

## COMPUTATION OF PLASTIC LIMIT USING MATHEMATICAL MODEL

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### ABSTRACT

*This paper presents a mathematical model that uses a British cone to predict the soil moisture-penetration relationship. This relationship is nonlinear for non-bentonite bearing soil and can be modelled using a sigmoidal mathematical function. The developed mathematical model gives a unique opportunity for computing and predicting the fall cone penetration of the soil even at a very low moisture content, at which, due to practical limitations, it is impossible to determine the soil penetration. The mathematical model was found to fit the moisture-penetration data reasonably well. Based on the data presented in this paper, both the liquid limit and plastic limit can be determined from the extended liquid limit flow curve by determining the plastic limit at a 1.2 mm penetration value. The computed plastic limit at a 1.2 mm penetration value was found to be in close agreement with the measured plastic limit.*

### INTRODUCTION

Determination of the plastic limit is very subjective and error-prone. It has been suggested by many researchers [1,2] to extrapolate the plastic limit from the cone penetrometer flow curve. The moisture content corresponding to 2 mm penetration has been suggested to be taken as the plastic limit of the soil. However, in practice, due to practical limitations, it is difficult or impossible to achieve soil penetration of less than 3 mm such that the plastic limit can be directly read-off from the cone penetration flow curve. Therefore, many researchers have proposed extrapolation procedures. Little has been done to develop a mathematical model that can be used to compute or predict the moisture-penetration relationship at a lower penetration value of less than 3 mm.

This paper presents a sigmoidal mathematical model that can be used to compute and predict

the soil's fall-cone penetration at any moisture content. The proposed mathematical model was found to fit the moisture-penetration data reasonably well except for bentonite bearing soil

### THE FALL CONE LIQUID LIMIT FLOW CURVE

In accordance with BS 1377: Part 2:1990 [3], the liquid limit fall cone flow curve is modelled as linear within the penetration range of  $20 \pm 5$  mm. This study indicates that this linear penetration is valid within a narrow range of moisture-penetration relationships. For the soil tested during this study, the test results suggest that within the moisture-penetration value of 20-4 mm, the liquid limit flow curve of non-bentonite bearing soil is highly nonlinear. However, the moisture-penetration relationship for bentonite bearing soil was found to be highly linear within the penetration range of 20-4 mm, suggesting that a separate mathematical model is required to define its moisture-penetration relationship.

The fall cone liquid limit flow curve of non-bentonite bearing soil is nonlinear. The fall cone liquid limit flow curve can be modelled using a nonlinear sigmoidal mathematical model shown in equation (1).

$$h = \frac{40}{1 + \left(\frac{x}{LL}\right)^n} \dots \dots (1)$$

Where 'h' is the penetration depth at any moisture content, 'x' is the moisture content, 'LL' is the liquid limit and 'n' is the flow curve constant. Differentiating equation (1) with respect to 'x', the slope of the flow curve at any moisture content can be computed using equation (2).

$$\frac{dh}{dx} = m = - \frac{40 \left(\frac{x}{LL}\right)^n n}{\left(1 + \left(\frac{x}{LL}\right)^n\right)^2} \dots \dots (2)$$

The average slope of the flow curve is determined during the liquid limit testing by establishing the moisture-penetration relationship within the moisture-penetration range of 25 -5 mm. Therefore, by replacing 'x' with Liquid Limit (LL), equation (2) is reduced to equation (3).

$$\frac{dh}{dx} = m = -\frac{10n}{LL} \dots \dots (3)$$

Making 'n' a subject, the equation (3) is transformed into equation (4).

$$n = -0.1 m LL \dots \dots (4)$$

Since the average slope of the flow curve at the liquid limit 'm' is known, then the value of the flow curve constant 'n' can be computed using equation (4). Substituting 'n' in equation (1) with equation (4), the equation (1) is transformed into equation (5)

$$h = \frac{40}{1 + \left(\frac{x}{LL}\right)^{-(0.1 LL m)}} \dots \dots (5)$$

From Equation (5), it can be seen that the fall cone liquid limit flow curve is controlled by the liquid limit and the average slope of the flow curve.

The accuracy of this proposed mathematical model was validated using test results of soil samples of varying liquid limits carried out during this study. The summary of the test results is shown in Figures 1,2, and 3. The model and the determined value of moisture and penetration are superimposed on the same graph. It can be seen that the mathematical model fits the moisture-penetration data very well while at the same time predicting penetration value at low moisture content. The predicted penetrations at the plastic limit for soils shown in Figures 1, 2 and 3 were found to be 3.48 mm, 1.28 mm, and 1.56 mm, respectively, with an average of 2.11 mm.

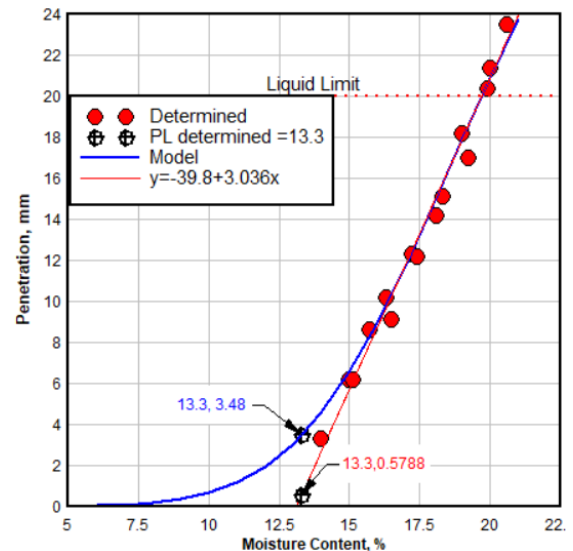


Figure 1: Reddish silty clay soil

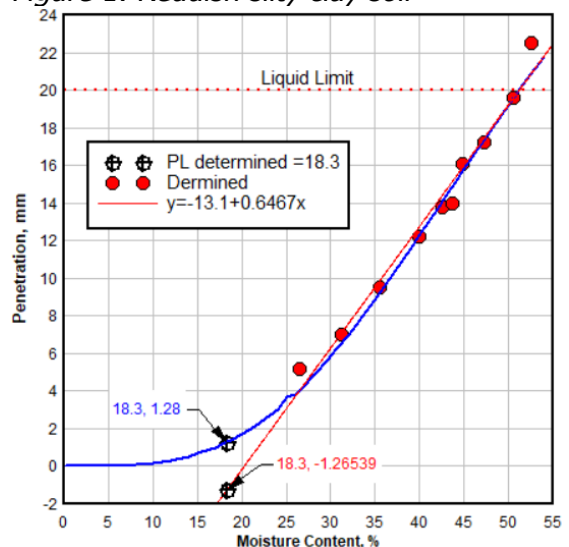


Figure 2: Brownish silty clay soil

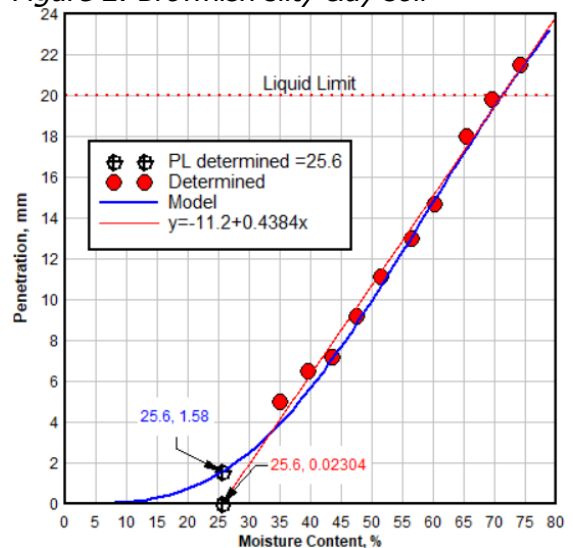


Figure 3: Clayey silty Soils

The proposed mathematical model was further validated using soil mixed with commercial

bentonite. The model fits the moisture-penetration data reasonably well for the soil mixed with 5% bentonite. However, the model deviates from the measured moisture-penetration values as the percentages of bentonite increase. The mathematical model predicts lower penetration values at higher bentonite content than the measured penetration value. It can be seen that the flow curve of the bentonite bearing soils becomes increasingly linear as bentonite content increases. The summary of the test results is shown in Figures 4,5,6, and 7.

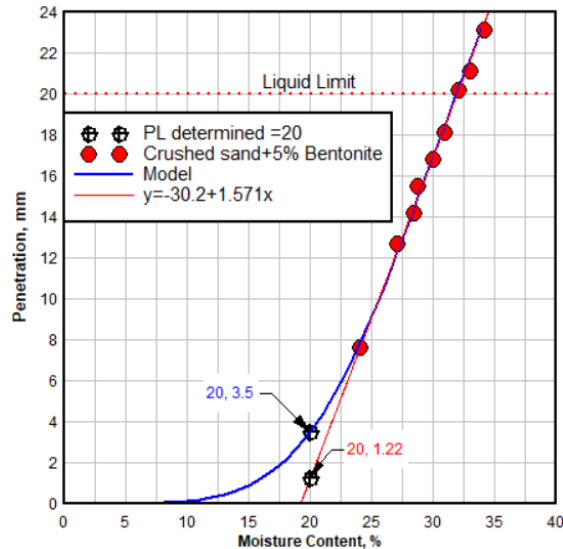


Figure 4: Crushed sand with 5% of bentonite

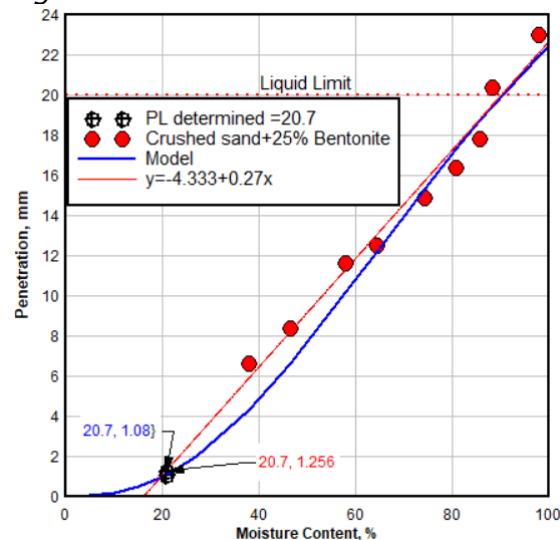


Figure 5: Crushed sand with 25% of bentonite

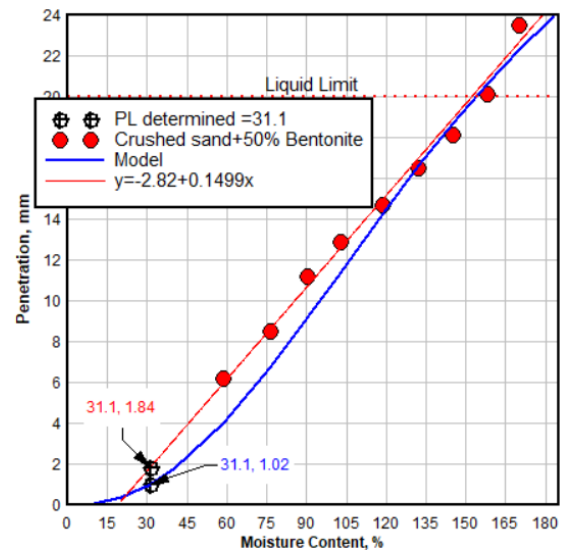


Figure 6: Crushed sand with 50% of bentonite

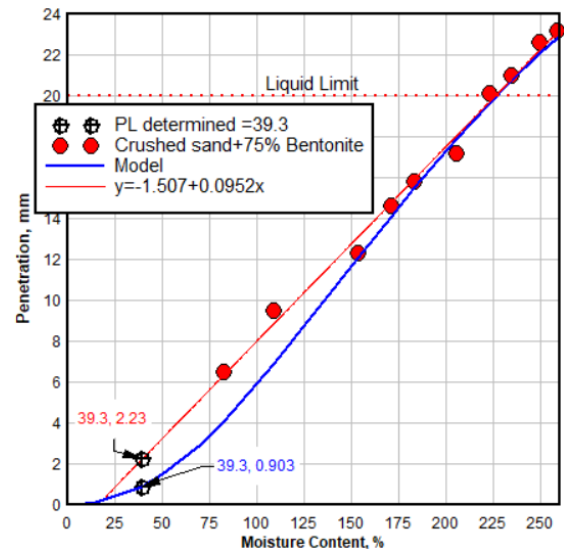


Figure 7: Crushed sand with 75% of bentonite

Feng (2001, 2004) [4,5] presented the liquid and plastic limit data for 30 soils (in 2001) and 21 soils (in 2004). The cone penetrometer liquid limit flow curves were fitted using a linear log-log model in the form of

$$\log w = \log c + m \log d \dots (6)$$

Where 'w' is the water content, 'c' is the water content at  $d=1$  mm, 'm' is the slope of the flow curve, and 'd' is the penetration depth. Selected soil data were generated from equation (6) and transformed into a linear model in the form shown in equation (7). The summary of transformed fall cone liquid limit flow curve data is shown in Figures 8-11.

$$y = mx + c \dots (7)$$

As for the soil tested during this study, the non-bentonite bearing fall cone liquid limit flow curve was nonlinear, while the bentonite bearing soil was linear. The model fitted the non-bentonite bearing soil and ca-montmorillonite soil reasonably well (Figure 11), suggesting that the proposed sigmoidal mathematical model can fit the fall cone liquid limit flow curve for all types of soil except bentonite bearing soils. The mathematical model was not validated using Na-montmorillonite clay soil. The modelled liquid limit flow curves are shown in Figures 8, 9, 10 and 11.

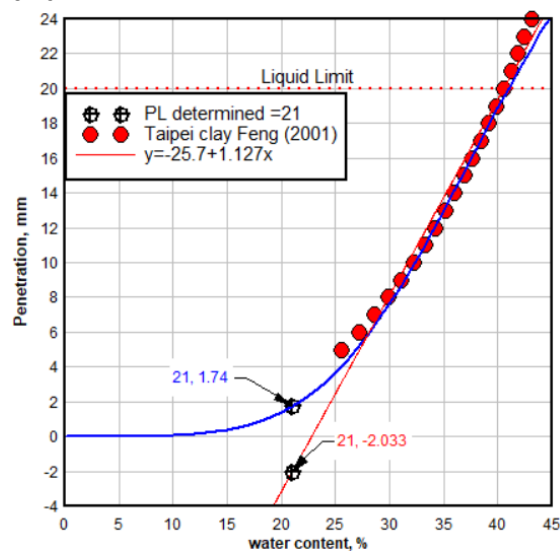


Figure 8: Flow curve for Taipei clay-sample 2 (Feng, 2001)

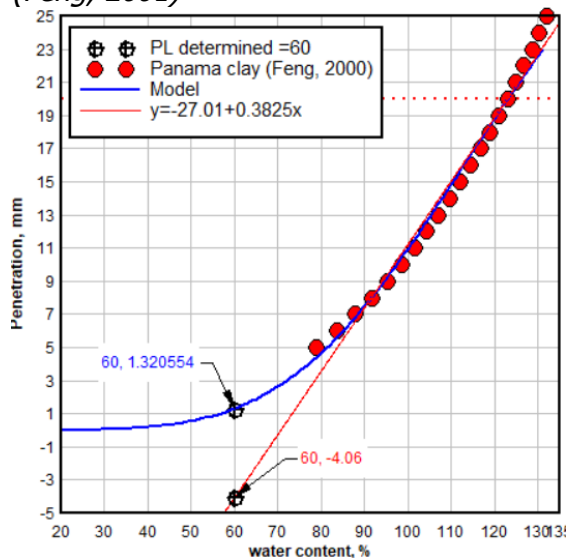


Figure 9: Flow curve for Panama clay-sample 7 (Feng, 2001)

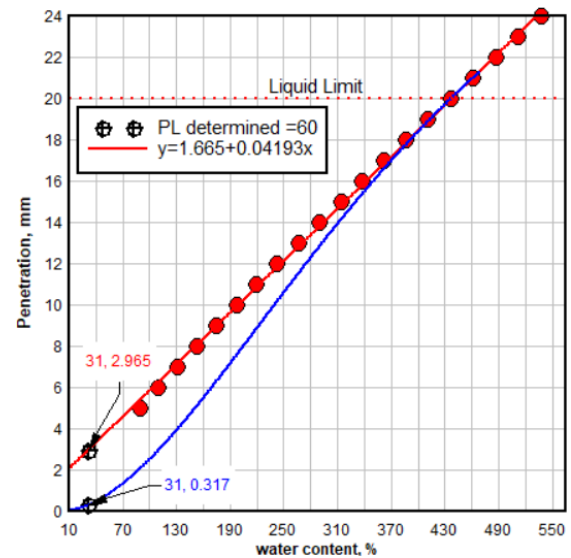


Figure 10: Flow curve for Bentonite B (Feng, 2004)

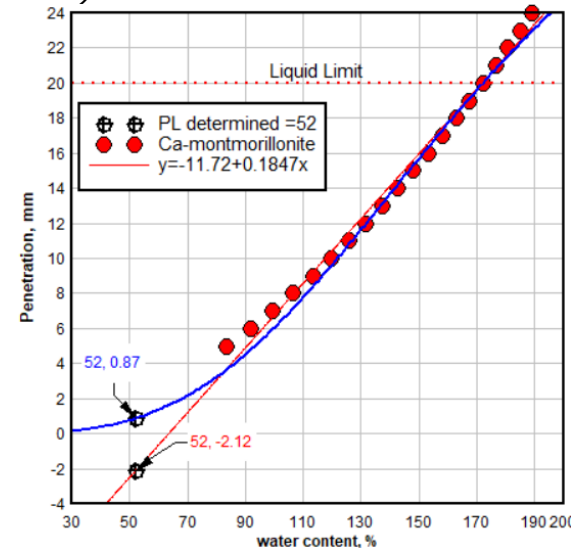


Figure 11: Flow curve for ca-montmorillonite (Feng, 2004)

## PENETRATION OF SOIL AT PLASTIC LIMIT

The penetration at the plastic limit can be computed using either equation (5) or (1) by substituting 'x' with 'PL'. The plastic limit computation was carried out for the soils tested during this study (3 soils), Feng(2001) (28 soils, two bentonite soil samples 9 and 21 were omitted from this analysis), Feng (2004) (19 soils, two bentonite samples-bentonite (B) and 50%B+50%K were omitted from this analysis). The summary of the computed penetration value at the plastic limit is shown in Figures 12 and 13. It can be seen that the penetration at the plastic limit is primarily confined within the narrow range of 0.5-2.0 mm, with an average penetration of

1.2 mm (3 penetration values were treated as outliers. The standard deviation of penetration data was 0.67 mm). Because the variability of the penetration data is minimal, as evidenced by the standard deviation, the average penetration at the plastic limit can be reasonably assumed to be the soil penetration at the plastic limit. Substituting the average penetration at the plastic limit of 1.2 mm in equations (1) and (5) and making the plastic limit the subject, the plastic limit can be computed using equations (8) or (9).

$$PL = LL e^{-\frac{3.476}{n}} \dots \dots (8)$$

$$PL = LL e^{-\frac{34.76}{LL m}} \dots \dots (9)$$

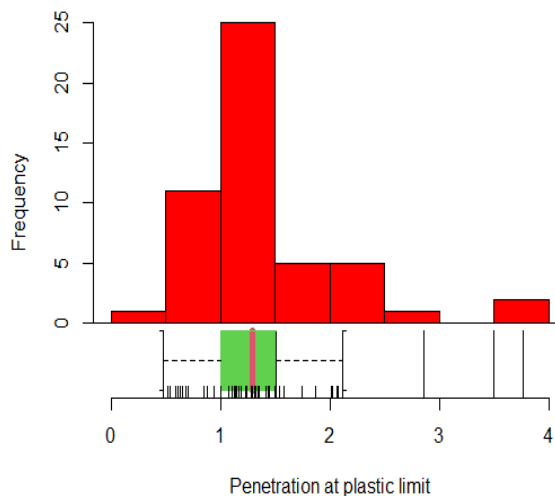


Figure 12: Histogram and boxplot showing the penetration at the plastic limit

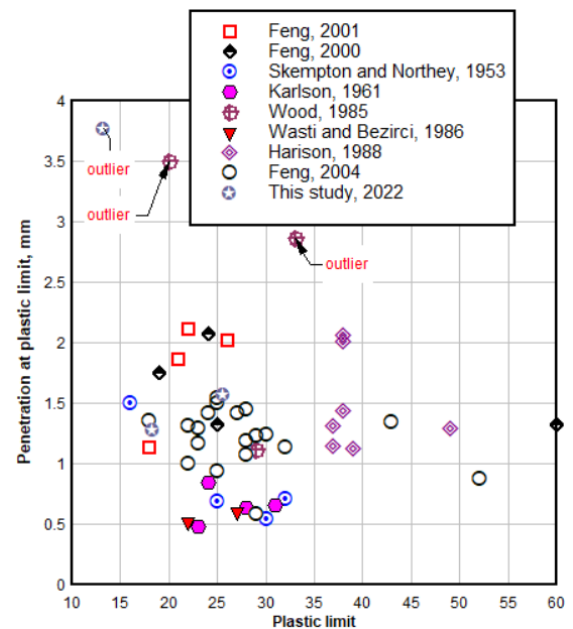


Figure 13: Computed penetration at plastic limit

### COMPUTATION OF PLASTIC LIMIT USING MATHEMATICAL MODEL

Computation of plastic limits of 50 soils using equation (8) or (9) indicates a very close agreement between the computed plastic limit and the measured plastic limit, as presented in Figures 14, 15 and 16.

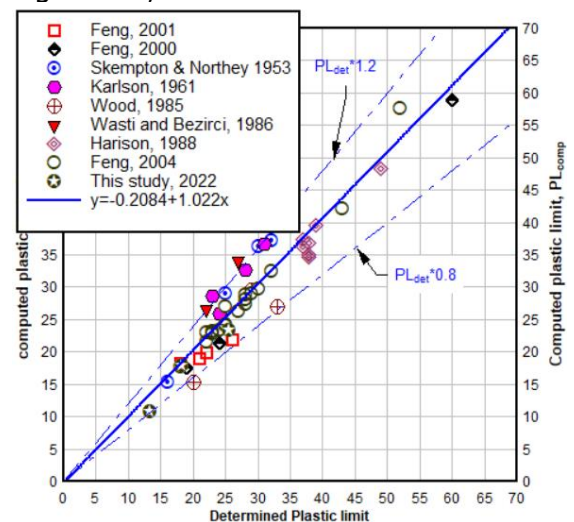


Figure 14: Comparison between measured and computed plastic limits (50 soil samples)

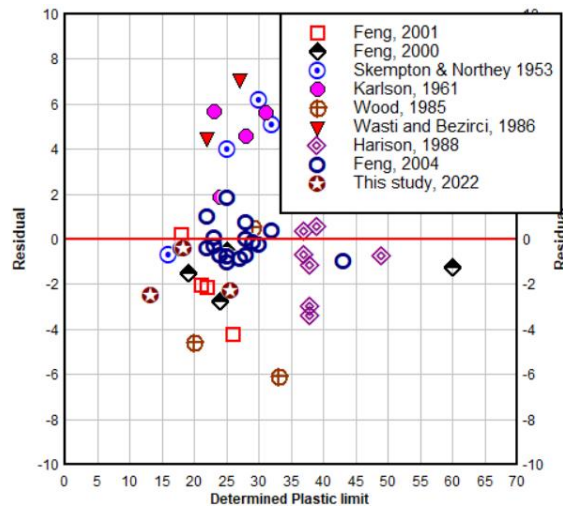


Figure 15: Difference between the measured, computed plastic limits (50 soil samples)

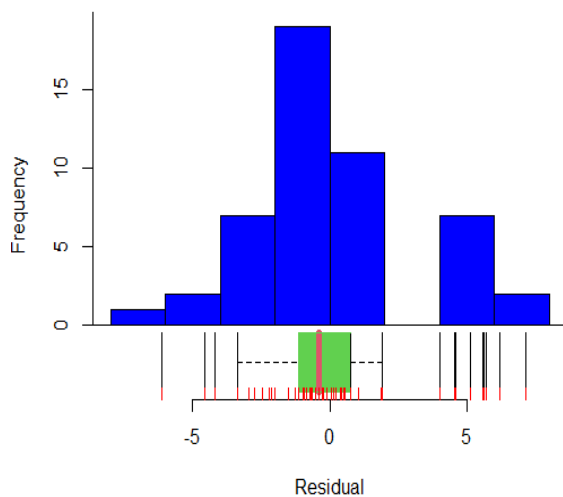


Figure 16: Residual histogram

## SUMMARY AND CONCLUSION

Based on the test results carried out during this study, as well as analysis of the Atterberg limits data from the literature, the following main conclusion can be drawn:

- The proposed sigmoidal mathematical model can be used to compute and predicts the penetration-moisture flow curve; therefore can be used for modelling the fall cone liquid limit flow curve.
- The fall cone liquid limit flow curve for non-bentonite bearing soil is nonlinear and can be modelled using the proposed sigmoidal mathematical model
- The fall cone liquid limit flow curve of bentonite bearing soil is highly linear within

the moisture-penetration range of 20-5 mm, thus cannot be modelled using the proposed sigmoidal mathematical model

- The plastic limit computed using the proposed mathematical model at a 1.2 mm penetration value is in close agreement with the measured plastic limit.
- Using the proposed sigmoidal mathematical model for computing the plastic limit will improve the repeatability and reproducibility of the plastic limit determination, which is ordinarily poor due to the subjectivity nature of determining the plastic limit using a conventional method of thread rolling.

## References:

1. **Towner, G.D (1973)**, *An examination of the fall cone method for the determination of some strength properties of remoulded agricultural soils*, Journal of soil science, Vol 24, NO.4, pp470-479
  2. **Harison, J.A, (1988)**, *Using the BS cone penetrometer for the determination of plastic limit of soils*, Geotechnique, Vol.38, No.3 pp433-438
  3. **BS 1377: Part 2: (1990)**: *Soil classification test*, British Standard Institution
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