Distributed Bragg reflector at normal incidence

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This documents briefly describes the interaction of a normally incident linearly polarised light with a distributed Bragg reflector as treated by the app, ***Normal_incidence_DBR***. It details the calculation method used by the app and the involved assumptions, which will help understand the results. This Python app is based on a script that I wrote during my postdoctoral research at Oxford University to simulate the response of bespoke mirrors for optical cavity coupling. Their measured spectral response matched closely with the simulated response.

1 Introduction

A distributed Bragg reflector (DBR), also known as *quarter-wave stack*, is a multilayered structure consisting of alternating layers of low and high refractive index materials deposited on a substrate [1, 2, 3]. These structures can obtain reflection greater than 99.99% over a wavelength range around a chosen central vacuum wavelength λ_0. The optical thickness of each layer is $\lambda_0/4$. The interference between waves reflected from each interface (Figure 1) results in well-defined wavelength stop-bands. For a given substrate with a refractive index n_S in a medium of refractive index n_0 (usually air), we can select the materials of refractive indices n_H and n_L, resp. to obtain the desired reflectivity and stop-band characteristics around a central wavelength of λ_0. Here, by refractive index it means the real part of the complex refractive index.

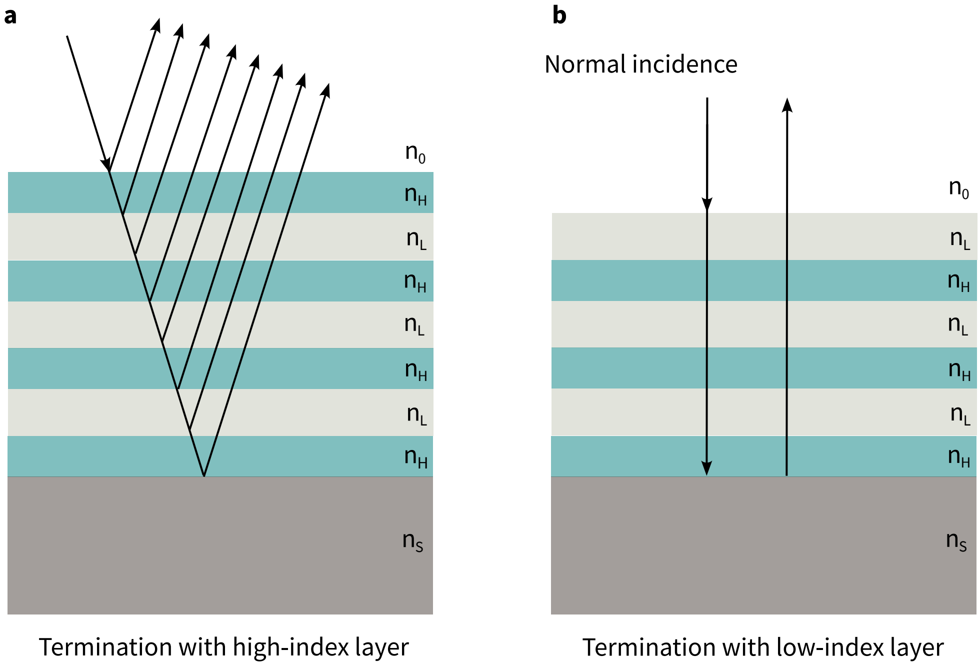

Figure 1 *Distributed Bragg reflector*
a shows rays of light incident at an angle on a quarter-wave stack, which has a high refractive index layer facing the ambient medium. The angle of incidence is zero in **b**, and the structure is terminated by a low index layer.

Commonly grown thin-films include SiO_2, TiO_2, Ta_2O_5, Al_2O_3, and Si_3N_4. The choice of the film material depends on its transparency window, process-dependent film properties, desired stop-band, and mechanical properties of the film. Deposition methods such as ion-beam sputtering, electron-beam evaporation, and magnetron sputtering are used to obtain high-quality films [4].

2 Reflectance and transmittance

At normal incidence, the transfer matrix M of p-pairs of high and low refractive index quarter-wave stacks (Figure 1) can be written as [1, 2, 3]

$$M = \begin{cases} \left(M_\mathrm{H} M_\mathrm{L}\right)^p M_\mathrm{H} & \text{high-index termination} \\ \left(M_\mathrm{H} M_\mathrm{L}\right)^p & \text{low-index termination} \end{cases} \tag{1}$$

where

$$M_\mathrm{H} = \begin{pmatrix} \cos(\phi) & \dfrac{\jmath \sin(\phi)}{n_\mathrm{H}} \\ \jmath\, n_\mathrm{H} \sin(\phi) & \cos(\phi) \end{pmatrix} \tag{2}$$

$$M_\mathrm{L} = \begin{pmatrix} \cos(\phi) & \dfrac{\jmath \sin(\phi)}{n_\mathrm{L}} \\ \jmath\, n_\mathrm{L} \sin(\phi) & \cos(\phi) \end{pmatrix}. \tag{3}$$

Here, $\phi = \dfrac{2\pi}{\lambda_0} n\, d$ is the phase change in a wave as it passes through a layer of refractive index n and physical thickness d. The transfer matrix M is of the form

$$M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \tag{4}$$

and the reflectance R and transmittance T are computed by the app as

$$R = \left(\mathrm{abs}\left(\frac{n_0 m_{11} - n_\mathrm{S} m_{22} + n_0 n_\mathrm{S} m_{12} - m_{21}}{n_0 m_{11} + n_\mathrm{S} m_{22} + n_0 n_\mathrm{S} m_{12} + m_{21}}\right)\right)^2 \tag{5}$$

$$T = \left(\frac{n_\mathrm{S}}{n_0}\right)\left(\mathrm{abs}\left(\frac{2 n_0}{n_0 m_{11} + n_\mathrm{S} m_{22} + n_0 n_\mathrm{S} m_{12} + m_{21}}\right)\right)^2. \tag{6}$$

The `abs` or complex modulus function gets rid of any residual complex values.

For the same number of layer pairs, terminating the stack with a high-index layer, as in Figure 1a, increases the reflectivity. However, some applications such as cavity-coupling of nano-emitters benefit from low-index layer termination, where if required, the reflectivity can be increased by adding more pairs.

3 Points to consider

1. The R and T formulas above do not consider losses due to absorption A and scattering S ($A = S = 0$), and $R + T + S + A = 1$. In high-quality sputtered films, these loses are very low in the range of $10^{-5} - 10^{-4}$ [5].
2. Although refractive index is a function of wavelength, the app only uses one value of respective refractive indices n_0, n_S, n_H, and n_L for the entire wavelength range of $\lambda_0 \pm 400$ nm. We can use the refractive indices at λ_0. Sellmeier's equation [6] has not been used here.

4 A very brief guide

This is a self-contained Python app, which runs on Windows without/with a Python installation.

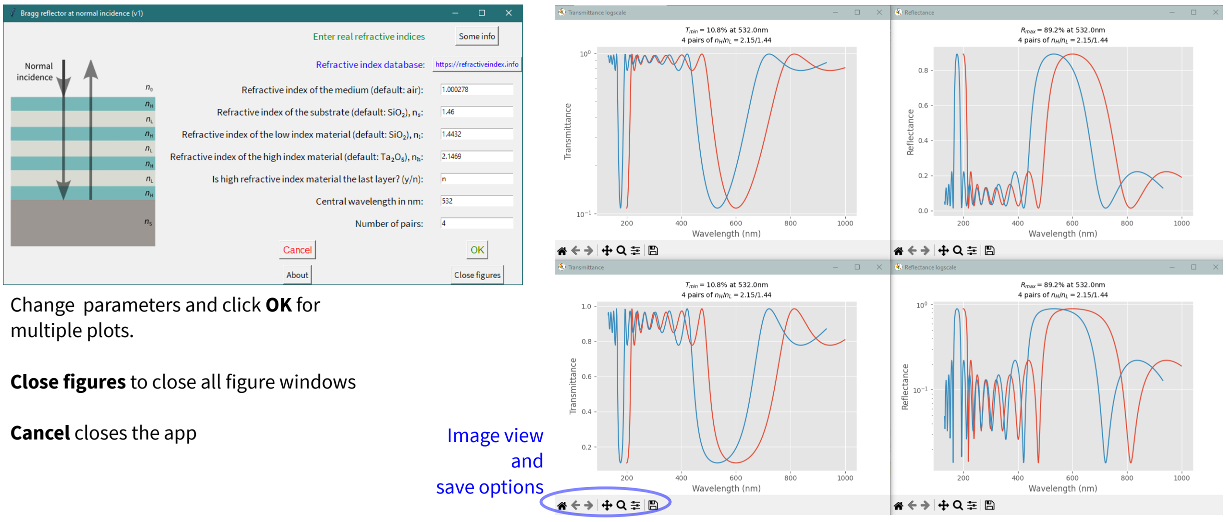

Figure 2 *App interface and output plots*

1) Download the 7-Zip compressed file from `https://archive.org/details/normal-incidence-dbr.-7z` and uncompress it on your Windows machine.

2) Go to the folder `Normal_incidence_DBR`, which contains all the dependencies. Double-click on `Normal_incidence_DBR.exe` to launch the app.

3) The app opens with default parameters (Figure 2), which can be changed accordingly. Click **OK** to generate the plots. The plot-title contains some important information.

4) Each **OK** click creates another line-plot in a different colour on the same axes. This can be used for comparing DBR performance, for example in DBRs with different number of pairs or with different materials. The title, however, corresponds to the latest plot.

5) Image tools at the bottom of each plot windows can be used to zoom-in, modify aspect ratio, and save the file in `svg`, `pdf`, `png`, or `tif` format.

6) Use **Close figures** to start fresh—restarting the `exe` takes time.

5 Working with Python source file

The Python source file can be downloaded from `https://github.com/ardatgithub/Normal_incidence_DBR/blob/main/Normal_incidence_DBR.py` and run directly. The initial execution is faster and the formulas can be updated as required. The dependencies include:

numpy
matplotlib
win32api
tkinter
webbrowser
PIL

Bibliography

[1] N. Kaiser and H. K. Pulker, Eds., *Optical Interference Coatings*. New York: Springer, 2003, vol. 88.

[2] E. Hecht, *Optics*, 5th ed. Essex, England: Pearson Education, 2017.

[3] F. L. Pedrotti, L. M. Pedrotti, and L. S. Pedrotti, *Introduction to Optics*, 3rd ed. Cambridge: Cambridge University Press, 2017.

[4] A. Piegari and F. Flory, Eds., *Optical Thin Films and Coatings: From Materials to Applications*. Woodhead Publishing, 2018.

[5] LAYERTEC GmbH, "Optical coating measurement report (private)," 2019.

[6] B. Saleh and M. Teich, *Fundamentals of Photonics*, 2nd ed. New York: John Wiley & Sons, Inc., 2007.