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ONE-DIMENSIONAL NON-LINEAR CAUCHY PROBLEM

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ABSTRACT. This note presents a one-dimensional nonlinear Cauchy problem solution.

Considering the one-dimensional nonlinear Cauchy problem, [1] (no initial or boundary conditions):

$$\frac{d}{dt}f(t,x,\nu,..)+f(t,x,\nu,..)\frac{d}{dx}f(t,x,\nu,..)-\nu\frac{d^2}{dx^2}f(t,x,\nu,..)=0, \tag{0.1}$$

where $x\in\mathbf{R}$, and $\nu\in\mathbf{R}_+$. Suppose $f(t,x,\nu,..)=\alpha(t,\nu,..)g(x,\nu,..)$, where $g(x,\nu,..)$ is the solution to

$$\mathbf{L}(g)=g\frac{dg}{dx}-\nu\frac{d^2g}{dx^2}=0, \tag{0.2}$$

substituting $f(t,x,\nu,..)$ into the equation (0.1) leaves us with the famous Bernoulli equation [2], with $n=2$ and $P(t)=Q(t)=1$:

$$\frac{d\alpha}{dt}+g^{-1}\Upsilon(\alpha^2-\alpha)=0, \tag{0.3}$$

which contains the solution to $t\in\mathbf{R}_+$:

$$\alpha(t,\nu,c_1)=(e^{-\frac{\Upsilon}{g}t}(c_1-e^{-\frac{\Upsilon}{g}t}))^{-1}, \tag{0.4}$$

it is trivial to show that:

$$\mathbf{L}(g(x,\nu,d_1,d_2))=\mathbf{L}(\sqrt{2}\sqrt{\nu}\sqrt{d_1}tan(\frac{\sqrt{2}\sqrt{d_1}x+\sqrt{2}\sqrt{d_1}d_2}{2\sqrt{\nu}}))=0, \tag{0.5}$$

where d_1 and d_2 are constants of integration, and thus

$$\Upsilon(x,\nu,d_1,d_2)=d_1tan(\frac{\sqrt{2}\sqrt{d_1}x+\sqrt{2}\sqrt{d_1}d_2}{2\sqrt{\nu}})sec^2(\frac{\sqrt{2}\sqrt{d_1}x+\sqrt{2}\sqrt{d_1}d_2}{2\sqrt{\nu}}), \tag{0.6}$$

let us take:

$$g(x,\nu,1,-\frac{\pi}{2})=\sqrt{2\nu}tan(\frac{1}{2}\sqrt{\frac{2}{\nu}}(x-\frac{\pi}{2})), \tag{0.7}$$

$$\Upsilon(x,\nu,1,-\frac{\pi}{2})=tan(\frac{1}{2}\sqrt{\frac{2}{\nu}}(x-\frac{\pi}{2}))sec^2(\frac{1}{2}\sqrt{\frac{2}{\nu}}(x-\frac{\pi}{2})), \tag{0.8}$$

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$$g^{-1}(x,\nu,1,-\frac{\pi}{2})\Upsilon(x,\nu,1,-\frac{\pi}{2}) = \frac{1}{\sqrt{2\nu}}sec^2(\frac{1}{2}\sqrt{\frac{2}{\nu}}(x-\frac{\pi}{2})), \tag{0.9}$$

hence

$$\alpha(t,\nu,1)g(x,\nu,1,-\frac{\pi}{2}) = (e^{-\frac{\Upsilon}{g}t}(1-e^{-\frac{\Upsilon}{g}t}))^{-1}tan(\frac{1}{2}\sqrt{\frac{2}{\nu}}(x-\frac{\pi}{2})), \tag{0.10}$$

(0.11)

$$f(t,x,\nu,1,1,-\frac{\pi}{2}) = (e^{-\frac{\Upsilon}{g}t}(1-e^{(-\frac{1}{\sqrt{2\nu}}sec^2(\frac{1}{2}\sqrt{\frac{2}{\nu}}(x-\frac{\pi}{2})))t}))^{-1}tan(\frac{1}{2}\sqrt{\frac{2}{\nu}}(x-\frac{\pi}{2})),$$

to exemplify:

$$f(0,x,\frac{1}{2},1,1,-\frac{\pi}{2}) = \infty tan(x-\frac{\pi}{2}), \tag{0.12}$$

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