THE ARITHMETIC ROOT OF THE DEGREE OF THE NATURAL EXPONENT

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Abstract. *In this article, some aspects of the development of students' creative activities in the calculation of exponential arithmetic roots, nth-order arithmetic roots, and 3rd-order roots are considered, and concrete properties and applications of each studied method in teaching algebra are discussed. examples are considered.*

Keywords: *degree, arithmetic root, square root, cube root n-degree root and its properties.*

Definition: the arithmetic root of a non-negative number with a natural exponent $n \geq 2$ a non-negative number whose degree n is equal to a. Arithmetic root of the nth degree of the number –a is defined as follows $\sqrt[n]{a}$ a number is called the root expression. If n=2, then $\sqrt[2]{a}$ is written instead. Arithmetic roots of the second degree are also called square roots, and roots of the third degree are called cube roots. In cases where it is clear that the word is about an arithmetic root of the nth degree, it is called "nth degree root" for short.

Using the definition, to prove that $\sqrt[n]{a}$ is equal to b;

1) $b \geq 0$

2) it is necessary to show that $b^n = a$

For example, $\sqrt[3]{64} = 4$, because 4>0 va 4^2=64.

From the definition of the arithmetic root, if it is $a \geq 0$, then it is $(\sqrt[n]{a}^{\,n}) = a, \sqrt[n]{a^n} = a$. In general, for any odd natural number 2k+1, when a<0, the equation x^{2k+1}=a has only one negative root. This root is defined as an arithmetic root as follows: $\sqrt[2k+1]{a}$ is called an odd root of a negative number.

Properties of arithmetic roots

n- a degree root has the following properties:

If $a \geq 0, b > 0$ and n, m are natural numbers, $n \geq 2, m \geq 2$, then the following equations are true:

1^0. $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

2^0. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

3^0. $(\sqrt[n]{a})^m = \sqrt[n]{a^m}$

4^0. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[n \cdot m]{a}$

In property 1, the number b can be equal to 0. In property 3, the number m can be any integer if a>0.

For example $\sqrt[n]{ab}=\sqrt[n]{a}\cdot\sqrt[n]{b}$ we prove. We use the definition of an arithmetic root.

1. $\sqrt[n]{a}\cdot\sqrt[n]{b}\ge 0$ because $a\ge 0$ va $b\ge 0$

2. $(\sqrt[n]{a}\cdot\sqrt[n]{b})^n=ab$ because $(\sqrt[n]{a}\cdot\sqrt[n]{b})^n=(\sqrt[n]{a})^n\cdot(\sqrt[n]{b})^n=a\cdot b$

Level with a rational indicator

In general, if n-natural number $n\ge 2$, m- integer and $\frac{m}{n}$ if is an integer, then a>0 the following equality is true:

$\sqrt[n]{a^m}=a^{\frac{m}{n}}$ according to the condition $\frac{m}{n}$ - integer, i.e. dividing m by n yields k integers.

In this case $\frac{m}{n}=k$ from equality $m=kn$ it follows that, using the properties of the degree and the arithmetic root, we obtain the following.

$\sqrt[n]{a^m}=\sqrt[n]{a^{nk}}=\sqrt[n]{(a^k)^n}=a^k=a^{\frac{m}{n}}$ if $\frac{m}{n}$ if not an integer, then $a^{\frac{m}{n}}$ (in this a>0) degree, $\sqrt[n]{a^m}=a^{\frac{m}{n}}$ the formula is defined to remain true, i.e. in this case $a^{\frac{m}{n}}=\sqrt[n]{a^m}$ is considered.

Now, if the reader has a<0, we accept the following equality

$$a^{\frac{m}{n}}=-(a)^{m\cdot\frac{1}{n}}=-\sqrt[n]{(a)^m} \qquad (1)$$

$m,n\in N;$

According to this introduced equality, if we ignore it, we will show the following errors.

$$-1=(-1)^1=(-1)^{\frac{2}{2}}=(-1)^{2\cdot\frac{1}{2}}=\sqrt{(-1)^2}=\sqrt{1}=1$$

And if we do not recognize equality (1) in general when a<0

$a=a^{\frac{1}{1}}=a^{\frac{m}{m}}=a^{m\cdot\frac{1}{m}}=\sqrt[m]{a^m}$; to be m=2k when necessary –a=a, a<0,b<0 we will come to; accordingly, a<0, b<0

$$\sqrt[n]{a^{m_1\cdot n}\cdot b^{m_2\cdot n}}=|a|^{m_1}\cdot|b|^{m_2} \qquad (2)$$

$n=2k;\ m_1=2k_1+1,\ m_2=2k_2+1$ let's talk about making it happen,

For example; $\sqrt[4]{(-2)^{12}\cdot(-3)^{20}}=\sqrt[4]{(-2)^{3\cdot 4}\cdot(-3)^{4\cdot 5}}=|-2|^3\cdot|-2|^5=8\cdot 243=1944;\ a<0;b<0$ if

$a-b=-(\sqrt{|a|})^2+(\sqrt{|b|})^2=(\sqrt{|b|})^2-(\sqrt{|a|})^2=(\sqrt{|b|}-\sqrt{|a|})\cdot(\sqrt{|b|}+\sqrt{|a|});$ so a<0 if $a=-(\sqrt{|a|})^2$.

Otherwise a<0,b<0 when $a-b=(\sqrt{|a|})^2-(\sqrt{|b|})^2=(\sqrt{|a|}-\sqrt{|b|})\cdot(\sqrt{|a|}+\sqrt{|b|});$

we will come to an error equality.

Let's give an example.

$1^0.\ \sqrt[4]{a^8\cdot b^{12}}=a^2\cdot|b|^3;$

$2^0.\ \sqrt[6]{a^{12}\cdot b^{18}}=a^2\cdot|b|^3;$

$3^0.\ \sqrt[3]{\sqrt{x^6\cdot y^{12}}}-(\sqrt[5]{x\cdot y^2})^5=\sqrt[3]{|x|^3\cdot y^6}-x\cdot y^2=|x|\cdot y^2-x\cdot y^2=x\cdot y^2-x\cdot y^2=0;x\geq 0$

Examples;

$$1^0.\ \left(\frac{(z^{\frac{2}{p}}+z^{\frac{2}{q}})^2-4z^{\frac{2}{p}+\frac{2}{q}}}{(z^{\frac{1}{p}}-z^{\frac{1}{q}})^2+4z^{\frac{1}{p}+\frac{1}{q}}}\right)^{\frac{1}{2}}=\left(\frac{z^{\frac{4}{p}}+2\cdot z^{\frac{2}{p}+\frac{2}{q}}+z^{\frac{4}{q}}-4z^{\frac{2}{p}+\frac{2}{q}}}{z^{\frac{2}{p}}-2\cdot z^{\frac{1}{p}+\frac{1}{q}}+z^{\frac{2}{q}}+4z^{\frac{1}{p}+\frac{1}{q}}}\right)^{\frac{1}{2}}=\left(\frac{z^{\frac{4}{p}}-2\cdot z^{\frac{2}{p}+\frac{2}{q}}+z^{\frac{4}{q}}}{z^{\frac{2}{p}}+2\cdot z^{\frac{1}{p}+\frac{1}{q}}+z^{\frac{2}{q}}}\right)^{\frac{1}{2}}=$$

$$\left(\frac{\left(z^{\frac{2}{p}}-z^{\frac{2}{q}}\right)^2}{\left(z^{\frac{1}{p}}+z^{\frac{1}{q}}\right)^2}\right)^{\frac{1}{2}}=\left(\frac{\left(z^{\frac{2}{p}}-z^{\frac{2}{q}}\right)}{\left(z^{\frac{1}{p}}+z^{\frac{1}{q}}\right)}\right)=\left(\frac{\left|\left(z^{\frac{1}{p}}+z^{\frac{1}{q}}\right)\left(z^{\frac{1}{p}}-z^{\frac{1}{q}}\right)\right|}{\left(z^{\frac{1}{p}}+z^{\frac{1}{q}}\right)}\right)=\left|z^{\frac{1}{p}}-z^{\frac{1}{q}}\right|$$

$$2^0.\ x\cdot\sqrt[3]{2x\sqrt{xy}-x\cdot\sqrt{3xy}}\cdot\sqrt[6]{x^3\cdot y(7+4\sqrt{3})}=x\cdot\sqrt[3]{2x^{\frac{3}{2}}\sqrt{y}-x^{\frac{3}{2}}\cdot\sqrt{3y}}\cdot\sqrt[6]{x^3\cdot y(7+4\sqrt{3})}=$$

$$=x\cdot x^{\frac{1}{2}}\cdot\sqrt[3]{2\sqrt{y}-\sqrt{3y}}\cdot x^{\frac{1}{2}}\cdot\sqrt[6]{y}\cdot\sqrt[6]{(7+4\sqrt{3})}=x^2\cdot y^{\frac{1}{6}}\cdot\sqrt[3]{2\sqrt{y}-\sqrt{3y}}\cdot\sqrt[6]{(2+\sqrt{3})^2}=$$

$$=x^2\cdot\left|\sqrt[3]{y}\right|\cdot\sqrt[3]{(2-\sqrt{3})}\cdot\sqrt[3]{(2+\sqrt{3})}=x^2\cdot\left|\sqrt[3]{y}\right|$$

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