International Journal of Modern Developments in Engineering and Science
Volume 2, Issue 4, April 2023
https://www.ijmdes.com | ISSN (Online): 2583-3138

Finite Difference Analysis of Stress Distribution Around an Elliptical Hole in a Plate Subjected to Axial Loading

Md. Mesbah Uddin*
Graduate Research Assistant, Department of Mechanical Engineering, Northern Illinois University, Illinois, United States of America

***Abstract*: The analysis of stress distribution around an internal elliptical hole under axial stress is a challenging task in engineering design and analysis due do the action of external. In recent years, the finite difference technique has been widely used to investigate the stress distribution in such structures. In this paper, stress distribution of two-dimensional rectangular plates with elliptical holes in elastic bodies have been analyzed using the finite difference method. The results show that stress concentration occurs near the boundary of the hole, and the presence of the hole weakens the plate. The most critical section of the plate is through the center of the hole and perpendicular to the loading direction. The points on the boundary of the hole on this section are the most susceptible to failure. These findings provide valuable insights for the design and optimization of engineering structures. Further research can extend these findings to other geometries, loading conditions, and material properties, to improve the safety and efficiency of engineering systems.**

***Keywords*: Finite difference scheme, stress distribution, numerical analysis.**

1. Introduction

As technology advances, machinery and structures become increasingly complex. Consequently, elasticity has become a classical topic, and its problems have become even more classical. However, stress analysis problems are still limited by many simplifying assumptions. To obtain more comprehensive and sophisticated analyses, researchers have turned to the Theory of Elasticity. Analysis based on this theory can provide more detailed and precise information about stress, strain, and deformation at any point within a body.

While the theories of elasticity were established earlier, practical problem-solving began with the introduction of the stress function. The stress function, developed earlier, had limited success. It was only used for the solution of two-dimensional problems in the form of a polynomial and applied to several problems in bending of beams of narrow rectangular cross-section. However, the elementary formulas of the strength of materials give correct values for normal and shearing stresses in a cantilever loaded at the free end.

Many researchers have used the stress function technique for analyzing the stress around a circular hole in a plate [1], stresses around a slender hole [2], stresses around a concentrated load on a straight boundary [3], stresses around a concentrated load on a beam, and tribological behavior of metal [4].

The formulation of two-dimensional elastic problems was employed, which was initially introduced by Uddin [5]. Idris [6] later utilized this formulation to obtain analytical solutions for various mixed boundary mode elastic problems. Ahmed extended its use by solving finite difference solutions for several mixed boundary value problems of simple rectangular bodies. It was discovered by Dow, Jones, and Harwood that the accuracy of the finite difference technique in investigating the distribution of stress along the boundary was superior to that of the finite element technique. Subsequently, Akanda [7] developed a new numerical scheme that enabled the solution of irregular-shaped elastic bodies with mixed-mode boundary conditions.

This paper focuses on the stress analysis of two-dimensional rectangular plates with elliptical holes in elastic bodies by finite difference method and changing different parameters for different geometric shapes of the internal hole. Though a rectangular plate is selected for analysis, this formulation can be used for other arbitrary shapes of plates. While a body has three dimensions, most practical problems of stress analysis can be reduced to two-dimensional problems under simplifying assumptions. The present proposal thus includes a wide range of problems of stress analysis of elastic bodies.

2. Method and Modelling

A. Governing Equation

The analysis of stresses in an elastic body is typically a three-dimensional problem. However, when dealing with plane stress or plane strain cases, the stress analysis of a three-dimensional body can be easily reduced into a two-dimensional problem. The problem under investigation in this study pertains to a plane strain problem. In the absence of any body forces, the equations governing the three-stress component σ_x, σ_y and σ_{xy} under the state of plane stress or plain strain are as follows:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = 0 \quad (1)$$

*Corresponding author: mesbahuddin1991@gmail.com

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \sigma_{xy}}{\partial x} = 0 \quad (2)$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)(\sigma_x + \sigma_y) = 0 \quad (3)$$

$$\frac{\partial^2 u}{\partial x^2} + \left(\frac{1-\mu}{2}\right)\frac{\partial^2 u}{\partial y^2} + \left(\frac{1+\mu}{2}\right)\frac{\partial^2 v}{\partial x \partial y} = 0 \quad (4)$$

$$\frac{\partial^2 v}{\partial y^2} + \left(\frac{1-\mu}{2}\right)\frac{\partial^2 v}{\partial x^2} + \left(\frac{1+\mu}{2}\right)\frac{\partial^2 u}{\partial x \partial y} = 0 \quad (5)$$

These two equations can be used for the solution. But it is still difficult to solve for two functions simultaneously. To overcome the difficulty, the two equations are transformed into a single equation with a single function. So, a new function called displacement potential function (ψ) is defined as a function of displacement components to reduce the no. of governing differential equations into a single equation like following:

$$u = \frac{\partial^2 \psi}{\partial x \partial y} \quad (6)$$

$$v = -\frac{1}{1+\mu}\left[(1-\mu)\frac{\partial^2 \psi}{\partial y^2} + 2\frac{\partial^2 \psi}{\partial x^2}\right] \quad (7)$$

Putting this value in ψ (x, y) it becomes:

$$\frac{\partial^4 \psi}{\partial x^4} + 2\frac{\partial^4 \psi}{\partial x^2 \partial y^2} + \frac{\partial^4 \psi}{\partial y^4} = 0 \quad (8)$$

So the problem is now reduced in such a way that a single function *(x,y)* has to be evaluated from the bi-harmonic which satisfying the boundary conditions that are specified at the boundary.

B. S Geometry and Boundary Condition

The geometry of the problem is shown in Fig. 1(a). The material taken assumed perfectly elastic and were given the properties of material with Poisson's ratio μ=0.3. Despite the choice, this procedure is also valid for any type of elastic material.

The boundary AB as shown in Fig. 1(b) is considered rigidly fixed. So, there will be no displacement in this part of the boundary and thus the boundary conditions are set as u_n=0.0, u_t =0.0. at the right boundary (CD) a uniform tensile load is applied. For this boundary for every nodal point the boundary conditions are set as $\sigma_n /E = 3\times10^{-4}$, $\sigma_t /E = 0.0$, where the symbol *E* denotes modulus of elasticity, E=200GPa. The top and bottom boundaries (boundary AD and BC) are free from stress. The boundary conditions for the free boundary are set as $\sigma_n /E = 0.0$, $\sigma_t /E = 0.0$. The surface of the internal hole is free from any external load; boundary conditions are, therefore, assigned as $\sigma_n /E = 0.0$, $\sigma_t /E = 0.0$. For the above problems distribution of stress and displacement components are obtained from the output of the computer program. From that result stresses and displacements of five different sections (as shown in Fig. 2(b)) are analyzed to study the effect of hole in plate.

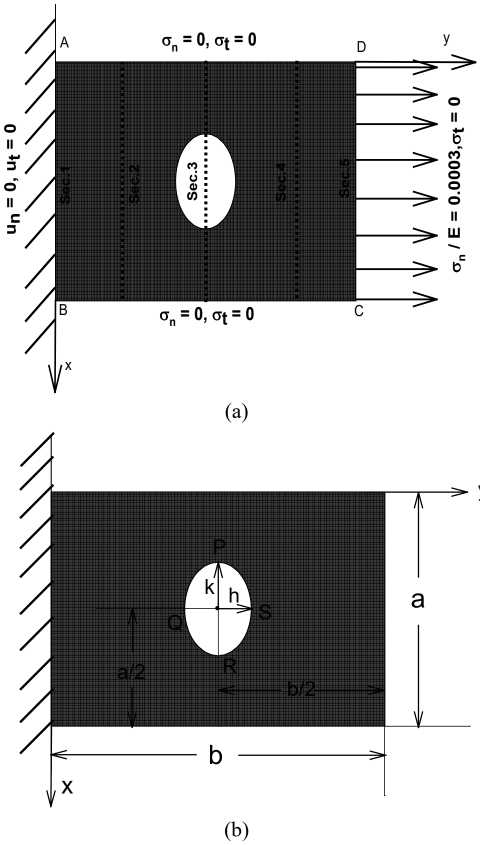
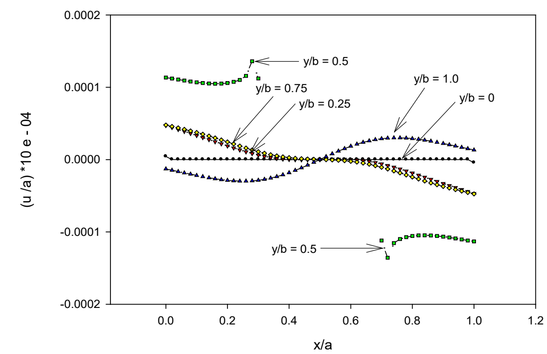


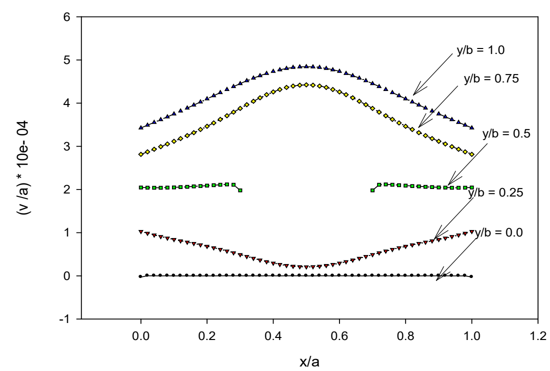
Fig. 1. Geometry of the problem, (a) Boundary Conditions applied for the problem, (b) The body is divided into 5 sections. (Sec 1: y/b=0, Sec 2: y/b=0.25, Sec 3: y/b=0.5, Sec 4: y/b=0.75, Sec 5: y/b=1.0)

3. Results and Discussions

Fig. 2 shows the distributions of u and v for the five sections described above, plotted against the distance from the top boundary, for h/k=0.5. The distributions of u for sections 1 and 5 and 2 and 4 are similar, indicating a symmetrical distribution of u around the hole. However, for the same loading condition, the values of u are higher in section 3, passing through the center of the hole. This is due to the flattening of the body under loading. Fig. illustrates the distribution of v for the five different sections for the body with hole size h/k=0.5, plotted against the distance from the top boundary. This figure shows a symmetrical distribution of v around the horizontal centerline (x/a=0.5). For any section between the hole and the loading boundary (section 5), the value of v is larger at the horizontal centerline. Conversely, for sections in the other zone i.e., between the fixed boundary and hole (section 1), the value of v is minimum at the horizontal centerline due to the flattening of the hole under loading. Throughout the body, the values of v are positive, and the values of v are higher in sections closer to the loading end and further from the fixed end. In section 3, the values of v for all nodal points are almost the same. However, for section 2 and 4, the values vary at each nodal point, with a higher variation than in sections 1 and 5. This variation is attributed to the presence of the hole. Without the hole, for a section perpendicular to the direction of loading, the values of v for every point would be the same. This effect diminishes at a distance larger than the radius of the hole.



(a)

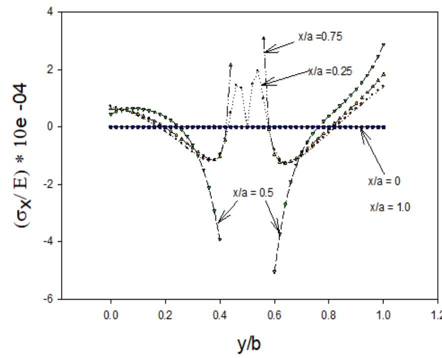


(b)

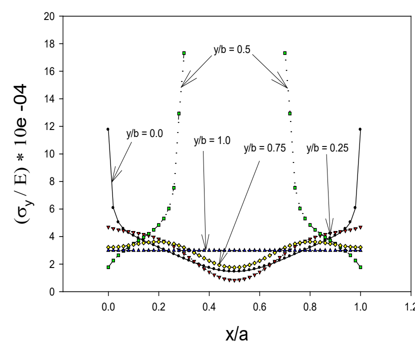
Fig. 2. Stress distribution of u (a) and v (b) in different 5 section for h=2.5 and k=5

In Fig. 3, the distribution of σ_x for different sections of the body with a hole size of h/k=0.5 is plotted against the distance from the left boundary. The stress component values for sections away from the hole are positive, while for some sections near the hole, they are negative. If there were no hole present, the whole body would experience compression along the x-axis, resulting in negative values of σ_x for every nodal point. Additionally, Fig. reveals that the values of the stress component at the upper and bottom boundaries are zero because the maximum material flow occurs at these regions, leading to a stress of zero. Furthermore, the values of σ_x for section 3 decrease as we move from the left or right boundary of the plate toward the hole boundary. This decrease is due to the greater material flow occurring in the vicinity of the hole boundary region, leading to a reduction in the stress component.

Fig. 3(b) displays the distribution of σ_y for the body with hole size h/k=0.5 plotted against the distance from the top boundary. This figure reveals that the values of σy for similar nodal points are the same at y/b=0.25 and y/b=0.75. The value of σ_y decreases as we approach the hole and then increases again. The distribution of σ_y for y/b=1.0 is a straight line, indicating no variation of σy on the free right surface (section 5). At the fixed end, i.e., y/b=0.0 (section 1), little variation occurs. The values of σ_y at the top and bottom boundaries for different sections, except section 3, are found to be larger than the values of the applied stress. In section 3, the values of σ_y at grid points on the upper and lower boundaries of the hole are almost equal to 4.2, which is the highest developed stress in the body. Fig. 3(b) demonstrates that among the sections, the stress component values are highest in section 3 (the section through the center of the hole and perpendicular to the direction of loading). Thus, section 3 is the most critical section of the plate.



(a)



(b)

Fig. 3. Stress distribution of σ_x (a) and σ_y (b) in different 5 section for h=2.5 and k=5

In Fig. 4, the distribution of τ_{xy} is shown for h/k=0.5 at the five sections depicted above. No change in τ_{xy} is observed when tensile loading is applied for y/b=0.0, 0.25, 0.75, and 1.0. However, at the boundary of the hole for y/b=0.5, two points are obtained. The upper part of section 3 of the hole yields a positive point, while the lower part of section 3 yields a negative point. These points indicate the presence of high values of τ_{xy} due to the hole's existence.

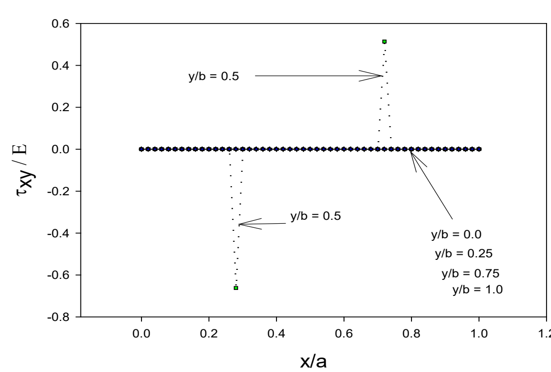


Fig. 4. Stress distribution of τ_{xy} in different 5 section for h=2.5 and k=5

4. Conclusion

The analysis of stress distribution in elastic bodies is a fundamental aspect of engineering mechanics. In the case of plates with internal holes, stress analysis is of particular importance due to their widespread applications in various fields of engineering. The present study focuses on the stress and displacement analysis of a rectangular plate with an internal elliptical hole under uniform tension loading conditions.

The results of this thesis work show that stress distribution changes around the hole boundary, with stress concentration occurring in the vicinity of the hole boundary. The presence of an internal hole also weakens the plate. The most critical section of a rectangular plate with an internal elliptical hole is a section through the center of the hole and perpendicular to the direction of loading. Additionally, the points on the hole boundary on this section are the most vulnerable to failure.

The findings of this study provide valuable insights into stress analysis of plates with internal holes, with implications for design and optimization of engineering structures. Further research can extend these findings to other geometries, loading conditions, and material properties, with the goal of improving the safety and efficiency of engineering systems.

References

[1] H. L. Cox, "The elasticity and strength of paper and other fibrous materials," 2nd ed.), John Wiley & Sons, 2005.

[2] M. Tuna and P. Trovalusci, "Stress distribution around an elliptic hole in a plate with 'implicit' and 'explicit' non-local models," in *Composite Structure*, vol. 256, 113003, Jan. 2021.

[3] S. K. Deb Nath and M. A. Salam Akanda, "A finite difference solution of a steel plate with an internal hole using the displacement potential approach," in *Journal of Strain Analysis*, vol. 44, no. 8, pp. 713–728, 2009.

[4] Uddin, Md. Mesbah, "Studying the Scratching Behavior of Nb/Nbc Nanolaminate Using Molecular Dynamics Atomistic Simulations," Northern Illinois University ProQuest Dissertations Publishing, 27999015, 2020.

[5] M. W. Uddin, "Finite difference solution of two-dimensional elastic problems with mixed boundary conditions," M.Sc. Thesis, Carleton University, Canada, 1966.

[6] A. B. M. Idris, "A new approach to the solution of mixed boundary value elastic problems," M.Sc. Thesis, Bangladesh University of Engineering and Technology, Bangladesh, 1993.

[7] M. A, S Akanda, S. R. Ahmed, M. R. Khan, and M. W. Uddin, "A finite difference scheme for mixed boundary value problems of arbitrary shaped elastic bodies," in *Advances in Engineering Software*, vol. 31, no. 3, pp. 173-184, 2000.