv\mu)t\right] \delta\left(\Omega \pm kv\mu\right) = \Delta t\, \pi\delta\left(\Omega \pm kv\mu\right)$$

- Finally,

$$\langle \Delta\mu\Delta\mu \rangle = \pi\Omega^2 \frac{1-\mu^2}{2} \Delta t \left(\frac{\delta B}{B_0}\right)^2 \delta\left(\Omega \pm kv\mu\right)$$

Pitch angle diffusion

$$\langle \Delta\mu \rangle = 0\,, \quad \langle \Delta\mu\Delta\mu \rangle = \pi\Omega^2 \frac{1-\mu^2}{2} \Delta t \left(\frac{\delta \mathrm{B}}{\mathrm{B}_0} \right)^2 \delta\left(\Omega \pm k v \mu\right)$$

- The mean value of the square of the pitch angle variation is proportional to the time lapse $\longrightarrow$ diffusion
- This is true only when the resonance condition is fulfilled:

$$\Omega \pm k v \mu = 0 \longrightarrow k = k_{\mathrm{res}} \equiv \frac{\Omega}{v\mu}$$

- The scattering depends on the power available at the resonant scale

$$\frac{\delta \mathrm{B}^2(k)}{\mathrm{B}_0^2} = \frac{\text{energy density in the turbulent field}}{\text{energy density in the regular field}} \equiv W(k)dk \equiv \mathcal{F}(k)$$

- What do you expect happening when $\mu \to 0$?

Diffusion coefficient in the presence of a spectrum of waves

- We can introduce now a diffusion coefficient in μ as

$$D_{\mu\mu}(k) = \frac{1}{2}\left\langle \frac{\Delta\mu\Delta\mu}{\Delta t} \right\rangle = \frac{\pi}{2}\Omega(1-\mu^2)\frac{\delta \mathrm{B}^2(k_{\mathrm{res}})}{\mathrm{B}_0^2} k_{\mathrm{res}}\delta\left(k - k_{\mathrm{res}}\right)$$

- which in the presence of a spectrum of waves becomes

$$D_{\mu\mu} = \frac{1}{2}\left\langle \frac{\Delta\mu\Delta\mu}{\Delta t} \right\rangle = \frac{\pi}{2}\Omega(1-\mu^2)\int dk \frac{\delta \mathrm{B}^2(k)}{\mathrm{B}_0^2} k\delta\left(k - k_{\mathrm{res}}\right)$$

- in terms of the dimensionless power in perturbations:

$$\boxed{D_{\mu\mu} = \frac{\pi}{2}\Omega(1-\mu^2)\mathcal{F}(k_{\mathrm{res}})}$$

- and the timescale necessary for deflection by $\mathcal{O}\left(\frac{\pi}{2}\right)$ may be estimated as

$$\tau_{\mathrm{D}} \sim \frac{1}{D_{\mu\mu}} \simeq \frac{1}{\Omega\mathcal{F}(k_{\mathrm{res}})}$$

notice that being Ω the gyration frequency in the unperturbed field, this expression is telling us that the particles must perform many gyrations in order to get a deflection by order unity.

Spatial diffusion

- The diffusion coefficient of particles in space, following the general definition is

$$D_{zz}(p) \simeq \frac{1}{3} v \lambda(p) \simeq \frac{1}{3} v v \tau_{\mathrm{D}}(p) = \frac{1}{3} v \frac{r_{\mathrm{L}}}{\mathcal{F}(k_{\mathrm{res}})}$$

- or in terms of the Bohm diffusion coefficient $D_{\mathrm{B}} = \frac{1}{3} v r_{\mathrm{L}}$

$$\boxed{D_{zz}(p) \simeq D_{\mathrm{B}}(p) \frac{1}{\mathcal{F}(k_{\mathrm{res}})}}$$

What B/C does imply on scattering micro-physics?

- By reproducing local measurements we obtained:

$$\underbrace{D(\mathrm{GV})/H \simeq 0.35 \times 10^{28}\ \mathrm{cm}^2/\mathrm{s}}_{\text{from B/C}} + \underbrace{H \simeq 5\ \mathrm{kpc}}_{\text{from Be/B}} \to D(\mathrm{GV}) \simeq 1.8 \times 10^{28}\ \mathrm{cm}^2/\mathrm{s}$$

- In terms of a diffusion coefficient:

$$D(E) = \frac{1}{3} r_{\mathrm{L}}(E) v \frac{1}{\mathcal{F}(k_{\mathrm{res}})} = \frac{1}{3} v \lambda_{\mathrm{diff}}(E) \quad \text{where} \quad k_{\mathrm{res}} = \frac{1}{r_{\mathrm{L}}(E)}$$

- implying that at $\sim$ GV:

$$\lambda_{\mathrm{diff}} \simeq \mathrm{pc}$$

remember this is (on average) how much a GV particle has to travel before to deflect by 90°

- the turbulence level required to do so

$$r_{\mathrm{L}}(\mathrm{GV}) \simeq 10^{12}\ \mathrm{cm} \to \mathcal{F}(k) \simeq \frac{r_{\mathrm{L}} c}{3 D_0} \simeq 6 \times 10^{-7} = \left(\frac{\delta B}{B_0}\right)^2_{k_{\mathrm{res}}}$$

notice that we prove a posteriori the validity of the perturbative (QLT) approach.

Another example of "Little things affect Big things"

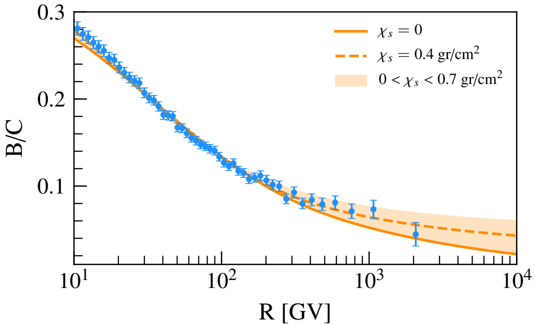
Transport ($\sim 10^{22}$ cm) $\longrightarrow$ mean free path ($\sim 10^{18}$ cm) $\longrightarrow$ waves lenght ($\sim 10^{13}$ cm)

Such a tiny perturbation at the scale of the Solar System stretches the transport time in the Galaxy from kyrs' to 100 Million of years!

What is the origin of these waves?

Additional effects not included in this picture

Evoli et al., PRD 99 (2019)



- Second-order Fermi acceleration in the ISM [Ptuskin et al., 2006, ApJ 642; Drury & Strong, 2017, A&A 597]
- Shock re-acceleration of secondary nuclei [Blasi, 2017, MNRAS 471; Bresci et al., 2019, MNRAS 488]
- Grammage at the sources [D'Angelo et al., 2016, PRD 94; Nava et al., 2016, MNRAS 461; Jacobs et al., 2022, JCAP 05]
- Secondary production at the sources [Blasi, 2009, PRL 103; Mertsch & Sarkar, 2014, PRD 90]
- ...

Conclusions

- Cosmic Ray transport in the Galaxies is complex!
- The numerical diffusion models are certainly a big step forward, but don't forget are based on simple notions.
- The framework is still incomplete... time for new ideas!
- Need to look at all the observational constraints and model them simultaneously.
- These ideas and models can be straightforwardly applied to any astrophysical magnetized object.

Exercise

- Assume a power-law spectrum of waves in our Galaxy

$$W(k) \propto k^{-\alpha} \quad \text{for } k > k_0$$

where $k_0 = 1/\text{pc}$ is roughly the turbulence correlation length, and $\alpha = 1/3$ in the Kolmogorov phenomenology, or $\alpha = 1/2$ in the Kraichnan one.

- Having measured a grammage of 5 gr/cm^2 at 10 GV, derive the turbulent over regular magnetic field ratio at the correlation-length scale, $\frac{\delta B}{B_0}(k_0)$, both for Kolmogorov and for Kraichnan turbulence.
- The proton flux measured by AMS-02 at 10 GV is 21.55 GV^{-1} m^{-2} s^{-1} sr^{-1}, derive the efficiency of conversion of SN energy to cosmic rays ξ.

Thank you!

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