



On Some Algorithms For Solving Symbolic 3-Plithogenic Equations

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Abstract:

The main goal of this paper is to study three different types of algebraic symbolic 3-plithogenic equations. The symbolic 3-plithogenic linear Diophantine equations, symbolic 3-plithogenic quadratic equations, and linear system of symbolic 3-plithogenic equations will be discussed and handled, where algorithms to solve the previous types will be presented and proved by transforming them to classical algebraic systems of equations.

Keywords: symbolic 3-plithogenic Diophantine equation, symbolic 3-plithogenic quadratic equation, linear system

Introduction and preliminaries

The concept of symbolic plithogenic algebraic structures was defined by Smarandache [1-5], and it was studied for the special case of $n=2$ by many authors [6-11].

In this work, we concentrate on the case of symbolic 3-plithogenic algebraic equations, we explain the solutions' algorithms in a similar way of the 2-plithogenic case.

We will not write the proofs, that is because they are similar to the classical case, and can be found in [12].

For basic definitions of 3-symbolic plithogenic rings and algebraic operations see [11].

Main Results

Definition.

Let $3 - SP_Z = \{a + bP_1 + cP_2 + dP_3; a, b, c, d \in Z\}$ be the symbolic 3-plithogenic ring of integers, the Diophantine equation with two variables is defined as follows:

$$AX + BY = C; A = a_0 + a_1P_1 + a_2P_2 + a_3P_3, B = b_0 + b_1P_1 + b_2P_2 + b_3P_3, C = c_0 + c_1P_1 + c_2P_2 + c_3P_3,$$

$$X = x_0 + x_1P_1 + x_2P_2 + x_3P_3, Y = y_0 + y_1P_1 + y_2P_2 + y_3P_3, a_i, b_i, c_i, x_i, y_i \in Z.$$

Theorem.

Let $AX + BY = C$ be the symbolic 3-plithogenic linear Diophantine equation with two variables, it is solvable if and only if the following linear Diophantine equations are solvable.

$$\begin{aligned} a_0x_0 + b_0y_0 &= c_0 \\ (a_0 + a_1)(x_0 + x_1) + (b_0 + b_1)(y_0 + y_1) &= c_0 + c_1 \\ (a_0 + a_1 + a_2)(x_0 + x_1 + x_2) + (b_0 + b_1 + b_2)(y_0 + y_1 + y_2) &= c_0 + c_1 + c_2 \\ (a_0 + a_1 + a_2 + a_3)(x_0 + x_1 + x_2 + x_3) + (b_0 + b_1 + b_2 + b_3)(y_0 + y_1 + y_2 + y_3) &= c_0 + c_1 + c_2 + c_3 \end{aligned}$$

The description of the algorithm.

To solve $AX + BY = C$ in $3 - SP_Z$, we must follow these steps.

Step1.

We compute $gcd(a_0, b_0), gcd(a_0 + a_1, b_0 + b_1), gcd(a_0 + a_1 + a_2, b_0 + b_1 + b_2)$.

If $gcd(a_0, b_0)/c_0, gcd(a_0 + a_1, b_0 + b_1)/c_0 + c_1, gcd(a_0 + a_1 + a_2, b_0 + b_1 + b_2)/c_0 + c_1 + c_2, gcd(a_0 + a_1 + a_2 + a_3, b_0 + b_1 + b_2 + b_3)/c_0 + c_1 + c_2 + c_3$ then it is solvable.

Step2.

We solve the equivalent system and get the values of $x_i, y_i; 0 \leq i \leq 2$.

3-symbolic plithogenic Quadratic equation.

Let $3 - SP_F$ be a symbolic 3-plithogenic field, the formula

$$AX^2 + BY^2 + C = 0; A = a_0 + a_1P_1 + a_2P_2 + a_3P_3, B = b_0 + b_1P_1 + b_2P_2 + b_3P_3, C = c_0 + c_1P_1 + c_2P_2 + c_3P_3,$$

$$X = x_0 + x_1P_1 + x_2P_2 + x_3P_3, Y = y_0 + y_1P_1 + y_2P_2 + y_3P_3, a_i, b_i, c_i, x_i, y_i \in F.$$

Is called the symbolic 3-plithogenic quadratic equation.

Theorem.

Let $AX^2 + BY^2 + C = 0$ be a symbolic 3-plithogenic quadratic equation over $3 - SP_F$, then it is solvable if and only if the following system is solvable:

$$\begin{cases} a_0x_0^2 + b_0y_0^2 + c_0 = 0 \dots (1) \\ (a_0 + a_1)(x_0 + x_1)^2 + (b_0 + b_1)(y_0 + y_1)^2 + (c_0 + c_1) = 0 \dots (2) \\ (a_0 + a_1 + a_2)(x_0 + x_1 + x_2)^2 + (b_0 + b_1 + b_2)(y_0 + y_1 + y_2)^2 + (c_0 + c_1 + c_2) = 0 \dots (3) \end{cases}$$

$$a_0x_0^2 + b_0y_0^2 + c_0 = 0 \dots (1)$$

$$(a_0 + a_1)(x_0 + x_1)^2 + (b_0 + b_1)(y_0 + y_1)^2 + (c_0 + c_1) = 0 \dots (2)$$

$$(a_0 + a_1 + a_2)(x_0 + x_1 + x_2)^2 + (b_0 + b_1 + b_2)(y_0 + y_1 + y_2)^2 + (c_0 + c_1 + c_2) = 0 \dots (3)$$

$$(a_0 + a_1 + a_2 + a_3)(x_0 + x_1 + x_2 + x_3)^2 + (b_0 + b_1 + b_2 + b_3)(y_0 + y_1 + y_2 + y_3)^2 + (c_0 + c_1 + c_2 + c_3) = 0 \dots (4)$$

The description of algorithm.

To solve $AX^2 + BY^2 + C = 0$ in $3 - SP_F$, follow these steps:

Step1.

Solve the equivalent classical system of quadratic equations. If (1), (2), and (3), (4) are solvable in the field F , then the symbolic 3-plithogenic quadratic equation is solvable.

Step2.

Discuss all possible cases of x_0, x_1, x_2, x_3 .

3-plithogenic Linear equations.

We begin the simplest case, a symbolic 3-plithogenic linear equation with one variable $A.X = B$.

This equation is solvable uniquely if and only if A is invertible and $X = A^{-1}B$.

Conclusion

In this paper, we have presented novel algorithms to solve many different types of 3-plithogenic algebraic equations (quadratic, linear, and linear Diophantine equations) by transforming them to classical systems of algebraic equations.

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