

## Mathematical Study on Impact of Temperature in Malaria Disease Transmission Dynamics

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**Abstract**

Malaria is one of the most common mosquito borne diseases. Temperature is an important factor which affects the life cycle of the mosquitoes and transmission dynamics of the malaria disease. In the present work, we use SEIR compartmental model for the human population and LSEI compartmental model for mosquito population taking temperature dependent parameters. Basic reproduction number,  $R_0$  of the model is computed using Next Generation Matrix Method. Stability of the disease free equilibrium and the existence of the endemic equilibrium point are discussed by basic reproduction number,  $R_0$ . Numerical results are carried out with different temperature levels. It is observed that temperature affects the transmission dynamics of malaria disease significantly.

**Keywords**

Malaria; Temperature; Basic Reproduction Number; Stability

**Introduction**

Change of climate mostly affects the transmission dynamics of vector born disease. The disease is transmitted by female Anopheles mosquitoes. It is spreading in almost all tropical and subtropical region of the world. About 3.2 million people are at risk of malaria. In 2015 there were 214 million new cases of malaria and 438000 deaths from malaria. There is not any effective and safe vaccine against malaria till date. Control e orts of malaria are based on the strategies of reducing the mosquitoes, using the anti-malaria drugs and personal protection against mosquito bites. But still million of people are not receiving the services [1].

Climate factors such as temperature, rainfall, humidity, wind and duration of daylight strongly affect the ecological and behavioral features of vector borne diseases [2]. In the late 19<sup>th</sup> century average global temperature was increased by about 0.5°C - 0.6°C and by 2100 it is estimated that global temperature will rise 1.0°C - 3.5°C [3,4].

Change of climate mostly impact on the transmission of vector borne diseases. For many diseases it is observed that the transmission of disease occurs within the lower end range and upper end range of temperature. These range of temperatures are respectively considered as 14°C- 18°C and 35°C- 40°C [4]. In the context of malaria, the female adult Anopheles mosquitoes feed more frequently in increasing temperature since blood is digested more quickly with higher temperature [8]. Githeko et al. (2015) explained that the mosquitos' feeds on blood every 4 days at 17°C and with a temperature increased to 25°C it feeds on blood every 2 days. The average lifespan of an adult mosquito is about 21 days and it is rapidly decreasing in 30°C- 32°C temperature [3]. Temperature levels above 34°C have a negative impact on the survival of both vectors and parasites and survival of adults reaches zero at around 40°C temperature [5,6]. Githeko et al. (2015) reported that the development of a gambiae larva stops when the ambient temperature is below 16°C and the temperature below 14°C leads to their deaths. Generally, the availability and productivity of mosquito breeding areas increases by rainfall. Although excess rainfall can rush out the Anopheles breeding areas.

Mathematical model is very useful tool to understand the dynamics of malaria disease transmission. The mathematical model for malaria disease transmission was studied by Ronald Ross in 1911 using SI compartmental model. His work was extended by Macdonald [7,8]. Aron and May described the properties of models defined by Ross-Macdonald. Anderson and May reviewed the model incorporating various parameters [9]. Chitnis et al. analyzed the bifurcation of a malaria model [10]. Mordecai predicted that the optimal temperature for malaria transmission occurs at 25°C [11]. In the present paper we consider SEIR model for human (host) and LSEI model for mosquito (vector) with temperature dependent model parameters.

**Formulation of the Model**

We follow the compartmental malaria model with temperature dependent model parameters to study the effect of temperature on malaria disease. The total human

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population  $N_h(t)$  at time  $t$  is divided into four epidemiological classes: the susceptible class  $S_h(t)$  (people who are healthy and may potentially get infected with malaria parasite), exposed class  $E_h(t)$  (people who are infected with malaria disease but cannot transmit to other), infectious class  $I_h(t)$  (people who are infected with malaria disease and able to transmit the disease) and recovered class  $R(t)$  (people who have recovered from malaria disease). Thus,

$$N_h(t) = S_h(t) + E_h(t) + I_h(t) + R(t)$$

We divide the mosquitoes into two stages: immature and mature (i.e. adult), since temperature affects both stages of the mosquitoes. The total mosquito population at time  $t$ ,  $N_m(t)$  is subdivided into four compartments: immature mosquitoes  $L(t)$  (egg, larva and pupa stages of mosquitoes), susceptible mosquitoes  $S_m(t)$  (mosquitoes that may potentially get infected with malaria parasite), exposed mosquitoes  $E_m(t)$  (mosquitoes infected with malaria that cannot transmit the disease) and infectious mosquitoes  $I_m(t)$  (mosquitoes infected with malaria that can transmit the disease). Then at any time  $t$ ,

$$N_m(t) = L(t) + S_m(t) + E_m(t) + I_m(t)$$

These state variables and their descriptions are presented in Table 1. Based on the description of state variables the SEIR-LSEI model is given by the following deterministic non-autonomous, system of nonlinear differential equation where  $T = T(t)$  denotes the ambient temperature at time  $t$ .

$$\frac{dS_h}{dt} = r + yR - a_h S_h - mS_h$$

$$As\ t \in \mathbb{R} \quad N_h \in \mathbb{R} \quad \frac{r}{m}$$

$$\frac{dI_h}{dt} = b E_h - (t + m + \rho) I_h$$

$$\frac{dR}{dt} = t I_h - (m + y) R$$

$$\frac{dL}{dt} = F(t) \left( \frac{\phi(T)}{k} (S_m + E_m + I_m) - (s(t) + w(t)) L \right) \quad (2.1)$$

$$\frac{dS_m}{dt} = s(t)L - a_m S_m - w(t) S_m$$

$$\frac{dE_m}{dt} = a_m S_m - (b_m(t) + w(t)) E_m$$

$$\frac{dI_m}{dt} = b_m(t) E_m - w(t) I_m$$

Where  $a_h = \frac{I}{N_h} \frac{b(t) I_m}{m}$  and  $a_m = \frac{I}{N_h} \frac{b(t) I_h}{m}$

The total size of vector population and host population are related by

$$\begin{aligned} N_h(t) &= S_h(t) + E_h(t) + I_h(t) + R(t) \\ N_m(t) &= L(t) + S_m(t) + E_m(t) + I_m(t) \end{aligned} \quad (2.2)$$

In the system (2.1), is the recruitment rate of human. All the recruited individuals are assumed to be uninfected new born when they join the susceptible class. (Figure 1). When an infectious mosquito bite a susceptible human, the human progresses to the exposed class.

Variables	Description
$S_h$	Number of susceptible human
$E_h$	Number of exposed humans with infection
$I_h$	Number of infectious humans with infection
$R$	Number of humans recovered from infection
$L$	Number of immature vectors
$S_m$	Number of susceptible vectors
$E_m$	Number of exposed vectors
$I_m$	Number of infectious vectors

Table 1: State Variables and their descriptions

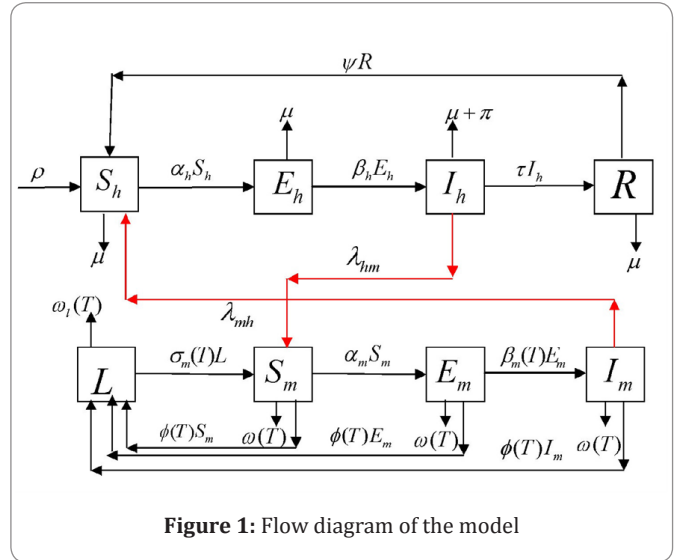


Figure 1: Flow diagram of the model

The infection rate  $\alpha_h$  to susceptible human is temperature dependent,  $\lambda_{mh}$  is the probability of malaria transmission to a susceptible human per bite from an infectious Anopheles mosquito and  $b(T)$  is the temperature dependent per capita biting rate of mosquitoes. Further  $\tau_h$  represents the progression rate of exposed individual to infectious class. The infected humans acquire temporary immunity at the rate of  $\tau$  to join the recovery class. The recovered humans lose the immunity at constant rate  $\psi$  and returns to the susceptible class. There is natural death rate in each host class and an additional disease related death rate  $\pi$  in the infectious class.

The female Anopheles mosquitoes take rest for a few days after obtaining blood meal from human host. These days the mosquitoes digest the blood and develop the eggs. This process depends on temperature. Typically eggs hatch within 2-3 days but in cold climate this may take up to 2-3 weeks [12]. The temperature dependent parameter  $\omega(T)$  is the egg deposition rate of mosquitoes from each class. We assume that the immature mosquito population is limited by carrying capacity  $k$ . So that the logistic growth rate for immature mosquitoes is represented by

$$\phi(T) \left( 1 - \frac{L}{k} \right) (S_m + E_m + I_m)$$

The parameters  $\sigma(T)$  and  $\omega_l(T)$  represent temperature dependent maturation rate and mortality rate of immature mosquitoes respectively [13,14]. The single compartment  $L(t)$  represents the three aquatic stage of the mosquitoes egg, larva and pupa. When mosquitoes become adult they enter to the susceptible compartment  $S_m(t)$ . Susceptible mosquitoes can become infected when they bite infectious human and move to the exposed class at the rate of temperature dependent parameter  $\alpha_m$  and  $\lambda_{hm}$  is the probability that a bite from a susceptible mosquito to an infectious human leads to the infection to the mosquito. The temperature dependent parameters  $\beta_m(T)$  and  $\omega(T)$  represent progression rate

of exposed mosquitoes and death rate of adult mosquito respectively [15]. Since mosquitoes never recover from infection after they are infected (because of short life-cycle of mosquitoes), the vector population does not include recover class.

### Temperature dependent parameters

Temperature affects on the rate of development of immature and adult mosquitoes and transmission of malaria. Low temperature strongly affects on the development of vector larvae. The ambient temperature determines the rate of the adult mosquitoes feed on human blood. Small increase in temperature would probably produce greater mosquitoes densities, higher biting rate and more rapid parasite development in the mosquito [16]. This relation gives that the biting rate of mosquito increases when the temperature increases and temperature levels above 34°C have negative impact on the survival of mosquitoes [5]. So, when the temperature increases above 28°C, the biting rate of mosquitoes decreases strictly and it will be zero in around 34°C (Figure 2). Mordecai et al. [11] have experimentally suggested the biting rate  $b(T)$  as a function defined by,

$$b = 0.000203T(T - 11.7)\sqrt{32.4 - T}; \tag{3.1}$$

The egg deposition rate for susceptible, exposed and infectious mosquitoes is given by [3]

$$f(T) = - 0.153T^2 + 8.61T - 97.7 \tag{3.2}$$

The population density of the mosquitoes increase in the range of temperature 17°C-28°C and egg deposition rate also increases in this range. Then the population density decreases after 28°C (Figure 3).

The temperature dependent parameters, progression rate of exposed vectors  $\beta_m(T)$ , natural mortality rate of immature and adult mosquitoes  $\omega_l(T)$  and  $\omega(T)$  respectively are defined as [17] (Table 2).

$$b_m(T) = - 0.00083T^2 + 0.044T - 0.487 \tag{3.3}$$

$$w(T) = - \log(- 0.000828T^2 + 0.0367T - 0.522) \tag{3.4}$$

$$w(T) = \frac{1}{8.560 + 20.654 \frac{e^{-0.000828T^2 + 0.0367T - 0.522}}{9.759} + 0.000827T} \tag{3.5}$$

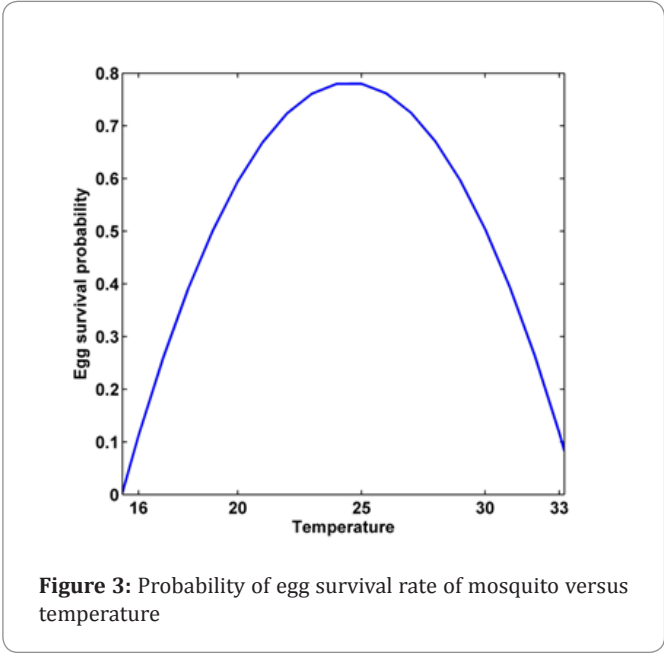


Figure 3: Probability of egg survival rate of mosquito versus temperature

The progression rate of mosquitoes  $\beta_m(T)$ , increases with temperature up to 28°C and then it decreases with increasing temperature. This result also reveals that there are more infected mosquitoes within the temperature range 17°C-28°C (Figure 4). Since the number of eggs laid by female mosquito is temperature dependent, the rate of life time egg laid  $B(T)$  is defined as [17].

$$B(T) = \frac{EFD(T)}{w(T)}$$

That is, the total number of eggs laid by a mosquito is equal to the product of number of eggs laid per female per day (EF D) and the average adult mosquito life-span, where

$$EFD(T) = - 0.153T^2 + 8.61T - 97.7$$

The probability that an egg survives to become an adult mosquito and the development time from egg to adult mosquito are represented by PEA(T) and TEA(T) respectively. Both the parameters are temperature dependent and defined as [17]

$$PEA(T) = - 0.000924T^2 + 0.453T - 4.77$$

$$TEA(T) = \frac{1}{- 0.00094T^2 + 0.049T - 0.552}$$

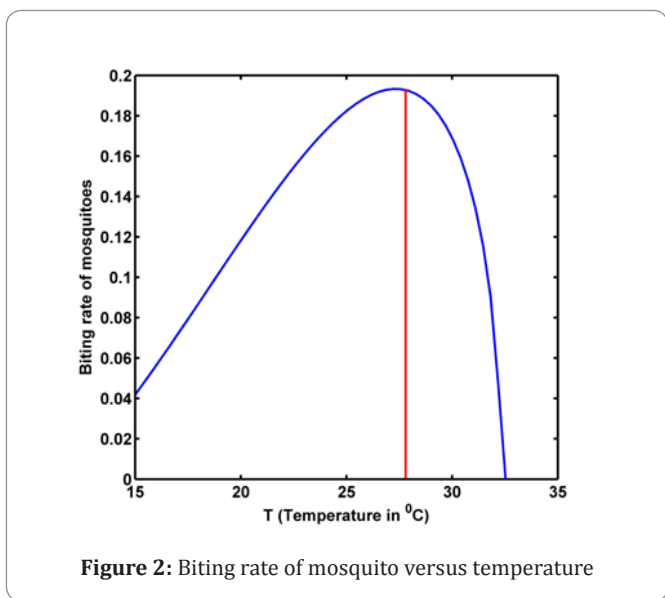


Figure 2: Biting rate of mosquito versus temperature

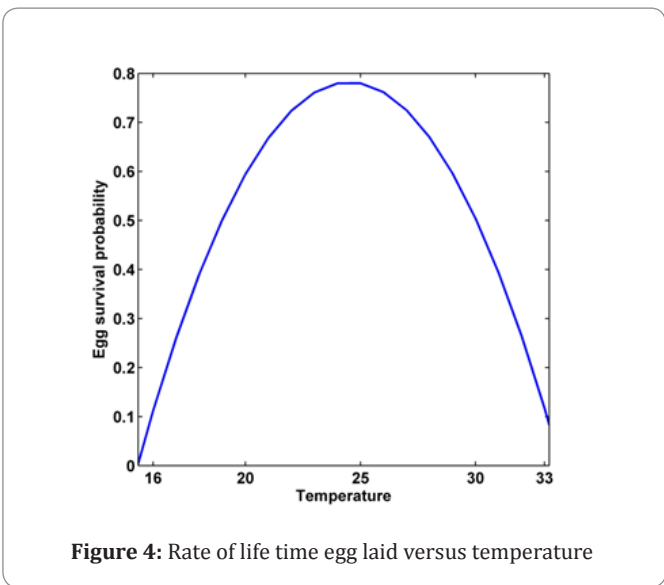


Figure 4: Rate of life time egg laid versus temperature

Parameters	Description	Dimensions
$\rho$	Recruitment rate of the humans	number $\times$ day <sup>-1</sup>
$\lambda_{mh}$	Transmission probability per contact for susceptible humans	dimensionless
$\lambda_{hm}$	Transmission probability per contact for susceptible vectors	dimensionless
$\beta_h$	Progression rate of exposed humans to infectious humans	day <sup>-1</sup>
$\mu$	Natural mortality rate for humans	day <sup>-1</sup>
$\pi$	Disease induced death rate	day <sup>-1</sup>
$\psi$	Rate of loss of immunity of humans	day <sup>-1</sup>
K	Carrying capacity of immature mosquitoes	dimensionless
b(T)	Temperature dependent biting rate of mosquitoes	day <sup>-1</sup>
$\phi(T)$	Temperature dependent egg deposition rate	day <sup>-1</sup>
$\sigma(T)$	Temperature dependent maturation rate from eggs to adult mosquitoes	day <sup>-1</sup>
$\omega(T)$	Temperature dependent natural mortality rate for adult mosquitoes	day <sup>-1</sup>
$\omega_l(T)$	Temperature dependent natural mortality rate for immature mosquitoes	day <sup>-1</sup>
$\beta_m(T)$	Progression rate of exposed mosquitoes	day <sup>-1</sup>
B(T)	The lifetime number of eggs laid	dimensionless
PEA(T)	The probability that an egg survives to become an adult mosquito	dimensionless
TEA(T)	The development time from egg to adult mosquito	dimensionless
EFD(T)	The number of eggs laid per female per day	day <sup>-1</sup>

**Table 2:** Parameters and Their Dimensions

Finally, the temperature dependent maturation rate from eggs to adult mosquito is given as [17]

$$s(T) = \frac{B(T)PEA(T)}{TEA(T)} \tag{3.6}$$

**Carrying capacity of human**

From the system of equations (2.1) we have

$$\frac{dN_h}{dt} = r - mN_h$$

$$As \ t \ \infty, \ N_h \ \rightarrow \ \frac{r}{m} \tag{4.1}$$

Hence the carrying capacity of human is  $\frac{p}{\mu}$

**Equilibrium Points and Stability Analysis**

**Disease free equilibrium point**

Disease free equilibrium point is a steady state solution of the system of equations (2.1) in the absence of infective population. In the present case,

$$E_h = I_h = R_h = E_m = I_m$$

So, the system of equations (2.1) has a disease free equilibrium point

$$= (S_h^*, E_h^*, I_h^*, R_h^*, L^*, E_m^*, I_m^*) = (\frac{\rho}{\mu}, 0, 0, 0, L^*, S_m^*, 0, 0)$$

Where

$$\frac{dL^*}{dt} = f(T) - \frac{L^* \phi(T)}{K} - (s(T) + w(T))L^*$$

$$\frac{dS_m^*}{dt} = s(T)L^* - w(T)S_m^*$$

**Basic Reproduction Number**

The basic reproduction number,  $R_0$  is the expected number of secondary infections produced by a single infective on its infectious life. According to the values of the basic reproduction number, disease can persist with  $R_0 > 1$  and the disease die out when  $R_0 < 1$ .

We compute the basic reproduction number  $R_0$ , associated with the disease free equilibrium point;

$$(\frac{p}{\mu}, 0, 0, 0, L^*, S_m^*, 0, 0)$$

using the next generation method [18]. Using first four equations of the above system of equations, a non-negative matrix F of the infection terms and the non-singular matrix V of the transition terms are given, respectively.

$$F = \begin{pmatrix} 0 & 0 & \frac{b(T)}{N_h} & 0 \\ 0 & 0 & 0 & 0 \\ \frac{b(T)S_m^*}{N_h} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$V = \begin{pmatrix} m + b_h & 0 & 0 & 0 \\ -b_h & t + m + p & 0 & 0 \\ 0 & 0 & b_m(T) + w(T) & 0 \\ 0 & 0 & -b_m(T) & w(T) \end{pmatrix}$$

Therefore, the basic reproduction number,

$$R_0 = r \{FV^{-1}\}$$

$$R_0 = \sqrt{\frac{b^2(T)b_h b_m(T) I_{mh} I_{hm} m S_m^*}{r w(T) (m + b_h) (b_m(T) + w(T)) (t + m + p)}}$$

Where

$$S_m^* = \frac{K s(T)}{w(T)} (1 - s(T) + w(T)) \frac{w(T)}{f(T) s(T)}$$

**Local stability of disease free equilibrium**

**Theorem 1:** The disease free equilibrium point for the system of equation (2.1) is locally asymptotically stable if  $R_0 < 1$  and unstable if  $R_0 > 1$ .

**Proof:** Jacobian matrix of the system of differential equations (2.1) at the disease free equilibrium point

$$J = \begin{pmatrix} (m + b_h) & 0 & 0 & 0 & I_{mh} b(T) \\ - (t + m + p) & 0 & 0 & 0 & 0 \\ t & - (y + m) & 0 & 0 & 0 \\ \frac{I_{hm} b(T) m S_m^*}{r} & 0 & - (b_m(T) + w(T)) & 0 & 0 \\ 0 & 0 & - b_m(T) & - w(T) & 0 \end{pmatrix}$$

The characteristic polynomial is:

$$I^4 + A_1 I^3 + A_2 I^2 + A_3 I + A_4 = 0 \tag{6.1}$$

$$A_1 = P_1 + P_2 + P_3 + P_4$$

$$A_2 = P_1(P_2 + P_3 + P_4) + P_2(P_3 + P_4) + P_3 P_4$$

$$A_3 = P_1 P_2 P_3 + P_1 P_2 P_4 + P_1 P_3 P_4 + P_2 P_3 P_4$$

$$A_4 = P_1 P_2 P_3 P_4 - B$$

Where

$$P_1 = w(T), P_2 = b_m(T) + w(T), P_3 = t + m + p, P_4 = m + b_h, P_5 = m + y$$

$$B = \frac{I_{hm} I_{hm} b^2(T) m b_h b_m(T) S_m^*}{r}$$

For the stability of the disease free equilibrium, we have to prove all the roots of the polynomial (6.1) lie in the left half of the complex plane. For the fourth order polynomial (6.1) to have the roots with negative real parts, the Routh-Hurwitz criteria are  $\det(H_i) > 0$  for  $i = 1, 2, 3, 4$

Also,

$$\det(H_1) = |A_1| = P_1 + P_2 + P_3 + P_4 > 0$$

$$\det(H_2) = \begin{vmatrix} A_1 & 1 \\ 0 & A_2 \end{vmatrix} = A_1 A_2$$

$$= (P_1 + P_2 + P_3 + P_4)(P_1(P_2 + P_3 + P_4) + P_2(P_3 + P_4) + P_3 P_4) > 0$$

$$\det(H_3) = \begin{vmatrix} A_1 & 1 & 0 \\ A_3 & A_2 & A_1 \\ 0 & 0 & A_3 \end{vmatrix} = A_3(A_1 A_2 - A_3) > 0$$

$$\det(H_4) = \begin{vmatrix} A_1 & 1 & 0 & 0 \\ A_3 & A_2 & A_1 & 1 \\ 0 & A_4 & A_3 & A_2 \\ 0 & 0 & 0 & A_4 \end{vmatrix}$$

By the Routh-Hurwitz criteria the disease free equilibrium point will be asymptotically stable if  $R_0 < 1$ , that is  $\det(H_i) > 0$  for  $i = 1, 2, 3, 4$ . Here, if  $R_0 < 1$ , then  $R_0^2 < 1$ ; And,

$$R_0^2 = \frac{b^2(T) b_h b_m(T) I_{mh} I_{hm} m S_m^*}{r P_1 P_2 P_3 P_4} < 1$$

$$\Rightarrow \frac{B}{P_1 P_2 P_3 P_4} < 1$$

$$\Rightarrow B < P_1 P_2 P_3 P_4$$

$$\text{Hence } \Rightarrow P_1 P_2 P_3 P_4 - B > 0$$

$$\det(H_4) > A_4(A_1 A_2 A_3 - A_1^2 A_4 - A_3^2)$$

$$= A_4(P_1 + P_2 + P_3 + P_4)(P_1(P_2 + P_3 + P_4) + P_2(P_3 + P_4) + P_3 P_4) - (P_1 + P_2 + P_3 + P_4)^2 (P_1 P_2 P_3 P_4 - B) - (P_1 P_2 P_3 + P_1 P_2 P_4 + P_1 P_3 P_4 + P_2 P_3 P_4)^2$$

In the above expression, all the negative terms get canceled. So,  $\det(H_4)$  is positive. Since all the determinants of Hurwitz matrices are positive, the polynomial (6.1) has the roots having negative real parts. Hence the disease free equilibrium point is stable if  $R_0 < 1$ . On the contrary, if  $R_0 > 1$  then  $P_1 P_2 P_3 P_4 - B < 0$ . So,  $A_4 < 0$  and hence all the roots of polynomial (6.1) cannot have negative real parts. It concludes that the disease free equilibrium point is unstable if  $R_0 > 1$ .

**Theorem 2:** (Existence of Endemic Equilibrium Point). The endemic equilibrium point of the system exists if  $R_0 > 1$ .

**Proof:** Solving the system of equations (2.1) the non-zero equilibrium point obtained is  $(S_h^e, E_h^e, I_h^e, R^e, L^e, S_m^e, E_m^e, I_m^e)$ , where

$$S_h^e = \frac{r(P + a M)}{R_0^2 P m}$$

$$E_h^e = \frac{P}{b_h} M$$

$$I_h^e = \frac{r(R_0^2 - 1)PPb}{ra b P + R^2 P(PPP - ytb)} = M$$

$$R^e = \frac{t M}{P_5}$$

$$L^e = \frac{K[f(T)s(T) - w(T)(s(T) + w(T))]}{f(T)s(T)} = V$$

$$S_m^e = \frac{s(T)V}{P + a M}$$

$$E_m^e = \frac{a s(T)MV}{P(P + a M)}$$

$$I_m^e = \frac{VMR m}{m r (P + a M)}$$

Where  $R_{om} = \frac{a r s(T)ab (T)}{mPP}$

The endemic equilibrium point exists if  $R_0^2 > 1$  i.e.  $R_0 > 1$ , since  $(P_3 P_4 P_5 - \psi \tau \beta_h) > 0$ .

### Numerical Results and Discussion

In the present work, we use SEIR-LSEI epidemic model of the malaria disease with temperature dependent parameters. The simulations are carried out to explore the effect of temperature in the transmission dynamics of the disease.

The parameter values for the simulation are considered as shown in the Table 3.

All the figures are drawn for  $T = 14^\circ\text{C}, 16^\circ\text{C}, 20^\circ\text{C}, 28^\circ\text{C}$  and  $32^\circ\text{C}$ . The following initial values are used for numerical computation,  $S_h(0) = 100000, E_h(0) = 4000, I_h(0) = 18000, R(0) = 18000, L(0) = 30000, S_m(0) = 130000, E_m(0) = 14000, I_m(0) = 4000, N_h = 140000, k = 40000$ .

Figure 5 shows the dynamics of susceptible human population in different temperature levels. It shows the positive impact of temperature on the transmission of disease. With the increase in temperature level from  $16^\circ\text{C}$ , more susceptible humans get infected of the disease. When infectious mosquito bites the susceptible human, it becomes infected and moves from susceptible class to the exposed class. Since biting rate of mosquito reaches its optimum value at  $28^\circ\text{C}$  temperature (Figure 2), the susceptible population size become minimum in this time. The population starts to increase due to the loss of immunity of recovered human and natural birth of the human population.

Figure 6 shows the dynamics of infectious human with different temperature levels. The population decreases with time because of the malaria induced death, natural death and development of immunities in host population. Infectious host population decreases in the temperature level below  $16^\circ\text{C}$  and above  $32^\circ\text{C}$  than in the  $28^\circ\text{C}$ . When infectious people get immunity they move to recovery class.

Larva cannot grow in the temperature below  $16^\circ\text{C}$  and in the temperature below  $14^\circ\text{C}$  larva cannot live. Consequently, the number of immature mosquitoes sharply decreases to zero in  $14^\circ\text{C}$  temperature level (Figure 7). It takes long time for immature mosquitoes to develop

to adult stage and egg deposition rate approaches to zero around  $16^\circ\text{C}$  temperature (Figure 3), hence, the number of immature mosquitoes decreases slowly and approaches to zero after 30 days (Figure 3). Temperature is the main factor to affect the development of immature mosquitoes. When temperature increases from  $16^\circ\text{C}$ , maturation rate of immature mosquitoes increases but time of maturation decreases. At around  $28^\circ\text{C}$  temperature, immature mosquitoes enter to the aquatic stages within 2-3 days. So, in this temperature, the number of immature mosquito population decreases sharply in initial stage. This is because of the increase of egg deposition causing to the increase in immature mosquitoes population reaching near to the carrying capacity (Figure 7).

The biting rate of mosquito is very low in the temperature below  $16^\circ\text{C}$  and is optimum in  $28^\circ\text{C}$  temperature (Figure 2). So, the number of susceptible mosquitoes decreases slowly in  $16^\circ\text{C}$  temperature and the number decreases rapidly in  $28^\circ\text{C}$  because of infection of the disease (Figure 8). The population of infectious vectors increases initially due to the interaction of susceptible vectors with the infectious hosts. Along with the increase in the biting rate, infectious mosquito population increases in  $20^\circ\text{C}$  and  $28^\circ\text{C}$  temperature levels.

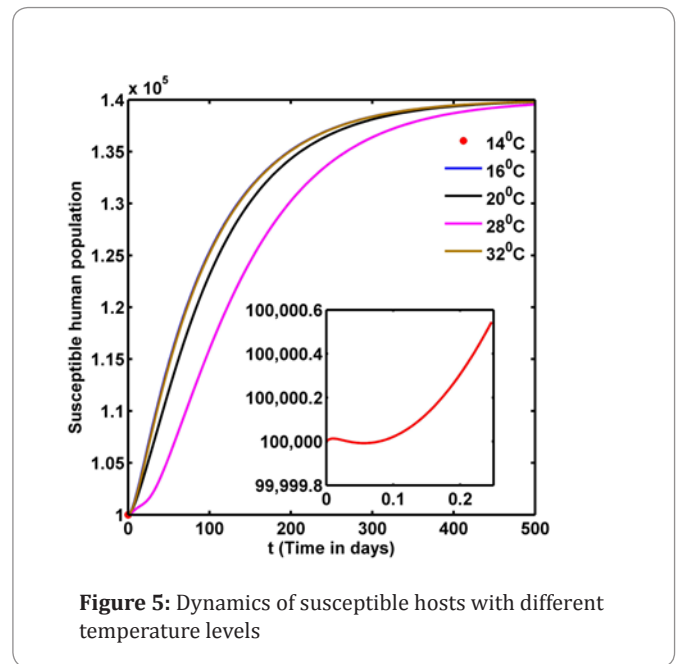


Figure 5: Dynamics of susceptible hosts with different temperature levels

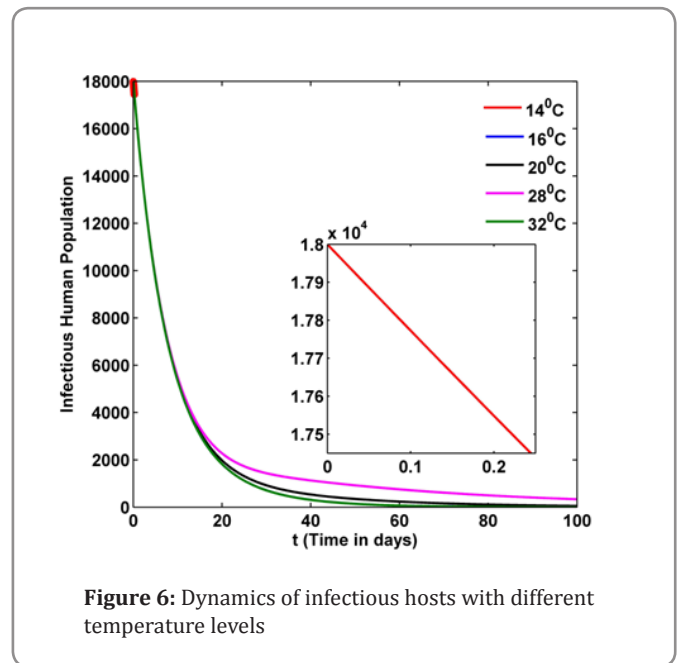


Figure 6: Dynamics of infectious hosts with different temperature levels

Parameters	$\rho$	$\beta_h$	$\psi$	$\tau$	$\mu$	$\pi$	$\lambda_{hm}$	$\lambda_{mh}$
Values	2.53	0.071	0.011	1/7	0.00001814	0.00000638	0.24	0.062

Table 3: Parameter Values

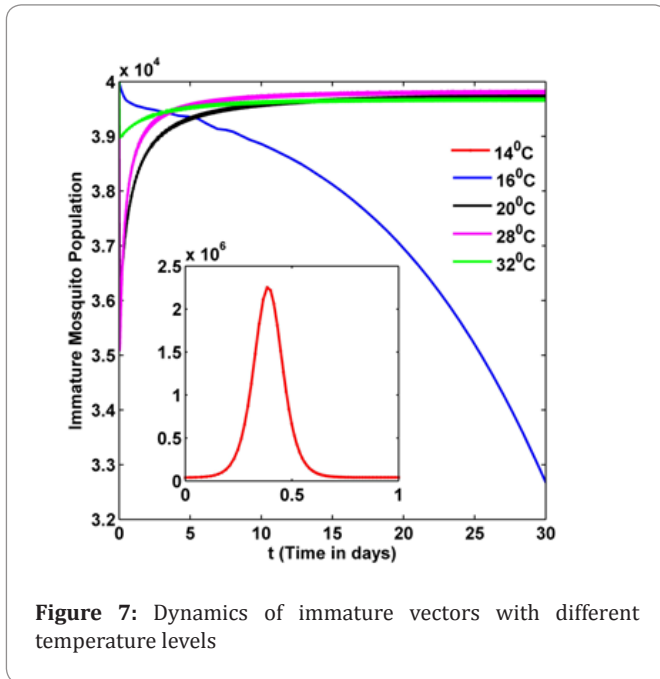


Figure 7: Dynamics of immature vectors with different temperature levels

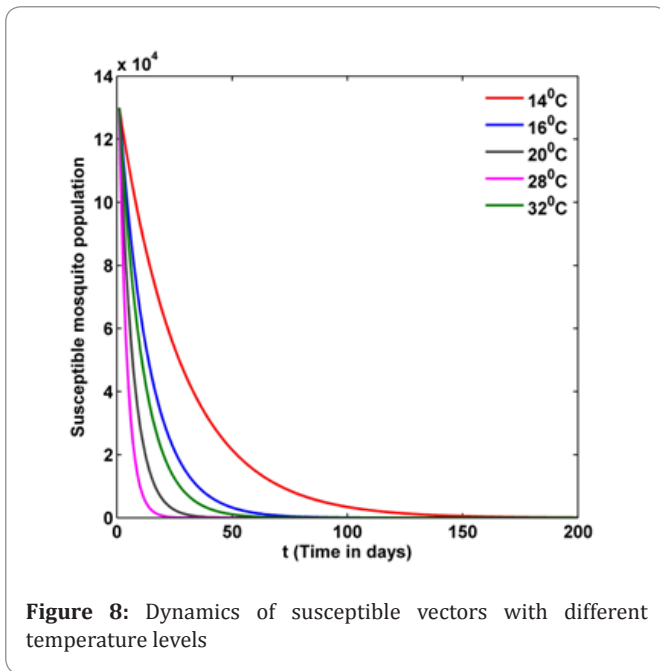


Figure 8: Dynamics of susceptible vectors with different temperature levels

But, more mosquitoes get infected in 28°C temperature than in 20°C because of the biting rate depending on the temperature. Afterwards, the population starts decreasing due to death of the mosquitoes. The infectious mosquito population decreases in the temperature levels 14°C, 16°C and 32°C as these temperature levels are not favorable for the survival of the mosquitoes.

**Conclusion**

Malaria is an infectious disease which is becoming more prevalent worldwide and is spreading in new population of the world. Climatic factors like temperature play a significant role in affecting vectorial capacities of the mosquitoes and then the transmission dynamics of

the disease.

(Figure 9) In the present work, we used epidemic compartmental vector host model, SEIR-LSEI to observe the impact of temperature in the transmission dynamics and prevalence of the disease. Parameters of the model are considered temperature dependent. The numerical results show that the temperature levels less than 16°C and greater than 32°C have negative impact on the survival of the mosquitoes and the transmission dynamics of the disease. So, the people living in this range of temperature are at low risk of the malaria disease. Meanwhile, it is observed from the simulations that large population get infected of the disease when the temperature is mild. Thus, mild temperature is favorable for the survival of the mosquitoes and transmission of the disease as well. The dimensionless number, basic reproduction number  $R_0$  which determines whether the disease dies out or persists is computed using Next Generation Matrix method. The results show that disease free equilibrium of the model is locally asymptotically stable when  $R_0 < 1$  and unstable when  $R_0 > 1$ . The number depends on the model parameters which depend on temperature. So, the temperature is responsible in determining the value of the number. Thus, the temperature has a significant impact on the transmission dynamics of the malaria disease.

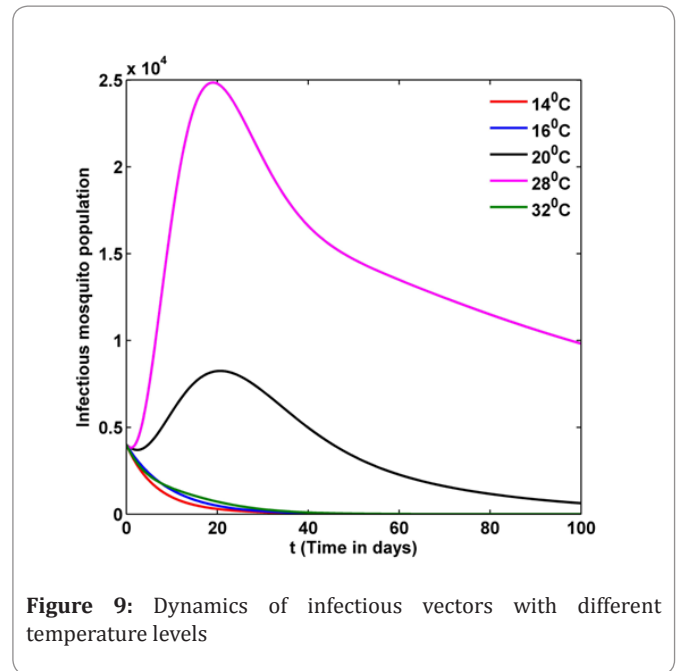


Figure 9: Dynamics of infectious vectors with different temperature levels

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