

Block-Wise Bordered and Pandiagonal Magic Squares Multiples of 3

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Abstract

*During past years author worked with **block-wise bordered** magic squares of even orders. It includes blocks of orders 4, 6, 8, 10, etc. Most of the cases are with equal sums magic squares. This type of work is an extension of classical bordered magic squares. In case of multiples of 4, the extension is made for pandiagonal magic squares [25]. For multiples of order 6 refer Taneja [26]. For the first time, we are presenting here bordered magic squares of odd number blocks. Specially in this work, we give bordered with different sum magic squares of order 3. Pandiagonal magic squares multiples of 3 are also given. These we get for all orders starting from order 3, except orders 18, 30, 42, etc. The work is up to order 36. Higher orders examples can be seen in Excel fileis attached with the work. It also include pandiagonal magic squares of equal sums up to order 117. The total work is up to order 120.*

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1 Introduction

During past years author [4, 5, 6, 7, 8, 9, 10] worked with **block-wise** magic squares from orders 12 to 47. Author [11, 12, 13, 14, 15, 16] also worked with **bordered** magic squares. The study on **bordered** magic squares is extended to **block-bordered** magic squares [17, 18, 19]. This is specially done for the magic squares of orders p and p , where p is a prime number. This study is still extended to **block-wise bordered** magic squares [20, 21, 22, 23]. Some conection with Pythagorean triples and area-representations are also made [25, 26, 27, 28, 29]. The main property of **bordered** magic squares is that if we remove external borders, still we get **sub-bordered** magic squares, i.e., each layer in itself lead us to magic squares. In many cases, the properties of **bordered** magic square are seperated by **even** and **odd** orders magic squares. In many cases, we get good properties for the **even** order **bordered** magic squares. In many cases, we have to use fractional numbers entries, specially to reach minimum perfect square sum of entries. For more study on **bordered** magic squares refer H. White’s web-site [3].

The aim of this work is to combine the study of **block-wise** and **bordered** magic squares. This kind of study still not seen by author. In this case we considers blocks of magic squares such as magic squares of order 4 and then put them in such a way that every time removing external borders, still we are left with magic squares. Based on this idea, we wrote with **block-wise bordered** magic squares of orders 108 and 104. Every time when we remove the external, we are left with **block-wise bordered** magic squares with minus order 8. For example, in case of order 108, removing external orders we are left with orders 100, 92, 84, etc. and in case of orders 104, removing external orders we are left with orders 96, 88, 80, etc. Thus alternatively we complete all order magic squares multiples of 4. The first two orders 4 and 8 are not **block-wise bordered** magic squares. From order 12 onwards, we always get **block-wise bordered** magic squares multiples of 4, i.e., of orders 12, 16, 20, etc. In all the situations the constructions of magic squares of order 4 are **pandiagonal** and of equal sums, while the **block-wise bordered** magic squares are not **pandiagonal**. In each case, if we redistribute the blocks of order 4 already constructed we reach to **pandiagonal** magic squares of orders 12, 16, 20, etc. but unfortunately they are no more **block-wise bordered** magic squares. Before proceeding further, let's classify the idea of bordered magic squares:

1.1 Classification of Bordered Magic Squares

- **Single Digit:** Bordered magic squares based on single digit [11, 12, 3].
- **Two Digits:** Bordered magic squares based on magic rectangles multiples of 2 [57, 58, 59, 60, 60].
- **Three Digits:** Bordered magic squares based on magic squares of order 3. This work.
- **Four Digits:** Bordered magic squares based on magic squares of order 4 [25].
- **Five Digits:** Bordered magic squares based on magic squares of order 5 (appearing soon).
- **Six Digits:** Bordered magic squares based on magic squares of order 6 [26], etc.

For the first time, we are presenting here bordered magic squares of odd number blocks. Specially in this work, we give bordered with different sum magic squares of order 3. The procedure to get these bordered magic squares is also given. Pandiagonal magic squares multiples of 3 are also given. These works for all orders starting from order 9, except orders 18, 30, 42, etc. Here, the work is up to order 36. Higher orders examples can be seen in **Excel file** is attached with this work.

2 Block-Wise Bordered Magic Squares of Orders 33 and 36

2.1 Magic Squares of Orders 33 and 36

Let's consider bordered magic square of orders 11 and 12 given by

11												671
112	102	104	106	108	9	8	6	4	2	110	671	
19	94	86	88	90	27	26	24	22	92	103	671	
17	35	80	74	76	41	40	38	78	87	105	671	
15	33	47	70	66	51	50	68	75	89	107	671	
13	31	45	55	64	57	62	67	77	91	109	671	
11	29	43	53	59	61	63	69	79	93	111	671	
115	97	83	73	60	65	58	49	39	25	7	671	
117	99	85	54	56	71	72	52	37	23	5	671	
119	101	44	48	46	81	82	84	42	21	3	671	
121	30	36	34	32	95	96	98	100	28	1	671	
12	20	18	16	14	113	114	116	118	120	10	671	
671	671	671	671	671	671	671	671	671	671	671	671	

12												870
133	124	126	141	143	22	20	18	5	3	1	134	870
17	113	108	38	106	40	36	26	120	24	114	128	870
14	35	48	42	102	104	91	53	93	47	110	131	870
13	111	45	86	84	57	90	58	60	100	34	132	870
8	33	46	56	71	76	65	78	89	99	112	137	870
6	118	51	62	66	77	72	75	83	94	27	139	870
129	23	101	64	80	67	74	69	81	44	122	16	870
130	115	96	82	73	70	79	68	63	49	30	15	870
135	29	95	85	61	88	55	87	59	50	116	10	870
136	117	98	103	43	41	54	92	52	97	28	9	870
138	31	37	107	39	105	109	119	25	121	32	7	870
11	21	19	4	2	123	125	127	140	142	144	12	870
870	870	870	870	870	870	870	870	870	870	870	870	870

Above two magic squares are with distributions:

$$D_{11 \times 11} := \{1, 2, \dots, 120, 121\}$$

$$D_{12 \times 12} := \{1, 2, \dots, 143, 144\}$$

These two magic squares are such that replacing the upper border still we are left with magic squares of sequential vales. Multiplying each entry by 9, we get

11												6039
1008	918	936	954	972	81	72	54	36	18	990	6039	
171	846	774	792	810	243	234	216	198	828	927	6039	
153	315	720	666	684	369	360	342	702	783	945	6039	
135	297	423	630	594	459	450	612	675	801	963	6039	
117	279	405	495	576	513	558	603	693	819	981	6039	
99	261	387	477	531	549	567	621	711	837	999	6039	
1035	873	747	657	540	585	522	441	351	225	63	6039	
1053	891	765	486	504	639	648	468	333	207	45	6039	
1071	909	396	432	414	729	738	756	378	189	27	6039	
1089	270	324	306	288	855	864	882	900	252	9	6039	
108	180	162	144	126	1017	1026	1044	1062	1080	90	6039	
6039	6039	6039	6039	6039	6039	6039	6039	6039	6039	6039	6039	

12												7830
1197	1116	1134	1269	1287	198	180	162	45	27	9	1206	7830
153	1017	972	342	954	360	324	234	1080	216	1026	1152	7830
126	315	432	378	918	936	819	477	837	423	990	1179	7830
117	999	405	774	756	513	810	522	540	900	306	1188	7830
72	297	414	504	639	684	585	702	801	891	1008	1233	7830
54	1062	459	558	594	693	648	675	747	846	243	1251	7830
1161	207	909	576	720	603	666	621	729	396	1098	144	7830
1170	1035	864	738	657	630	711	612	567	441	270	135	7830
1215	261	855	765	549	792	495	783	531	450	1044	90	7830
1224	1053	882	927	387	369	486	828	468	873	252	81	7830
1242	279	333	963	351	945	981	1071	225	1089	288	63	7830
99	189	171	36	18	1107	1125	1143	1260	1278	1296	108	7830
7830	7830	7830	7830	7830	7830	7830	7830	7830	7830	7830	7830	

The distributions of these two magic squares is given by

$$D_{11 \times 11} := \{9, 18, \dots, 1080, 1089\}$$

$$D_{12 \times 12} := \{9, 18, \dots, 1287, 1296\}.$$

In both the cases the difference between entries is 9. Now in each case replace the entries by magic squares of order 3 formed by the entries as given below:

$$\begin{aligned} 9 &\rightarrow 1, 2, \dots, 9 \\ 18 &\rightarrow 10, 11, \dots, 18 \\ 27 &\rightarrow 19, 20, \dots, 27 \\ \dots &\rightarrow \dots \dots \\ 1287 &\rightarrow 1979, 1980, \dots, 1287 \\ 1296 &\rightarrow 1288, 1289 \dots, 1296 \end{aligned}$$

This lead us to following two magic squares of order 33 and 36.

33																											17985							
1005	1000	1007	915	910	917	933	928	935	951	946	953	969	964	971	78	73	80	69	64	71	51	46	53	33	28	35	15	10	17	987	982	989	17985	
1006	1004	1002	916	914	912	934	932	930	952	950	948	970	968	966	79	77	75	70	68	66	52	50	48	34	32	30	16	14	12	988	986	984	17985	
1001	1008	1003	911	918	913	929	936	931	947	954	949	965	972	967	74	81	76	65	72	67	47	54	49	29	36	31	11	18	13	983	990	985	17985	
168	163	170	843	838	845	771	766	773	789	784	791	807	802	809	240	235	242	231	226	233	213	208	215	195	190	197	825	820	827	924	919	926	17985	
169	167	165	844	842	840	772	770	768	790	788	786	808	806	804	241	239	237	232	230	228	214	212	210	196	194	192	826	824	822	925	923	921	17985	
164	171	166	839	846	841	767	774	769	785	792	787	803	810	805	236	243	238	227	234	229	209	216	211	191	198	193	821	828	823	920	927	922	17985	
150	145	152	312	307	314	717	712	719	663	658	665	681	676	683	366	361	368	357	352	359	339	334	341	699	694	701	780	775	782	942	937	944	17985	
151	149	147	313	311	309	718	716	714	664	662	660	682	680	678	367	365	363	358	356	354	340	338	336	700	698	696	781	779	777	943	941	939	17985	
146	153	148	308	315	310	713	720	715	659	666	661	677	684	679	362	369	364	353	360	355	335	342	337	695	702	697	776	783	778	938	945	940	17985	
132	127	134	294	289	296	420	415	422	627	622	629	591	586	593	456	451	458	447	442	449	609	604	611	672	667	674	798	793	800	960	955	962	17985	
133	131	129	295	293	291	421	419	417	628	626	624	592	590	588	457	455	453	448	446	444	610	608	606	673	671	669	799	797	795	961	959	957	17985	
128	135	130	290	297	292	416	423	418	623	630	625	587	594	589	452	459	454	443	450	445	605	612	607	668	675	670	794	801	796	956	963	958	17985	
114	109	116	276	271	278	402	397	404	492	487	494	573	568	575	510	505	512	555	550	557	600	595	602	690	685	692	816	811	818	978	973	980	17985	
115	113	111	277	275	273	403	401	399	493	491	489	574	572	570	511	509	507	556	554	552	601	599	597	691	689	687	817	815	813	979	977	975	17985	
110	117	112	272	279	274	398	405	400	488	495	490	569	576	571	506	513	508	551	558	553	596	603	598	686	693	688	812	819	814	974	981	976	17985	
96	91	98	258	253	260	384	379	386	474	469	476	528	523	530	546	541	548	564	559	566	618	613	620	708	703	710	834	829	836	996	991	998	17985	
97	95	93	259	257	255	385	383	381	475	473	471	529	527	525	547	545	543	565	563	561	619	617	615	709	707	705	835	833	831	997	995	993	17985	
92	99	94	254	261	256	380	387	382	470	477	472	524	531	526	542	549	544	560	567	562	614	621	616	704	711	706	830	837	832	992	999	994	17985	
1032	1027	1034	870	865	872	744	739	746	654	649	656	537	532	539	582	577	584	519	514	521	438	433	440	348	343	350	222	217	224	60	55	62	17985	
1033	1031	1029	871	869	867	745	743	741	655	653	651	538	536	534	583	581	579	520	518	516	439	437	435	349	347	345	223	221	219	61	59	57	17985	
1028	1035	1030	866	873	868	740	747	742	650	657	652	533	540	535	578	585	580	515	522	517	434	441	436	344	351	346	218	225	220	56	63	58	17985	
1050	1045	1052	888	883	890	762	757	764	483	478	485	501	496	503	636	631	638	645	640	647	465	460	467	330	325	332	204	199	206	42	37	44	17985	
1051	1049	1047	889	887	885	763	761	759	484	482	480	502	500	498	637	635	633	646	644	642	466	464	462	331	329	327	205	203	201	43	41	39	17985	
1046	1053	1048	884	891	886	758	765	760	479	486	481	497	504	499	632	639	634	641	648	643	461	468	463	326	333	328	200	207	202	38	45	40	17985	
1068	1063	1070	906	901	908	393	388	395	429	424	431	411	406	413	726	721	728	735	730	737	753	748	755	375	370	377	186	181	188	24	19	26	17985	
1069	1067	1065	907	905	903	394	392	390	430	428	426	412	410	408	727	725	723	736	734	732	754	752	750	376	374	372	187	185	183	25	23	21	17985	
1064	1071	1066	902	909	904	389	396	391	425	432	427	407	414	409	722	729	724	731	738	733	749	756	751	371	378	373	182	189	184	20	27	22	17985	
1086	1081	1088	267	262	269	321	316	323	303	298	305	285	280	287	852	847	854	861	856	863	879	874	881	897	892	899	249	244	251	6	1	8	17985	
1087	1085	1083	268	266	264	322	320	318	304	302	300	286	284	282	853	851	849	862	860	858	880	878	876	898	896	894	250	248	246	7	5	3	17985	
1082	1089	1084	263	270	265	317	324	319	299	306	301	281	288	283	848	855	850	857	864	859	875	882	877	893	900	895	245	252	247	2	9	4	17985	
105	100	107	177	172	179	159	154	161	141	136	143	123	118	125	1014	1009	1016	1023	1018	1025	1041	1036	1043	1059	1054	1061	1077	1072	1079	87	82	89	17985	
106	104	102	178	176	174	160	158	156	142	140	138	124	122	120	1015	1013	1011	1024	1022	1020	1042	1040	1038	1060	1058	1056	1078	1076	1074	88	86	84	17985	
101	108	103	173	180	175	155	162	157	137	144	139	119	126	121	1010	1017	1012	1019	1026	1021	1037	1044	1039	1055	1062	1057	1073	1080	1075	83	90	85	17985	
17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985

$$D_{33 \times 33} := \{1, 2, \dots, 1088, 1089\}$$

$$D_{36 \times 36} := \{1, 2, \dots, 1295, 1296\}.$$

These two magic squares are bordered magic squares with border made by magic squares of order 3. If you remove the external border still we are left with magic squares of lower order. Let's see how it works.

2.2 Magic Squares of Orders 27 and 30

These magic squares are obtained from above by the application of the formula $\frac{a^2 - b^2}{2}$, $a > b$. Removing the external border of order 3 and then subtracting $\frac{33^2 - 27^2}{2} := 180$ and $\frac{36^2 - 30^2}{2} := 198$ respectively from magic squares of orders 33 and 36, we get magic squares of orders 27 and 30 given by

27																											9855	
663	658	665	591	586	593	609	604	611	627	622	629	60	55	62	51	46	53	33	28	35	15	10	17	645	640	647	9855	
664	662	660	592	590	588	610	608	606	628	626	624	61	59	57	52	50	48	34	32	30	16	14	12	646	644	642	9855	
659	666	661	587	594	589	605	612	607	623	630	625	56	63	58	47	54	49	29	36	31	11	18	13	641	648	643	9855	
132	127	134	537	532	539	483	478	485	501	496	503	186	181	188	177	172	179	159	154	161	519	514	521	600	595	602	9855	
133	131	129	538	536	534	484	482	480	502	500	498	187	185	183	178	176	174	160	158	156	520	518	516	601	599	597	9855	
128	135	130	533	540	535	479	486	481	497	504	499	182	189	184	173	180	175	155	162	157	515	522	517	596	603	598	9855	
114	109	116	240	235	242	447	442	449	411	406	413	276	271	278	267	262	269	429	424	431	492	487	494	618	613	620	9855	
115	113	111	241	239	237	448	446	444	412	410	408	277	275	273	268	266	264	430	428	426	493	491	489	619	617	615	9855	
110	117	112	236	243	238	443	450	445	407	414	409	272	279	274	263	270	265	425	432	427	488	495	490	614	621	616	9855	
96	91	98	222	217	224	312	307	314	393	388	395	330	325	332	375	370	377	420	415	422	510	505	512	636	631	638	9855	
97	95	93	223	221	219	313	311	309	394	392	390	331	329	327	376	374	372	421	419	417	511	509	507	637	635	633	9855	
92	99	94	218	225	220	308	315	310	389	396	391	326	333	328	371	378	373	416	423	418	506	513	508	632	639	634	9855	
78	73	80	204	199	206	294	289	296	348	343	350	366	361	368	384	379	386	438	433	440	528	523	530	654	649	656	9855	
79	77	75	205	203	201	295	293	291	349	347	345	367	365	363	385	383	381	439	437	435	529	527	525	655	653	651	9855	
74	81	76	200	207	202	290	297	292	344	351	346	362	369	364	380	387	382	434	441	436	524	531	526	650	657	652	9855	
690	685	692	564	559	566	474	469	476	357	352	359	402	397	404	339	334	341	258	253	260	168	163	170	42	37	44	9855	
691	689	687	565	563	561	475	473	471	358	356	354	403	401	399	340	338	336	259	257	255	169	167	165	43	41	39	9855	
686	693	688	560	567	562	470	477	472	353	360	355	398	405	400	335	342	337	254	261	256	164	171	166	38	45	40	9855	
708	703	710	582	577	584	303	298	305	321	316	323	456	451	458	465	460	467	285	280	287	150	145	152	24	19	26	9855	
709	707	705	583	581	579	304	302	300	322	320	318	457	455	453	466	464	462	286	284	282	151	149	147	25	23	21	9855	
704	711	706	578	585	580	299	306	301	317	324	319	452	459	454	461	468	463	281	288	283	146	153	148	20	27	22	9855	
726	721	728	213	208	215	249	244	251	231	226	233	546	541	548	555	550	557	573	568	575	195	190	197	6	1	8	9855	
727	725	723	214	212	210	250	248	246	232	230	228	547	545	543	556	554	552	574	572	570	196	194	192	7	5	3	9855	
722	729	724	209	216	211	245	252	247	227	234	229	542	549	544	551	558	553	569	576	571	191	198	193	2	9	4	9855	
87	82	89	141	136	143	123	118	125	105	100	107	672	667	674	681	676	683	699	694	701	717	712	719	69	64	71	9855	
88	86	84	142	140	138	124	122	120	106	104	102	673	671	669	682	680	678	700	698	696	718	716	714	70	68	66	9855	
83	90	85	137	144	139	119	126	121	101	108	103	668	675	670	677	684	679	695	702	697	713	720	715	65	72	67	9855	
9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855

30																											13515				
816	811	818	771	766	773	141	136	143	753	748	755	159	154	161	123	118	125	33	28	35	879	874	881	15	10	17	825	820	827	13515	
817	815	813	772	770	768	142	140	138	754	752	750	160	158	156	124	122	120	34	32	30	880	878	876	16	14	12	826	824	822	13515	
812	819	814	767	774	769	137	144	139	749	756	751	155	162	157	119	126	121	29	36	31	875	882	877	11	18	13	821	828	823	13515	
114	109	116	231	226	233	177	172	179	717	712	719	735	730	737	618	613	620	276	271	278	636	631	638	222	217	224	789	784	791	13515	
115	113	111	232	230	228	178	176	174	718	716	714	736	734	732	619	617	615	277	275	273	637	635	633	223	221	219	790	788	786	13515	
110	117	112	227	234	229	173	180	175	713	720	715	731	738	733	614	621	616	272	279	274	632	639	634	218	225	220	785	792	787	13515	
798	793	800	204	199	206	573	568	575	555	550	557	312	307	314	609	604	611	321	316	323	339	334	341	699	694	701	105	100	107	13515	
799	797	795	205	203	201	574	572	570	556	554	552	313	311	309	610	608	606	322	320	318	340	338	336	700	698	696	106	104	102	13515	
794	801	796	200	207	202	569	576	571	551	558	553	308	315	310	605	612	607	317	324	319	335	342	337	695	702	697	101	108	103	13515	
96	91	98	213	208	215	303	298	305	438	433	440	483	478	485	384	379	386	501	496	503	600	595	602	690	685	692	807	802	809	13515	
97	95	93	214	212	210	304	302	300	439	437	435	484	482	480	385	383	381	502	500	498	601	599	597	691	689	687	808	806	804	13515	
92	99	94	209	216	211	299	306	301	434	441	436	479	486	481	380	387	382	497	504	499	596	603	598	686	693	688	803	810	805	13515	
861	856	863	258	253	260	357	352	359	393	388	395	492	487	494	447	442	449	474	469	476	546	541	548	645	640	647	42	37	44	13515	
862	860	858	259	257	255	358	356	354	394	392	390	493	491	489	448	446	444	475	473	471	547	545	543	646	644	642	43	41	39	13515	
857	864	859	254	261	256	353	360	355	389	396	391	488	495	490	443	450	445	470	477	472	542	549	544	641	648	643	38	45	40	13515	
6	1	8	708	703	710	375	370	377	519	514	521	402	397	404	465	460	467	420	415	422	528	523	530	195	190	197	897	892	899	13515	
7	5	3	709	707	705	376	374	372	520	518	516	403	401	399	466	464	462	421	419	417	529	527	525	196	194	192	898	896	894	13515	
2	9	4	704	711	706	371	378	373	515	522	517	398	405	400	461	468	463	416	423	418	524	531	526	191	198	193	893	900	895	13515	
834	829	836	663	658	665	537	532	539	456	451	458	429	424	431	510	505	512	411	406	413	366	361	368	240	235	242	69	64	71	13515	
835	833	831	664	662	660	538	536	534	457	455	453	430	428	426	511	509	507	412	410	408	367	365	363	241	239	237	70	68	66	13515	
830	837	832	659	666	661	533	540	535	452	459	454	425	432	427	506	513	508	407	414	409	362	369	364	236	243	238	65	72	67	13515	
60	55	62	654	649	656	564	559	566	348	343	350	591	586	593	294	289	296	582	577	584	330	325	332	249	244	251	843	838	845	13515	
61	59	57	655	653	651	565	563	561	349	347	345	592	590	588	295	293	291	583	581	579	331	329	327	250	248	246	844	842	840	13515	
56	63	58	650	657	652	560	567	562	344	351	346	587	594	589	290	297	292	578	585	580	326	333	328	245	252	247	839	846	841	13515	
852	847	854	681	676	683	726	721	728	186	181	188	168	163	170	285	280	287	627	622	629	267	262	269	672	667	674	51	46	53	13515	
853	851	849	682	680	678	727	725	723	187	185	183	169	167	165	286	284	282	628	626	624	268	266	264	673	671	669	52	50	48	13515	
848	855	850	677	684	679	722	729	724	182	189	184	164	171	166	281	288	283	623	630	625	263	270	265	668	675	670	47	54	49	13515	
78	73	80	132	127	134	762	757	764	150	145	152	744	739	746	780	775	782	870	865	872	24	19	26	888	883	890	87	82	89	13515	
79	77	75	133	131	129	763	761	759	151	149	147	745	743	741	781	779	777	871	869	867	25	23	21	889	887	885	88	86	84	13515	
74	81	76	128	135	130	758	765	760	146	153	148	740	747	742	776	783	778	866	873	868	20	27	22	884	891	886	83	90	85	13515	
13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515

Above two magic squares of orders 27 and 30 formed by blocks of magic squares of order 3 are with distributions

$$D_{33 \times 33} := \{1, 2, \dots, 728, 729\}$$

$$D_{36 \times 36} := \{1, 2, \dots, 899, 900\}$$

2.3 Magic Squares of Orders 21 and 24

These magic squares are obtained from above by the application of the formula $\frac{a^2 - b^2}{2}$, $a > b$. Removing the external border of order 3, and then subtracting $\frac{37^2 - 21^2}{2} := 144$ and $\frac{30^2 - 24^2}{2} := 162$ respectively from magic squares of orders 27 and 30, we get magic squares of orders 21 and 24 given by

21																			4641		
393	388	395	339	334	341	357	352	359	42	37	44	33	28	35	15	10	17	375	370	377	4641
394	392	390	340	338	336	358	356	354	43	41	39	34	32	30	16	14	12	376	374	372	4641
389	396	391	335	342	337	353	360	355	38	45	40	29	36	31	11	18	13	371	378	373	4641
96	91	98	303	298	305	267	262	269	132	127	134	123	118	125	285	280	287	348	343	350	4641
97	95	93	304	302	300	268	266	264	133	131	129	124	122	120	286	284	282	349	347	345	4641
92	99	94	299	306	301	263	270	265	128	135	130	119	126	121	281	288	283	344	351	346	4641
78	73	80	168	163	170	249	244	251	186	181	188	231	226	233	276	271	278	366	361	368	4641
79	77	75	169	167	165	250	248	246	187	185	183	232	230	228	277	275	273	367	365	363	4641
74	81	76	164	171	166	245	252	247	182	189	184	227	234	229	272	279	274	362	369	364	4641
60	55	62	150	145	152	204	199	206	222	217	224	240	235	242	294	289	296	384	379	386	4641
61	59	57	151	149	147	205	203	201	223	221	219	241	239	237	295	293	291	385	383	381	4641
56	63	58	146	153	148	200	207	202	218	225	220	236	243	238	290	297	292	380	387	382	4641
420	415	422	330	325	332	213	208	215	258	253	260	195	190	197	114	109	116	24	19	26	4641
421	419	417	331	329	327	214	212	210	259	257	255	196	194	192	115	113	111	25	23	21	4641
416	423	418	326	333	328	209	216	211	254	261	256	191	198	193	110	117	112	20	27	22	4641
438	433	440	159	154	161	177	172	179	312	307	314	321	316	323	141	136	143	6	1	8	4641
439	437	435	160	158	156	178	176	174	313	311	309	322	320	318	142	140	138	7	5	3	4641
434	441	436	155	162	157	173	180	175	308	315	310	317	324	319	137	144	139	2	9	4	4641
69	64	71	105	100	107	87	82	89	402	397	404	411	406	413	429	424	431	51	46	53	4641
70	68	66	106	104	102	88	86	84	403	401	399	412	410	408	430	428	426	52	50	48	4641
65	72	67	101	108	103	83	90	85	398	405	400	407	414	409	425	432	427	47	54	49	4641
4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641

24																							6924		
69	64	71	15	10	17	555	550	557	573	568	575	456	451	458	114	109	116	474	469	476	60	55	62	6924	
70	68	66	16	14	12	556	554	552	574	572	570	457	455	453	115	113	111	475	473	471	61	59	57	6924	
65	72	67	11	18	13	551	558	553	569	576	571	452	459	454	110	117	112	470	477	472	56	63	58	6924	
42	37	44	411	406	413	393	388	395	150	145	152	447	442	449	159	154	161	177	172	179	537	532	539	6924	
43	41	39	412	410	408	394	392	390	151	149	147	448	446	444	160	158	156	178	176	174	538	536	534	6924	
38	45	40	407	414	409	389	396	391	146	153	148	443	450	445	155	162	157	173	180	175	533	540	535	6924	
51	46	53	141	136	143	276	271	278	321	316	323	222	217	224	339	334	341	438	433	440	528	523	530	6924	
52	50	48	142	140	138	277	275	273	322	320	318	223	221	219	340	338	336	439	437	435	529	527	525	6924	
47	54	49	137	144	139	272	279	274	317	324	319	218	225	220	335	342	337	434	441	436	524	531	526	6924	
96	91	98	195	190	197	231	226	233	330	325	332	285	280	287	312	307	314	384	379	386	483	478	485	6924	
97	95	93	196	194	192	232	230	228	331	329	327	286	284	282	313	311	309	385	383	381	484	482	480	6924	
92	99	94	191	198	193	227	234	229	326	333	328	281	288	283	308	315	310	380	387	382	479	486	481	6924	
546	541	548	213	208	215	357	352	359	240	235	242	303	298	305	258	253	260	366	361	368	33	28	35	6924	
547	545	543	214	212	210	358	356	354	241	239	237	304	302	300	259	257	255	367	365	363	34	32	30	6924	
542	549	544	209	216	211	353	360	355	236	243	238	299	306	301	254	261	256	362	369	364	29	36	31	6924	
501	496	503	375	370	377	294	289	296	267	262	269	348	343	350	249	244	251	204	199	206	78	73	80	6924	
502	500	498	376	374	372	295	293	291	268	266	264	349	347	345	250	248	246	205	203	201	79	77	75	6924	
497	504	499	371	378	373	290	297	292	263	270	265	344	351	346	245	252	247	200	207	202	74	81	76	6924	
492	487	494	402	397	404	186	181	188	429	424	431	132	127	134	420	415	422	168	163	170	87	82	89	6924	
493	491	489	403	401	399	187	185	183	430	428	426	133	131	129	421	419	417	169	167	165	88	86	84	6924	
488	495	490	398	405	400	182	189	184	425	432	427	128	135	130	416	423	418	164	171	166	83	90	85	6924	
519	514	521	564	559	566	24	19	26	6	1	8	123	118	125	465	460	467	105	100	107	510	505	512	6924	
520	518	516	565	563	561	25	23	21	7	5	3	124	122	120	466	464	462	106	104	102	511	509	507	6924	
515	522	517	560	567	562	20	27	22	2	9	4	119	126	121	461	468	463	101	108	103	506	513	508	6924	
6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924

Above two magic squares of orders 21 and 24 formed by blocks of magic squares of order 3 are with distributions

$$D_{21 \times 21} := \{1, 2, \dots, 440, 441\}$$

$$D_{24 \times 24} := \{1, 2, \dots, 575, 576\}.$$

2.4 Magic Squares of Orders 15 and 18

These magic squares are obtained from above by the application of the formula $\frac{a^2 - b^2}{2}$, $a > b$. Removing the external border of order 3, and then subtracting $\frac{21^2 - 15^2}{2} := 108$ and $\frac{24^2 - 18^2}{2} := 126$ respectively from magic squares of orders 21 and 24, we get magic squares of orders 15 and 18 given

by

15															1695
195	190	197	159	154	161	24	19	26	15	10	17	177	172	179	1695
196	194	192	160	158	156	25	23	21	16	14	12	178	176	174	1695
191	198	193	155	162	157	20	27	22	11	18	13	173	180	175	1695
60	55	62	141	136	143	78	73	80	123	118	125	168	163	170	1695
61	59	57	142	140	138	79	77	75	124	122	120	169	167	165	1695
56	63	58	137	144	139	74	81	76	119	126	121	164	171	166	1695
42	37	44	96	91	98	114	109	116	132	127	134	186	181	188	1695
43	41	39	97	95	93	115	113	111	133	131	129	187	185	183	1695
38	45	40	92	99	94	110	117	112	128	135	130	182	189	184	1695
222	217	224	105	100	107	150	145	152	87	82	89	6	1	8	1695
223	221	219	106	104	102	151	149	147	88	86	84	7	5	3	1695
218	225	220	101	108	103	146	153	148	83	90	85	2	9	4	1695
51	46	53	69	64	71	204	199	206	213	208	215	33	28	35	1695
52	50	48	70	68	66	205	203	201	214	212	210	34	32	30	1695
47	54	49	65	72	67	200	207	202	209	216	211	29	36	31	1695
1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695

18															2925			
285	280	287	267	262	269	24	19	26	321	316	323	33	28	35	51	46	53	2925
286	284	282	268	266	264	25	23	21	322	320	318	34	32	30	52	50	48	2925
281	288	283	263	270	265	20	27	22	317	324	319	29	36	31	47	54	49	2925
15	10	17	150	145	152	195	190	197	96	91	98	213	208	215	312	307	314	2925
16	14	12	151	149	147	196	194	192	97	95	93	214	212	210	313	311	309	2925
11	18	13	146	153	148	191	198	193	92	99	94	209	216	211	308	315	310	2925
69	64	71	105	100	107	204	199	206	159	154	161	186	181	188	258	253	260	2925
70	68	66	106	104	102	205	203	201	160	158	156	187	185	183	259	257	255	2925
65	72	67	101	108	103	200	207	202	155	162	157	182	189	184	254	261	256	2925
87	82	89	231	226	233	114	109	116	177	172	179	132	127	134	240	235	242	2925
88	86	84	232	230	228	115	113	111	178	176	174	133	131	129	241	239	237	2925
83	90	85	227	234	229	110	117	112	173	180	175	128	135	130	236	243	238	2925
249	244	251	168	163	170	141	136	143	222	217	224	123	118	125	78	73	80	2925
250	248	246	169	167	165	142	140	138	223	221	219	124	122	120	79	77	75	2925
245	252	247	164	171	166	137	144	139	218	225	220	119	126	121	74	81	76	2925
276	271	278	60	55	62	303	298	305	6	1	8	294	289	296	42	37	44	2925
277	275	273	61	59	57	304	302	300	7	5	3	295	293	291	43	41	39	2925
272	279	274	56	63	58	299	306	301	2	9	4	290	297	292	38	45	40	2925
2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925

Above two magic squares of orders 15 and 18 formed by blocks of magic squares of order 3 are with distributions

$$D_{15 \times 15} := \{1, 2, \dots, 224, 225\}$$

$$D_{18 \times 18} := \{1, 2, \dots, 323, 324\}.$$

2.5 Magic Squares of Orders 9 and 12

These magic squares are obtained from above by the application of the formula $\frac{a^2 - b^2}{2}$, $a > b$. Removing the external border of order 3, and then subtracting $\frac{15^2 - 9^2}{2} := 72$ and $\frac{18^2 - 12^2}{2} := 90$ respectively from magic squares of orders 15 and 18, we get magic squares of orders 9 and 12 given by

9										369
	69	64	71	6	1	8	51	46	53	369
	70	68	66	7	5	3	52	50	48	369
	65	72	67	2	9	4	47	54	49	369
	24	19	26	42	37	44	60	55	62	369
	25	23	21	43	41	39	61	59	57	369
	20	27	22	38	45	40	56	63	58	369
	33	28	35	78	73	80	15	10	17	369
	34	32	30	79	77	75	16	14	12	369
	29	36	31	74	81	76	11	18	13	369
	369	369	369	369	369	369	369	369	369	369

12													870
60	55	62	105	100	107	6	1	8	123	118	125	870	
61	59	57	106	104	102	7	5	3	124	122	120	870	
56	63	58	101	108	103	2	9	4	119	126	121	870	
15	10	17	114	109	116	69	64	71	96	91	98	870	
16	14	12	115	113	111	70	68	66	97	95	93	870	
11	18	13	110	117	112	65	72	67	92	99	94	870	
141	136	143	24	19	26	87	82	89	42	37	44	870	
142	140	138	25	23	21	88	86	84	43	41	39	870	
137	144	139	20	27	22	83	90	85	38	45	40	870	
78	73	80	51	46	53	132	127	134	33	28	35	870	
79	77	75	52	50	48	133	131	129	34	32	30	870	
74	81	76	47	54	49	128	135	130	29	36	31	870	
870	870	870	870	870	870	870	870	870	870	870	870	870	

Above two magic squares of orders 9 and 12 formed by blocks of magic squares of order 3 are with distributions

$$D_{15 \times 15} := \{1, 2, \dots, 80, 81\}$$

$$D_{18 \times 18} := \{1, 2, \dots, 143, 144\}.$$

Remark 1. *Whole the work is done up to order 120. Due to lack of space, an excel file is attached with the work including pandiagonal magic squares. Except orders 6, 18, 40, 42, etc. we can write pandiagonals of other orders multiples of 3. The section below give pandiagonal magic squares up to order 36 except the orders 18 and 30. In some case these orders are with semi-magic squares of equal orders.*

3 Pandiagonal Magic Squares Multiples 3

This section brings pandiagonal magic squares multiples 3, It includes magic squares of order 9, 12, 15, 21, 24 27, 33 and 36. The details are excluded as these are studied extensively in author’s previous works [7, 9, 22].

3.1 Pandiagonal Magic Square of Order 9

Below is a pandiagonal magic squares of order 9.

	pan	369	369	369	369	369	369	369	369	369	369
369	22	71	30	27	64	32	20	69	34	369	
369	35	21	67	28	23	72	33	25	65	369	
369	66	31	26	68	36	19	70	29	24	369	
369	40	8	75	45	1	77	38	6	79	369	
369	80	39	4	73	41	9	78	43	2	369	
369	3	76	44	5	81	37	7	74	42	369	
369	58	53	12	63	46	14	56	51	16	369	
369	17	57	49	10	59	54	15	61	47	369	
	48	13	62	50	18	55	52	11	60	369	
	369	369	369	369	369	369	369	369	369	369	369

The blocks of order 3 are **semi-magic squares** with equal sums, i.e., $SM_{3 \times 3} := 123$

3.2 Pandiagonal Magic Square of Order 12

Below is a pandiagonal magic squares of order 12.

	pan	870	870	870	870	870	870	870	870	870	870	870
870	55	45	68	96	106	83	13	27	2	126	112	137
870	69	56	43	82	95	108	3	14	25	136	125	114
870	44	67	57	107	84	94	26	1	15	113	138	124
870	18	28	5	121	111	134	60	46	71	91	105	80
870	4	17	30	135	122	109	70	59	48	81	92	103
870	29	6	16	110	133	123	47	72	58	104	79	93
870	132	118	143	19	33	8	90	100	77	49	39	62
870	142	131	120	9	20	31	76	89	102	63	50	37
870	119	144	130	32	7	21	101	78	88	38	61	51
870	85	99	74	54	40	65	127	117	140	24	34	11
870	75	86	97	64	53	42	141	128	115	10	23	36
	98	73	87	41	66	52	116	139	129	35	12	22
12	870	870	870	870	870	870	870	870	870	870	870	870

3.3 Pandiagonal Magic Square of Orders 15

Below is a pandiagonal magic squares of order 15.

	pan	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	
1695	186	65	88	187	74	78	188	75	76	189	64	86	190	62	87	1695
1695	80	193	66	89	183	67	90	181	68	79	191	69	77	192	70	1695
1695	73	81	185	63	82	194	61	83	195	71	84	184	72	85	182	1695
1695	36	200	103	37	209	93	38	210	91	39	199	101	40	197	102	1695
1695	95	43	201	104	33	202	105	31	203	94	41	204	92	42	205	1695
1695	208	96	35	198	97	44	196	98	45	206	99	34	207	100	32	1695
1695	6	215	118	7	224	108	8	225	106	9	214	116	10	212	117	1695
1695	110	13	216	119	3	217	120	1	218	109	11	219	107	12	220	1695
1695	223	111	5	213	112	14	211	113	15	221	114	4	222	115	2	1695
1695	156	50	133	157	59	123	158	60	121	159	49	131	160	47	132	1695
1695	125	163	51	134	153	52	135	151	53	124	161	54	122	162	55	1695
1695	58	126	155	48	127	164	46	128	165	56	129	154	57	130	152	1695
1695	171	20	148	172	29	138	173	30	136	174	19	146	175	17	147	1695
1695	140	178	21	149	168	22	150	166	23	139	176	24	137	177	25	1695
	28	141	170	18	142	179	16	143	180	26	144	169	27	145	167	1695
	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695

The blocks of order 3 are **semi-magic squares** with equal sums, i.e., $SM_{3 \times 3} := 339$

3.4 Pandiagonal Magic Square of Orders 21

Below is a pandiagonal magic squares of order 21.

	pan	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	
4641	246	111	306	232	123	308	250	110	303	233	125	305	252	109	302	234	122	307	247	112	304	4641
4641	300	243	120	312	245	106	299	240	124	314	242	107	298	239	126	311	244	108	301	241	121	4641
4641	117	309	237	119	295	249	114	313	236	116	296	251	113	315	235	118	297	248	115	310	238	4641
4641	288	363	12	274	375	14	292	362	9	275	377	11	294	361	8	276	374	13	289	364	10	4641
4641	6	285	372	18	287	358	5	282	376	20	284	359	4	281	378	17	286	360	7	283	373	4641
4641	369	15	279	371	1	291	366	19	278	368	2	293	365	21	277	370	3	290	367	16	280	4641
4641	183	90	390	169	102	392	187	89	387	170	104	389	189	88	386	171	101	391	184	91	388	4641
4641	384	180	99	396	182	85	383	177	103	398	179	86	382	176	105	395	181	87	385	178	100	4641
4641	96	393	174	98	379	186	93	397	173	95	380	188	92	399	172	97	381	185	94	394	175	4641
4641	225	405	33	211	417	35	229	404	30	212	419	32	231	403	29	213	416	34	226	406	31	4641
4641	27	222	414	39	224	400	26	219	418	41	221	401	25	218	420	38	223	402	28	220	415	4641
4641	411	36	216	413	22	228	408	40	215	410	23	230	407	42	214	412	24	227	409	37	217	4641
4641	162	69	432	148	81	434	166	68	429	149	83	431	168	67	428	150	80	433	163	70	430	4641
4641	426	159	78	438	161	64	425	156	82	440	158	65	424	155	84	437	160	66	427	157	79	4641
4641	75	435	153	77	421	165	72	439	152	74	422	167	71	441	151	76	423	164	73	436	154	4641
4641	267	342	54	253	354	56	271	341	51	254	356	53	273	340	50	255	353	55	268	343	52	4641
4641	48	264	351	60	266	337	47	261	355	62	263	338	46	260	357	59	265	339	49	262	352	4641
4641	348	57	258	350	43	270	345	61	257	347	44	272	344	63	256	349	45	269	346	58	259	4641
4641	204	132	327	190	144	329	208	131	324	191	146	326	210	130	323	192	143	328	205	133	325	4641
4641	321	201	141	333	203	127	320	198	145	335	200	128	319	197	147	332	202	129	322	199	142	4641
	138	330	195	140	316	207	135	334	194	137	317	209	134	336	193	139	318	206	136	331	196	4641
	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641

The blocks of order 3 are **semi-magic squares** with equal sums, i.e., $SM_{3 \times 3} := 663$

3.5 Pandiagonal Magic Square of Orders 24

Below is a pandiagonal magic squares of order 24.

pan	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
6924	253	231	278	336	358	311	25	51	2	540	514	563	258	232	281	331	357	308	30	52	5	535	513	560	6924	
6924	279	254	229	310	335	360	3	26	49	562	539	516	280	257	234	309	332	355	4	29	54	561	536	511	6924	
6924	230	277	255	359	312	334	50	1	27	515	564	538	233	282	256	356	307	333	53	6	28	512	559	537	6924	
6924	36	58	11	529	507	554	264	238	287	325	351	302	31	57	8	534	508	557	259	237	284	330	352	305	6924	
6924	10	35	60	555	530	505	286	263	240	303	326	349	9	32	55	556	533	510	285	260	235	304	329	354	6924	
6924	59	12	34	506	553	531	239	288	262	350	301	327	56	7	33	509	558	532	236	283	261	353	306	328	6924	
6924	552	526	575	37	63	14	324	346	299	241	219	266	547	525	572	42	64	17	319	345	296	246	220	269	6924	
6924	574	551	528	15	38	61	298	323	348	267	242	217	573	548	523	16	41	66	297	320	343	268	245	222	6924	
6924	527	576	550	62	13	39	347	300	322	218	265	243	524	571	549	65	18	40	344	295	321	221	270	244	6924	
6924	313	339	290	252	226	275	541	519	566	48	70	23	318	340	293	247	225	272	546	520	569	43	69	20	6924	
6924	291	314	337	274	251	228	567	542	517	22	47	72	292	317	342	273	248	223	568	545	522	21	44	67	6924	
6924	338	289	315	227	276	250	518	565	543	71	24	46	341	294	316	224	271	249	521	570	544	68	19	45	6924	
6924	181	207	158	408	382	431	97	75	122	468	490	443	186	208	161	403	381	428	102	76	125	463	489	440	6924	
6924	159	182	205	430	407	384	123	98	73	442	467	492	160	185	210	429	404	379	124	101	78	441	464	487	6924	
6924	206	157	183	383	432	406	74	121	99	491	444	466	209	162	184	380	427	405	77	126	100	488	439	465	6924	
6924	108	82	131	457	483	434	192	214	167	397	375	422	103	81	128	462	484	437	187	213	164	402	376	425	6924	
6924	130	107	84	435	458	481	166	191	216	423	398	373	129	104	79	436	461	486	165	188	211	424	401	378	6924	
6924	83	132	106	482	433	459	215	168	190	374	421	399	80	127	105	485	438	460	212	163	189	377	426	400	6924	
6924	480	502	455	109	87	134	396	370	419	169	195	146	475	501	452	114	88	137	391	369	416	174	196	149	6924	
6924	454	479	504	135	110	85	418	395	372	147	170	193	453	476	499	136	113	90	417	392	367	148	173	198	6924	
6924	503	456	478	86	133	111	371	420	394	194	145	171	500	451	477	89	138	112	368	415	393	197	150	172	6924	
6924	385	363	410	180	202	155	469	495	446	120	94	143	390	364	413	175	201	152	474	496	449	115	93	140	6924	
6924	411	386	361	154	179	204	447	470	493	142	119	96	412	389	366	153	176	199	448	473	498	141	116	91	6924	
6924	362	409	387	203	156	178	494	445	471	95	144	118	365	414	388	200	151	177	497	450	472	92	139	117	6924	
6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924

3.6 Pandiagonal Magic Square of Orders 27

Below is a pandiagonal magic squares of order 27.

pan	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	
9855	622	449	24	636	434	25	626	451	18	637	438	20	642	439	14	643	444	8	630	455	10	647	445	3	632	459	4	9855
9855	17	645	433	2	646	447	19	639	437	6	641	448	7	635	453	12	629	454	23	631	441	13	624	458	27	625	443	9855
9855	456	1	638	457	15	623	450	5	640	452	16	627	446	21	628	440	22	633	442	9	644	435	26	634	436	11	648	9855
9855	649	44	402	663	29	403	653	46	396	664	33	398	669	34	392	670	39	386	657	50	388	674	40	381	659	54	382	9855
9855	395	672	28	380	673	42	397	666	32	384	668	43	385	662	48	390	656	49	401	658	36	391	651	53	405	652	38	9855
9855	51	379	665	52	393	650	45	383	667	47	394	654	41	399	655	35	400	660	37	387	671	30	404	661	31	389	675	9855
9855	460	503	132	474	488	133	464	505	126	475	492	128	480	493	122	481	498	116	468	509	118	485	499	111	470	513	112	9855
9855	125	483	487	110	484	501	127	477	491	114	479	502	115	473	507	120	467	508	131	469	495	121	462	512	135	463	497	9855
9855	510	109	476	511	123	461	504	113	478	506	124	465	500	129	466	494	130	471	496	117	482	489	134	472	490	119	486	9855
9855	514	152	429	528	137	430	518	154	423	529	141	425	534	142	419	535	147	413	522	158	415	539	148	408	524	162	409	9855
9855	422	537	136	407	538	150	424	531	140	411	533	151	412	527	156	417	521	157	428	523	144	418	516	161	432	517	146	9855
9855	159	406	530	160	420	515	153	410	532	155	421	519	149	426	520	143	427	525	145	414	536	138	431	526	139	416	540	9855
9855	352	179	564	366	164	565	356	181	558	367	168	560	372	169	554	373	174	548	360	185	550	377	175	543	362	189	544	9855
9855	557	375	163	542	376	177	559	369	167	546	371	178	547	365	183	552	359	184	563	361	171	553	354	188	567	355	173	9855
9855	186	541	368	187	555	353	180	545	370	182	556	357	176	561	358	170	562	363	172	549	374	165	566	364	166	551	378	9855
9855	190	314	591	204	299	592	194	316	585	205	303	587	210	304	581	211	309	575	198	320	577	215	310	570	200	324	571	9855
9855	584	213	298	569	214	312	586	207	302	573	209	313	574	203	318	579	197	319	590	199	306	580	192	323	594	193	308	9855
9855	321	568	206	322	582	191	315	572	208	317	583	195	311	588	196	305	589	201	307	576	212	300	593	202	301	578	216	9855
9855	244	611	240	258	596	241	248	613	234	259	600	236	264	601	230	265	606	224	252	617	226	269	607	219	254	621	220	9855
9855	233	267	595	218	268	609	235	261	599	222	263	610	223	257	615	228	251	616	239	253	603	229	246	620	243	247	605	9855
9855	618	217	260	619	231	245	612	221	262	614	232	249	608	237	250	602	238	255	604	225	266	597	242	256	598	227	270	9855
9855	55	341	699	69	326	700	59	343	693	70	330	695	75	331	689	76	336	683	63	347	685	80	337	678	65	351	679	9855
9855	692	78	325	677	79	339	694	72	329	681	74	340	682	68	345	687	62	346	698	64	333	688	57	350	702	58	335	9855
9855	348	676	71	349	690	56	342	680	73	344	691	60	338	696	61	332	697	66	334	684	77	327	701	67	328	686	81	9855
9855	82	719	294	96	704	295	86	721	288	97	708	290	102	709	284	103	714	278	90	725	280	107	715	273	92	729	274	9855
9855	287	105	703	272	106	717	289	99	707	276	101	718	277	95	723	282	89	724	293	91	711	283	84	728	297	85	713	9855
9855	726	271	98	727	285	83	720	275	100	722	286	87	716	291	88	710	292	93	712	279	104	705	296	94	706	281	108	9855
9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855

The blocks of order 3 are **semi-magic squares** with equal sums, i.e., $SM_{3 \times 3} := 1095$

3.7 Pandiagonal Magic Square of Orders 33

Below is a pandiagonal magic squares of order 33.

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