



ANIQ FANLARNI O'QITISHDA INNOVATSION TEXNOLOGIYALARDAN VA NOAN'ANAVIY USULLARDAN FOYDALANISH.

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Kelajak uchun har tomonlama etuk mutaxassis kadrlar tayyorlashning mohiyati, zamonaviy fan va texnikaning rivojlanish talablariga mos barkamol avlodni tarbiyalash masalalari izchillik bilan tashkil etilib, bu boradagi dolzarb masalalar va ularni amalga oshirish chora tadbiri milliy dasturda belgilab berilgan.

Shu ma'noda "trigonometrik funksiya" larni o'rganishni klassik bo'limgan (noan'anaviy) usulini misol tariqasida keltirishni lozim topdik.

Algebraik tenglama va funksiyalarni yaxshi o'rgangan o'quvchilar ham trigonometrik funksiyalarni o'rganishda ba'zi bir qiyinchiliklarga duch kelishi mumkin. Muammoning asosiy sabablaridan biri, mualliflarning fikricha adabiyotlarning yo'qligida yoki bo'lsa ham juda kamligidadir.

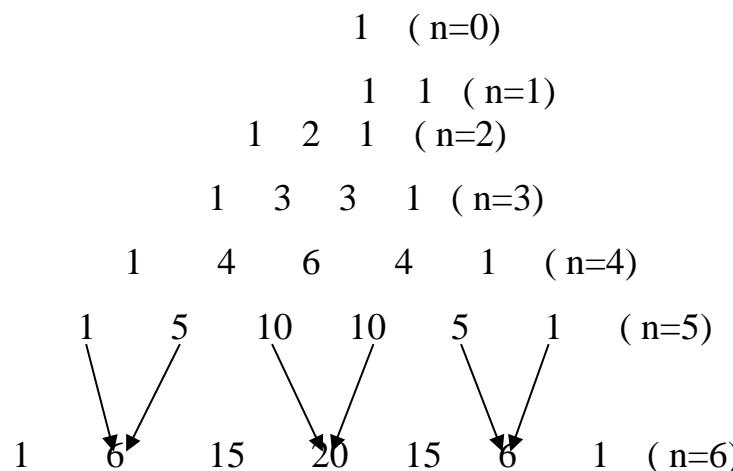
Karrali trigonometrik funksiyalarni oddiy trigonometrik funksiyalarga aylantirishda ikki burchak yig'indisining sinusi va kosinuslaridan foydalanadi. Bu an'anaviy usul o'quvchilardan ko'proq vaqt talab qiladi.

Hozirgi fan va texnika taraqqiyoti o'quvchilardan muammolarni tez va o'ta to'g'ri bajarishni talabqiladi. Masalan, $\cos 3x$ karrali funksiyani oddiy trigonometrik funksiya ko'rinishida keltiraylik.

$\cos 3x = \cos(x+2x) = \cos x \cos 2x - \sin x \sin 2x$ va ba'zi almashtirishlardan so'ng $4\cos^3 x - 3\cos x$ ifodaga tengligi kelib chiqadi. Bu klassik usul bilan ayniyatni isbotlashda o'quvchilar vaqtini yutqazish bilan birga soddalashtirish jarayonida xatolikka yo'l qo'yishi ham mumkin.

Biz taklif qilgan noan'anaviy usul vaqtini tejash bilan birlgilikda algebraik ayniyat bilan trigonometrik funksiyaning uzviy bog'liliginini asoslaydi.

Ko'phadlarni koeffitsentlarini aniqlashda Paskal uchburchagidan foydalanamiz.



Misol tariqasida $(a + b)^n$ darajasini ko'phad ko'rinishga keltiraylik. Bu hadlarni $a = \cos x$, $b = \sin x$ bilan almashtirsak va $n = 2$ bo'lganda Paskal uchburchagi

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

$$(a + b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

O'quvchi $\cos nx$ va $\sin nx$ formulalarni oddiy trigonometrik funksiyaga almashtirishning bu usulini quyidagi sxemada ko'rish mumkin (1-sxema):

$$\begin{aligned} \cos 2x &= 1\cos^2 x - 1\sin^2 x \\ &+ \quad \quad \quad - \\ \sin 2x &= 2\cos x \sin x \end{aligned}$$

$$\begin{aligned} \cos 3x &= 1\cos^3 x - 3\cos x \sin^2 x \\ &+ \quad \quad \quad + 1\sin^3 x \\ \sin 3x &= 3\cos^2 x \sin x \end{aligned}$$

$$\begin{aligned} \cos 4x &= 1\cos^4 x - 6\cos^2 x \sin^2 x - 1\sin^4 x \\ &+ \quad \quad \quad + \\ \sin 4x &= 4\cos^3 x \sin x + 4\cos x \sin^3 x \end{aligned}$$

Bu ma'lumotdan kelib chiqib quyidagi formulalarni taklif qilamiz [1];

$$\cos nx = a_1 \cos^n x - a_3 \cos^{n-2} x - a_5 \sin^2 x + \dots$$



$$\sin nx = a_2 \cos^{n-1} x \sin x - a_4 \cos^{n-3} x \sin^3 x + \dots$$

Bu usul yordamida qiyinroq bo'lgan $\operatorname{tg} nx$ funksiyasini ham oddiy funksiya ko'rinishiga keltirishimiz mumkin:

Xususiy holdat $\operatorname{tg} 2x = \frac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x}$; $\operatorname{tg} 3x = \frac{3 \operatorname{tg} x - \operatorname{tg}^3 x}{1 - 3 \operatorname{tg}^2 x}$; $\operatorname{tg} 4x = \frac{4 \operatorname{tg} x - 4 \operatorname{tg}^3 x}{1 - 6 \operatorname{tg}^2 x + \operatorname{tg}^4 x}$ va hakozo.

1- Misol. Ayniyatni isbot qiling.

$$\cos\left(\frac{5}{2}\pi - 6\alpha\right) \sin^3(\pi - 2\alpha) - \cos(6\alpha - \pi) \sin^3\left(\frac{\pi}{2} - 2\alpha\right) = \cos^3 4\alpha$$

$$\text{Yechish: } \cos\left(\frac{5}{2}\pi - 6\alpha\right) \sin^3(\pi - 2\alpha) - \cos(6\alpha - \pi) \sin^3\left(\frac{\pi}{2} - 2\alpha\right) =$$

$$= \cos\left(\frac{5}{2}\pi - 6\alpha\right) (\sin(\pi - 2\alpha))^3 - \cos(\pi - 6\alpha) \left(\sin\left(\frac{\pi}{2} - 2\alpha\right)\right)^3 =$$

$$= [\sin 3x = 3 \sin x - 4 \sin^3 x, \cos 3x = 4 \cos^3 x - 3 \cos x] = \sin 6\alpha \sin^3 2\alpha + \\ + \cos 6\alpha \cos^3 2\alpha = \sin 3(2\alpha) \sin^3 2\alpha + \cos 3(2\alpha) \cos^3 2\alpha = (3 \sin 2\alpha -$$

$$- 4 \sin^3 2\alpha) \sin^3 2\alpha + (4 \cos^3 2\alpha - 3 \cos 2\alpha) \cos^3 2\alpha = 3 \sin^4 2\alpha - 4 \sin^6 2\alpha +$$

$$+ 4 \cos^6 2\alpha - 3 \cos^4 2\alpha = -3(\cos^4 2\alpha - \sin^4 2\alpha) + 4(\cos^6 2\alpha - \sin^6 2\alpha) =$$

$$= -3(\cos^2 2\alpha - \sin^2 2\alpha)(\cos^2 2\alpha + \sin^2 2\alpha) + 4(\cos^2 2\alpha - \sin^2 2\alpha) \times$$

$$\times (\cos^4 2\alpha + \cos^2 2\alpha \sin^2 2\alpha + \sin^4 2\alpha) = -3(\cos^2 2\alpha - \sin^2 2\alpha) +$$

$$4(\cos^2 2\alpha - \sin^2 2\alpha)((\cos^4 2\alpha + \sin^4 2\alpha) + \cos^2 2\alpha \sin^2 2\alpha) = -3 \cos 4\alpha +$$

$$4 \cos 4\alpha ((\cos^2 2\alpha + \sin^2 2\alpha)^2 - 2 \cos^2 2\alpha \sin^2 2\alpha + \cos^2 2\alpha \sin^2 2\alpha) =$$

$$-3 \cos 4\alpha + 4 \cos 4\alpha (1 - \cos^2 2\alpha \sin^2 2\alpha) = -3 \cos 4\alpha + 4 \cos 4\alpha - \cos 4\alpha \times$$

$$\times (4 \sin^2 2\alpha \cos^2 2\alpha) = \cos 4\alpha - \cos 4\alpha (4 \sin^2 2\alpha \cos^2 2\alpha) = \cos 4\alpha - \cos 4\alpha \times$$

$$\sin^2 4\alpha = \cos 4\alpha (1 - \sin^2 4\alpha) = \cos 4\alpha \cos^2 4\alpha = \cos^3 4\alpha.$$

2- misol. Tenglamani yeching.

$$2 \cos 13x + 3 \cos 3x + 3 \cos 5x - 8 \cos x \cos^3 4x = 0$$

$$\text{Yechish. } \cos 3\alpha = 4 \cos^3 \alpha - 3 \cos \alpha, \cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2},$$

$$\cos \alpha - \cos \beta = 2 \sin \frac{\alpha + \beta}{2} \sin \frac{\beta - \alpha}{2}$$

$$2 \cos 13x + 3(\cos 3x + \cos 5x) - 8 \cos x \cos^3 4x = 0$$

$$\Leftrightarrow 2 \cos 13x + 3 \cdot 2 \cos 4x \cos x - 8 \cos x \cos^3 4x = 0 \Leftrightarrow$$



$$\Leftrightarrow 2\cos 13x - 2\cos x(4\cos^3 4x - 3\cos 4x) = 0 \Leftrightarrow$$

$$\Leftrightarrow 2\cos 13x - 2\cos x \cos 12x = 0 \Leftrightarrow 2\cos 13x - (\cos 13x + \cos 11x) = 0 \Leftrightarrow$$

$$\Leftrightarrow -2\sin 12x \sin x = 0$$

1) $\sin 12x = 0, 12x = \pi k, x_1 = \frac{\pi k}{12}, k \in \mathbb{Z}$

2) $\sin x = 0, x_2 = \pi n, n \in \mathbb{Z}$ chet ildiz

Javob. $x = \frac{\pi k}{12}, k \in \mathbb{Z}$

Shunday qilib, karrali trigonometrik funksiyalarni oddiy trigonometrik funksuyaga aylantirishning noan'anaviy va an'anaviy usulga qaraganda bir qancha qulayliklarga ega. Shulardan biri vaqtini tejash bilan birga o'quvchilarda sxemalar bilan ishlash mahoratini shakllantiradi. Shuningdek, trigonometrik funksiyalarni algebraik ayniyatlar bilan uzviy bog'liqligini ta'minlaydi. Bir vaqtda karrali trigonometrik funksiyalarni ayni $\cos nx, \sin nx$ larni oddiy trigonometrik funksiyalarga o'tkazish mumkin. (1-sxema)

Foydalanilgan adabiyotlar:

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