

# Input-output analytics for urban systems: explorations in policy and planning

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## Summary

We develop an inter-region input-output economic model of future challenges and appropriate policies. We initially focus on the UK—developing an analytic tool for the ‘levelling up’ agenda: to examine the characteristics of economically ‘good’ versus ‘poor’ cities and to explore policies aimed to shift cities from poor to good. Our method has potential application to equivalent challenges at varying scales and in other countries. The analysis is distinctive: estimate 48 city input-output accounts and the trading relationships between them via a novel incorporation of travel costs. Temporal extensions facilitate a scalable method to model future scenarios of policy outcomes.

**KEYWORDS:** input-output modelling, spatial interaction modelling, inter-city trade, levelling-up

## 1. Introduction

All aspects of urban policy are interdependent as illustrated by **Figure 1**, which shows relationships between the main elements of a city. The red, left-hand side represents demography: individuals and households who live, work, and use a variety of services in an urban environment. The green, right-hand side represents a city economy: flows of housing, employment, products, and services. These sub-systems are linked through the transport and communications sub-systems denoted in blue. There are mathematical models available for each of the sub-systems, as well as comprehensive models for whole city systems. These models can be built at various spatial scales—from ‘whole’ cities in a regional or national system, to fine-grained spatial structures within a single city.<sup>1</sup> We develop a scalable model of the right-hand side: economic relationships within or between cities, with novel temporal and spatial components for ‘whole country’ analysis with an initial focus on levelling-up policy outcomes.

Despite a wide range of current urban sub-models, few estimate economic policy scenarios. Our extensions to Leontief’s input-output model (1986) addresses this need. Inter-nation input-output models are well established,<sup>2</sup> using good data on national demographics and trade—but there is comparatively little work on inter-city input-output models, and poor data on inter-city trade. To tackle the levelling-up agenda and contribute to this gap in research, we develop an inter-city input-output model to estimate future scenarios of policies targeting inter-city trade.

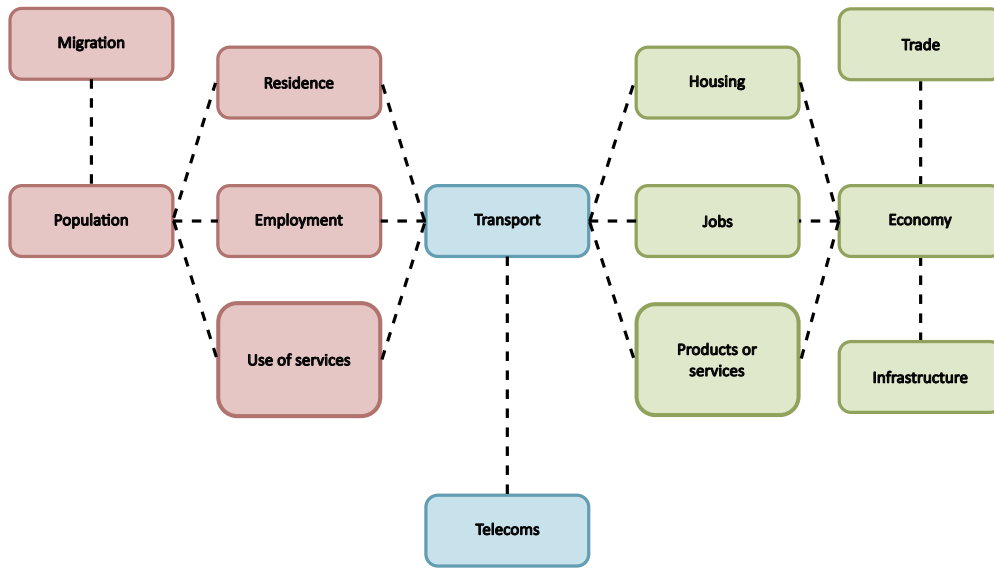
Our model, rooted in some heroic assumptions and innovative sub-models to estimate missing data (Evans and Levy, 2016), will offer insights on potential levelling-up agenda outcomes and lay foundations for further research.

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<sup>1</sup> For example, QUANT: see Batty and Milton (2021); SPENSER: see Birkin (2021).

<sup>2</sup> See <https://www.oecd.org/sti/ind/input-outputtables.htm>



**Figure 1:** The main elements of a comprehensive city model

## 2. Inter-city input-output accounts

Our model focuses on the economic dynamics of city systems. Our accounting framework builds on a sketch in Oléron-Evans (2016), which expands on Levy, Oléron-Evans and Wilson (2016). The model specification roots in input-output accounts for UK cities, including estimates of trade flows between each city pair and the rest of the world.

### 2.1. Input-output model variables

The key model variables are listed in **Table 1**, where subscripts  $i$  and  $j$  denote cities and superscripts  $m$  and  $n$  denote sectors. Note that for any variable where subscript  $i$  is replaced by an asterisk '\*', this means summation over all  $i$ .

**Table 1:** The main input-output model variables at one point in time

Variable	Description
$P_i$	population of city (or region) $i$
$Q_i^m$	employment in sector $m$ in city $i$
$X_i^m$	total production of sector $m$ in city $i$ : $m = 1, 2, 3, \dots, S$ and $i = 1, 2, 3, \dots, N$
$F_i^m$	final demands for $m$ in $i$
$e_i^m$	total domestic exports of $m$ from $i$ to other cities (excluding international exports)
$m_i^m$	total domestic imports of $m$ into $i$ from other cities (excluding international imports)
$E_i^m$	international exports of $m$ from city $i$
$M_i^m$	international imports of $m$ to city $i$
$a_i^{mn}$	the amount of $m$ needed to produce a unit of $n$ in $i$
$x_i^{mn}$	the total demand for sector $m$ to produce a unit of sector $n$ in $i$
$y_{ij}^m$	the trade flow (domestic export) of $m$ from city $i$ to city $j$
$c_{ij}^k$	the cost of transporting one unit from city $i$ to city $j$ , by transport mode $k$

## 2.2. Core input-output model

We define  $x_i^{mn}$  as intermediate demand (demand for sector  $m$  in city  $i$  to produce a unit of  $n$ ) in relation to  $a_i^{mn}$  (the amount of  $m$  needed to produce a unit of  $n$  in  $i$ ), such that:

$$x_i^{mn} = a_i^{mn} X_i^n \quad (1)$$

The core accounting equation can then be written as:

$$X_i^m + m_i^m + M_i^m = F_i^m + e_i^m + E_i^m + \sum_n a_i^{mn} X_i^n \quad (2)$$

The left-hand sum  $X_i^m + m_i^m + M_i^m$  is the *supply* of sector  $m$  in city  $i$ : where  $X_i^m$  is total production of good  $m$  in city  $i$ ,  $m_i^m$  is domestic imports of good  $m$  to city  $i$  and  $M_i^m$  is international imports of good  $m$  to city  $i$ . The right-hand sum  $F_i^m + e_i^m + E_i^m + \sum_n a_i^{mn} X_i^n$  is the *demand* for (or *use* of) good  $m$  in city  $i$  for sector  $n$ .  $F_i^m$  is the final demand for good  $m$  in city  $i$ ,  $e_i^m$  is inter-city exports of good  $m$  from city  $i$ ,  $E_i^m$  is international exports of  $m$  from  $i$ , and  $\sum_n a_i^{mn} X_i^n$  is the sum of all intermediate demands ( $x_i^{mn}$  as specified in Equation 1) for city  $i$  producing good  $n$  via good  $m$ .

## 2.3. Core data and model estimation

We use data on 48 cities provided by the UK Office of National Statistics (ONS) and NOMIS<sup>3</sup> following Centre for Cities specifications of city Primary Urban Areas (PUAs). ONS input-output tables are calculated from national inter-sector Supply and Use tables, and 2018 is the latest available. Beginning with 2017, we validate our 2018 model projection by combining national and PUA level data on age distributions and employment per sector to estimate city level components of Equation 2. Specifically:

- a national input-output table<sup>4</sup>
- population,  $P_i$  (working age 15-64)
- employment by city  $i$  and sector  $m$ ,  $Q_i^m$
- international trade flows,  $E_*^m$  and  $M_*^m$

We estimate  $y_{ij}^m$ —inter-city export  $e_i^m$  and import  $m_i^m$  flows—by filling the remaining terms in Equation 2 for cities with Equations 3–6, where national tallies are scaled by city populations:

$$X_i^m = X_*^m Q_i^m / Q_*^m \quad (3)$$

$$F_i^m = F_*^m Q_i^m / Q_*^m \quad (4)$$

$$E_i^m = E_*^m Q_i^m / Q_*^m \quad (5)$$

$$M_i^m = M_*^m Q_i^m / Q_*^m \quad (6)$$

We then set an initial state of  $e_i^m = 0$  and  $m_i^m = p M_i^m$  (where  $0 < p < 1$ ) and iterate to model convergence.

## 2.4. Spatial input-output model extensions

We extend the above inter-city input-output model by incorporating domestic transportation costs into the calculation of  $y_{ij}^m$ :

$$y_{ij}^m = b_{ij}^m m_j^m \quad (7)$$

<sup>3</sup> NOMIS is a platform for ONS data: <https://www.nomisweb.co.uk/>.

<sup>4</sup> For tractability we assume city technical coefficients equal national:  $a_i^{mn} = a_*^{mn}$ .

where  $i \neq j$  and  $j \neq 0$ .  $b_{ij}^m$  is a doubly-constrained spatial interaction term following Wilson (1970):

$$b_{ij}^m = KA_i^m B_j^m Q_i^m Q_j^m \exp(-\beta c_{ij}^k) \quad (8)$$

where:

$$A_i^m = 1/\sum_j B_j^m Q_j^m \exp(-\beta c_{ij}^k) \quad (9)$$

$$B_j^m = 1/\sum_i A_i^m Q_i^m \exp(-\beta c_{ij}^k) \quad (10)$$

$$K = 1/\sum_j b_{ij}^m \quad (11)$$

For Equation 8 we estimate  $c_{ij}^k$ —travel costs between cities  $i$  and  $j$  by mode  $k$ —as the distance between PUA centroids. We then expand the previous estimation process by iteratively calculating  $y_{ij}^m$  via Equation 7, then  $e_i^m$  and  $m_i^m$  via Equations 12 and 13, until the model converges (tuning  $\beta$ —weight of transport—to optimise convergence).

$$\sum_j y_{ij}^m = e_i^m \quad (12)$$

$$m_i^m = F_i^m + e_i^m + E_i^m + \sum_n a_i^{mn} X^n - X_i^m - M_i^m \quad (13)$$

## 2.5. Future scenarios and policy implications

As demonstrated above, our spatial input-output model for UK city trade is calculated from 2017 ONS and NOMIS data. Following Dearden and Wilson (2015), we generate a baseline future scenario time series from ONS population projections, tuning parameters to fit the 2018 national input-output table.

That provides a framework to model leveling up policy scenarios *relative* to our validated baseline. The scenario estimates help gauge long-term policy outcomes on targeted cities and sectors, and the impact on the rest of the UK. Further: the framework could be applied to other polices such as transport (Northern Powerhouse Rail and High Speed 2). Similarly, our model could be scaled to assess policies at other levels, such as investment in public transport within a single city or even rail infrastructure between nations.

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## Biographies

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