

ALGEBRAIK SHAKILDAGI KOMPLEKS SONLAR USTIDA AMALLARNI MAPLE TIZIMIDA BAJARISH

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ABSTRACT

Kompleks son haqida asosiy tushunchalar va ta'riflar berilgan. Algebraik shakldagi kompleks sonlar ustida qo'shish va ayirish amallari Maple tizimida bajarilgan.

KIRISH. Hozirgi vaqtda har qanday jiddiy hisob-kitoblar, qoida tariqasida, kompyuterlarda va birinchi navbatda, shaxsiy kompyuterlarda amalga oshiriladi. Ushbu maqolada Maple dasturidan foydalanib kompleks sonlar uchun tuzilgan matematik modellarning samarali va undan foydalanib tahlil va qaror qabul qilishda ahamiyatli ekanligini ko'rsatamiz.

1.Asosiy ta'riflar.

1-ta'rif. a va b -haqiqiy sonlar uchun yozilgan $z = a + ib$ ko'rinishidagi ifodaga kompleks son deb aytiladi.

Bunda $i = \sqrt{-1}$ ($i^2 = -1$) tenglik bilan aniqlanuvchi mavhum birlik deb ataluvchi birlik. z kompleks sonning haqiqiy va mavhum qismlari quyidagicha belgilanadi:

$$\operatorname{Re} z = a, \quad \operatorname{Im} z = b.$$

Xususiyl holda, agar $a = 0$ bo'lsa, u holda $z = 0 + ib$ sonni sof mavhum son, agar $b = 0$ bo'lsa, u holda $z = a + i \cdot 0 = a$, ya'ni haqiqiy son hosil bo'ladi. Shunday qilib, haqiqiy va mavhum sonlar z kompleks sonlarning xususiyl hollaridir.

Kompleks sonning $z = a + bi$ ko'rinishdagi yozuvi uning algebraik shakli deyiladi.

2-ta'rif. Agar ikkita $z_1 = a_1 + ib_1$ va $z_2 = a_2 + ib_2$ kompleks sonlarning haqiqiy qismlari va mavhum qismlari o'zaro teng bo'lsa, bu kompleks sonlar teng, ya'ni $z_1 = z_2$ bo'ladi, ($\operatorname{Re} z_1 = \operatorname{Re} z_2$ va $\operatorname{Im} z_1 = \operatorname{Im} z_2$ bo'lsa, $z_1 = z_2$ hisoblanadi).



3-ta'rif. $z = a + ib$ kompleks sonning haqiqiy va mavhum qismi nolga teng bo'lsagina, u nolga teng bo'ladi, ya'ni agar $a = 0$ va $b = 0$ bo'lsagina $z = 0$, va aksincha.

4-ta'rif. $z = a + ib$ va $\bar{z} = a - ib$ kompleks sonlar *qo'shma kompleks sonlar* deyiladi.

5-ta'rif. $z_1 = a + ib$ va $z_2 = -a - ib$ kompleks sonlar *qarama-qarshi kompleks sonlar* deyiladi.

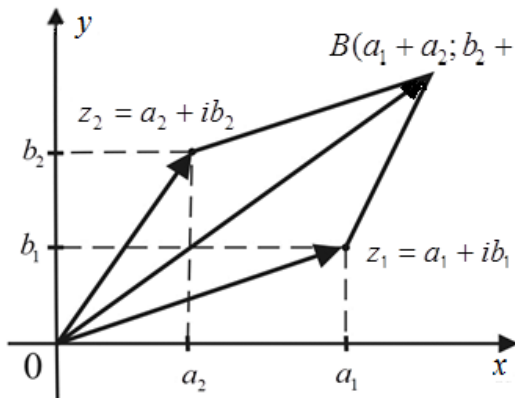
2.Algebraik shakildagi kompleks sonlar ustida amallar

Kompleks sonlarni qo'shish

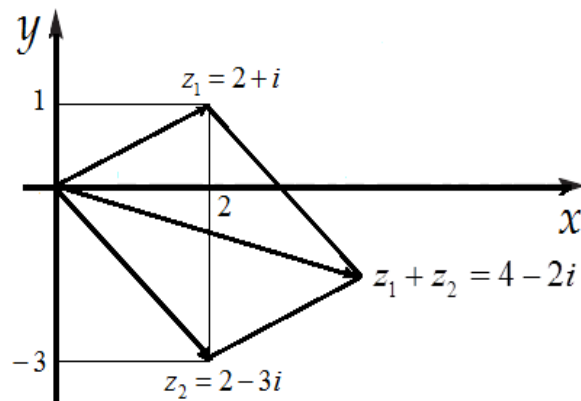
Ikki $z_1 = a_1 + ib_1$ va $z_2 = a_2 + ib_2$ kompleks sonning *yig'indisi* deb,

$$z_1 + z_2 = (a_1 + ib_1) + (a_2 + ib_2) = (a_1 + a_2) + i(b_1 + b_2)$$

tenglik bilan aniqlanuvchi kompleks songa aytiladi. Bu formuladan vektorlar bilan ifodalangan kompleks sonlarni qo'shish vektorlarni qo'shish qoidasi bo'yicha bajarilishi kelib chiqadi (1-rasm).



1-rasm.



1a-rasm.

1-misol. Ushbu $z_1 = 2 + i$ va $z_2 = 2 - 3i$ kompleks sonlarning yig'indisini toping.

Yechish. $z_1 + z_2 = (2 + i) + (2 - 3i) = (2 + 2) + i(1 - 3) = 4 - 2i$.

(1a-rasm)

Maple dasturi

> restart;

> x1:=2;y1:=1; x2:=2;y2:=-3:

$z_1 = 2 + i$ kompleks sonni aniqlash:

> z1:=x1+y1*I; z1 := 2 + I

> y1:=Im(z1);x1:=Re(z1); y1 := 1 x1 := 2



> **polar(z1);** $\text{polar}\left(\sqrt{5}, \arctan\left(\frac{1}{2}\right)\right)$

$z_2 = 2 - 3i$ kompleks sonni aniqlash:

> **z2:=x2+y2*I;** $z_2 := 2 - 3I$

> **y2:=Im(z2);x2:=Re(z2);** $y_2 := -3$ $x_2 := 2$

> **polar(z2);** $\text{polar}\left(\sqrt{13}, -\arctan\left(\frac{3}{2}\right)\right)$

$z_3 = z_1 + z_2 = 4 - 2i$ kompleks sonni aniqlash:

> **z3:=z1+z2;** $z_3 := 4 - 2I$

> **y3:=Im(z3);x3:=Re(z3);** $y_3 := -2$ $x_3 := 4$

> **polar(z3);** $\text{polar}\left(2\sqrt{5}, -\arctan\left(\frac{1}{2}\right)\right)$

Kompleks sonlarni vektorlarini qurish:

> **with(plottools):**

Vz1:=arrow([0,0], [x1,y1],.1,.2,.4, color=blue):

**Vz1a:=arc([0,0],.5,0..arctan(Im(z1),Re(z1)),
color=red):**

Vz2:=arrow([0,0], [x2,y2],.1,.2,.4, color=green):

**Vz2a:=arc([0,0],2,0..arctan(Im(z2),Re(z2)),
color=red):**

Vz3:=arrow([0,0], [x3,y3],.2,.3,.4, color=red):

**Vz3a:=arc([0,0],3,0..arctan(Im(z3),Re(z3)),
color=red):**

Vz13:=arrow([x1,y1],[x3,y3],.1,.2,.4,color=yellow):

Vz23:=arrow([x2,y2],[x3,y3],.1,.2,.4,color=yellow):

plots[display](Vz1,Vz1a,Vz2,Vz2a,Vz3,Vz3a,Vz13,

Vz23, axes=normal,view=[-4..4,-4..4], scaling=constrained);

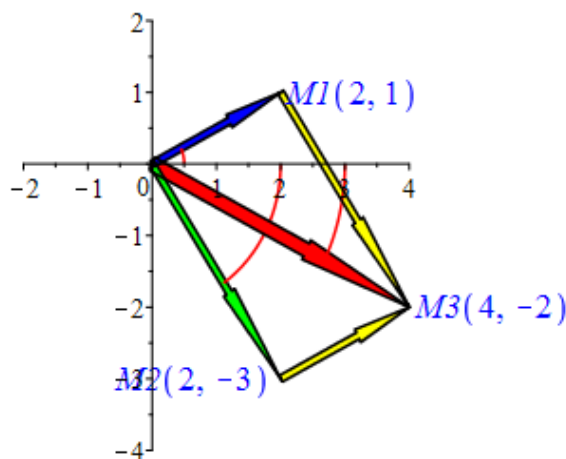
Kompleks sonlarning nuqtalarini qirish.

> **with(plots): Nuqta1:=textplot([[x1,y1,'M1(x1,y1)']], color=blue,align=Right,
font=[TIMES,ROMAN,14]):**

> **Nuqta2:=textplot([[x2,y2,'M2(x2,y2)']], color=blue, align=Left,
font=[TIMES,ROMAN,14]):**

> **Nuqta3:=textplot([[x3,y3,'M3(x3,y3)']], color=blue, align=Right,
font=[TIMES,ROMAN,14]):**

> **KSA:=plots[display](Vz1,Vz1a,Vz2,Vz2a,Vz3,Vz3a, Vz13,
Vz23,Nuqta1,Nuqta2,Nuqta3,axes=normal,
view=[-2..4,-4..2],scaling=constrained); (1a-rasm)**



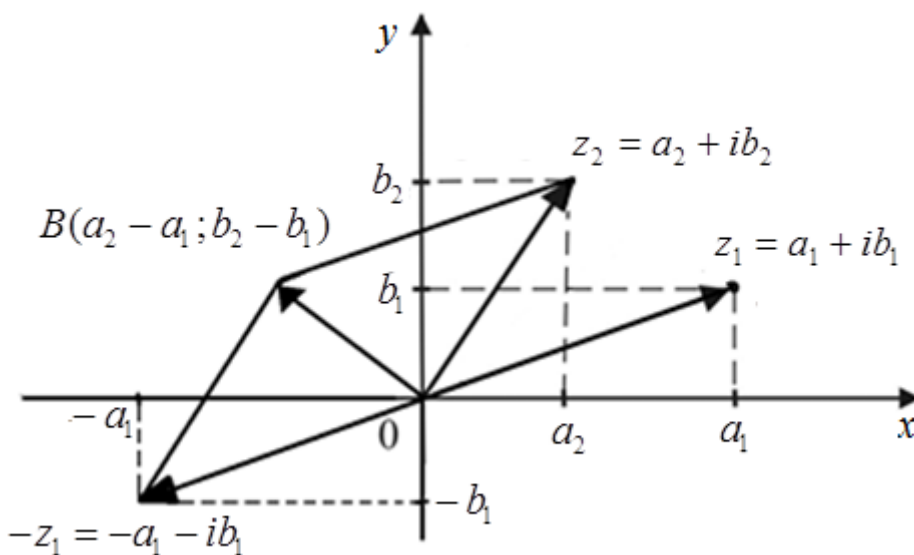
1a-rasm.

Kompleks sonlarni ayirish.

Ikkita $z_1 = a_1 + ib_1$ va $z_2 = a_2 + ib_2$ kompleks sonning ayirmasi deb, shunday songa aytiladiki, u z_2 ga qo'shilganda yig'indida z_1 kompleks son hosil bo'ladi (2-rasm). Demak,

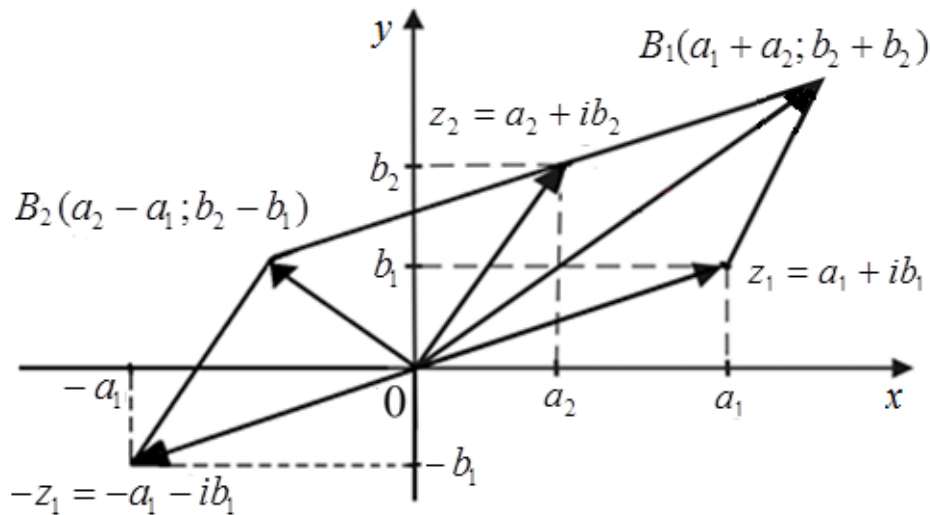
$$z_2 - z_1 = (a_2 + ib_2) - (a_1 + ib_1) = (a_2 - a_1) + i(b_2 - b_1).$$

Demak, grafigini qurishda $z_2 - z_1$ ayirma $z_2 + (-z_1)$ yig'indi bo'lishini e'tiborga olamiz.



2-rasm.

Ikkita $z_1 = a_1 + ib_1$ va $z_2 = a_2 + ib_2$ kompleks sonning yig'indisi va ayirmasini bitta koordinatalar sistemasida quyidagicha quramiz (3-rasm).



3-rasm.

Shuni ta'kidlab o'tamiz, ikki kompleks son ayirmasining moduli kompleks tekislikda shu sonlarni ifodalovchi nuqtalar orasidagi masofaga teng:

$$|z_2 - z_1| = \sqrt{(a_2 - a_1)^2 + (b_2 - b_1)^2}.$$

2-misol. Ushbu $z_1 = 4 - 3i$ va $z_2 = 2 + 6i$ kompleks sonlarning yig'indis va ayirmasini toping.

Yechish.

$$z_2 + z_1 = (2 + 6i) + (4 - 3i) = (2 + 4) + i(6 - 3) = 6 + 3i.$$

$$z_2 - z_1 = (2 + 6i) - (4 - 3i) = (2 - 4) + i(6 + 3) = -2 + 9i.$$

Bu kompleks sonlarni ayirmasini Maple dasturida qurish ucun

$$z_2 - z_1 = z_2 + (-z_1)$$

qoidani e'tiborga olamiz(2-rasm).

Maple dasturi

> restart;

$$z_1 = 4 - 3i \text{ kompleks sonni aniqlash:}$$

> x1:=4;y1:=-3; x2:=2;y2:=6:

> z1:=x1+y1*I; $z1 := 4 - 3I$

> x1:=Re(z1); y1:=Im(z1); $x1 := 4 \quad y1 := -3$

> polar(z1); $\text{polar}\left(5, -\arctan\left(\frac{3}{4}\right)\right)$

$$z_2 = 2 + 6i \text{ kompleks sonni aniqlash:}$$

> z2:=x2+y2*I; $z2 := 2 + 6I$



> $x_2 := \text{Re}(z_2)$; $y_2 := \text{Im}(z_2)$; $x_2 := 2$ $y_2 := 6$

> $\text{polar}(z_2)$; $\text{polar}(2\sqrt{10}, \arctan(3))$

$z_3 = z_2 + z_1 = 6 + 3i$ kompleks sonni aniqlash:

> $z_3 := z_2 + z_1$; $z_3 := 6 + 3i$

> $y_3 := \text{Im}(z_3)$; $x_3 := \text{Re}(z_3)$; $y_3 := 3$ $x_3 := 6$

> $\text{polar}(z_3)$; $\text{polar}\left(3\sqrt{5}, \arctan\left(\frac{1}{2}\right)\right)$

$z_4 = z_2 - z_1 = -2 + 9i$ kompleks sonni aniqlash:

> $z_4 := z_2 - z_1$; $z_4 := -2 + 9i$

> $y_4 := \text{Im}(z_4)$; $x_4 := \text{Re}(z_4)$; $y_4 := 9$ $x_4 := -2$

> $\text{polar}(z_4)$; $\text{polar}\left(\sqrt{85}, -\arctan\left(\frac{9}{2}\right) + \pi\right)$

Kompleks sonlarning yig'indisi va ayirmasini qurish:

> **with(plottools):**

Vz1:=arrow([0,0], [x1,y1], .1, .2, .4, color=blue):

Vz1a:=arc([0,0],1,0..arctan(Im(z1),Re(z1)), color=red):

Vz2:=arrow([0,0],[x2,y2], .1, .2,.4,color=green):

**Vz2a:=arc([0,0],2,0..arctan(Im(z2),Re(z2)),
color=red):**

Vz3:=arrow([0,0], [x3,y3], .2, .3, .4, color=red):

**Vz3a:=arc([0,0],3,0..arctan(Im(z3),Re(z3)),
color=red):**

Vz13:=arrow([x1,y1], [x3,y3], .1, .2, .4, color=yellow):

Vz23:=arrow([x2,y2], [x3,y3], .1, .2, .4, color=yellow):

Vz4:=arrow([0,0], [x4,y4], .2, .3, .4,color=red):

**Vz4a:=arc([0,0],3,0..arctan(Im(z4),Re(z4)),
color=red):**

Vz1q:=arrow([0,0], [-x1,-y1], .2, .3, .4, color=yellow):

Vz14:=arrow([-x1,-y1], [x4,y4], .1, .2, .4, color=yellow):

Vz24:=arrow([x2,y2], [x4,y4], .1, .2, .4, color=yellow):

plots[display](Vz1,Vz1a,Vz2,Vz2a,Vz3,Vz3a,Vz13,

Vz23,Vz4,Vz4a,Vz14,Vz24,Vz1q, axes=normal,

view=[-4..6,-4..10],scaling=constrained);(4-rasm)

Kompleks sonlarning yig'indisini koordinatalari bilan qurish:

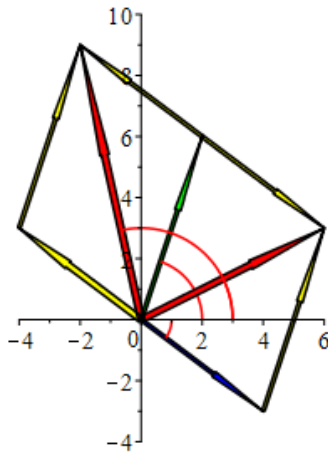
> **with(plots): Nuqta1:=textplot([[x1,y1,'M(x1,y1)']], color=blue,align=Right,
font=[TIMES,ROMAN,14]):**

> **Nuqta2:=textplot([[x2,y2,'M(x2,y2)']], color=blue,align=Right,
font=[TIMES,ROMAN,14]):**

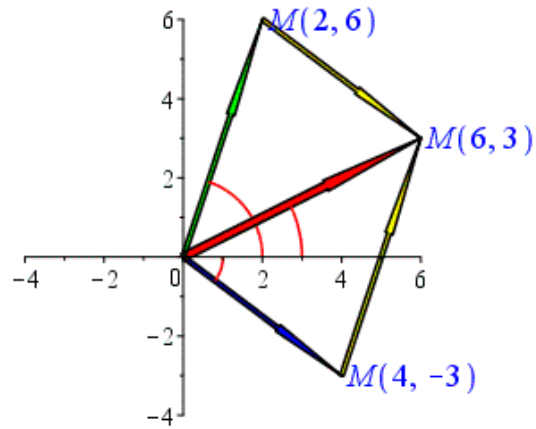
> **Nuqta3:=textplot([[x3,y3,'M(x3,y3)']], color=blue,align=Right,
font=[TIMES,ROMAN,14]):**

> **KSA:=plots[display](Vz1,Vz1a,Vz2,Vz2a,Vz3,Vz3a,**

Vz13, Vz23, Nuqta1, Nuqta2, Nuqta3, axes=normal, view=[-4..6,-4..6], scaling=constrained); (5-rasm)



4-rasm.



5-rasm.

Xulosa. Maple dasturining imkoniyatlarini kompleks sinlar uchun qo'llanishi o'quvchida tasviriy fikrlash, masalani yechishning programmalash va animatsiyalash imkonoyatini hamda kompleks sonlarni koeffitsientlariga qarab tez va aniq qurish va qo'shishda Maple dasturini qo'llash usullari ko'rsatilgan.

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