BMA Knockout Swap Model

The BMA Ratio Swap with BMA Knockout is a two-legged BMA ratio swap where one leg pays a contract specified fixed rate and the other leg pays Libor times a contract specified ratio (plus a contract specified constant spread).

On any contract payment date both the fixed and Libor coupon payments may cancel if the historical average BMA rate observed during the coupon period is above (or below, as specified by the contract) the knockout strike.

For example, consider a contract where one party receives the fixed rate and pays 68% of 1M Libor. Both payments are cancelled if on a coupon date the 1M average of the 1W observations of the BMA rate (i.e., the average of the prior 4 observations of 1W BMA rate) is greater 5%.

If we consider a deal called Libor Swap with BMA Knock-In where the knockout condition is defined by a maximum level for average BMA, the coupon payments at time *i S* are the following:

The payoff of the fixed leg of the Libor Swap with BMA Knock-In is

$$c\alpha_i^{fixed} N_i^{BMA} I_{\{B_{S_I} < K\}},$$

The payoff of the LIBOR leg of the Libor Swap with BMA Knock-In is

$$\alpha_i N_i^{Libor} \rho_{ratio} L_{T_i}[T_i, S_i] I_{\{B_{S_I} < K\}} + s\ \alpha_i N_i^{Libor} I_{\{B_{S_I} < K\}}.$$

Similarly, we have for a swap called Libor Swap with BMA Knock-Out where the coupon cancellation condition is defined by a minimum level for average BMA at time i S :

The payoff of the fixed leg of the Libor Swap with BMA Knock-Out is

$$c\alpha_i^{fixed} N_i^{BMA} I_{\{B_{S_I} > K\}},$$

The payoff of the LIBOR leg of the Libor Swap with BMA Knock-Out is

$$\alpha_i N_i^{Libor} \rho_{ratio} L_{T_i}[T_i, S_i] I_{\{B_{S_I} > K\}} + s\ \alpha_i N_i^{Libor} I_{\{B_{S_I} > K\}}.$$

There is a dual switch between Libor Swap with BMA Knock-In and Libor Swap with BMA Knock-Out. To avoid the tedious repetition, we only review Libor Swap with BMA Knock-Out below.

Now we want to get the payoff of the equation above. Under the forward measure T , we assume that BMA rate and Libor rate are following two different driftless Geometry Brownian Motions:

$$\frac{dL_{T_i}}{L_{T_i}} = \sigma_i^L dX_{T_i}$$

$$\frac{dB_{S_i}}{B_{S_i}} = \sigma_i^B dY_{S_i}$$

We have the following three formulas which are used to get the payoff of Libor Swap with BMA Knock-Out equation

$$\text{Prob}\{B_{S_i} > K\} = N(d_1),$$

$$\text{Prob}\{B_{S_i} < K\} = N(-d_1),$$

$$E^{S_i}(L_{T_i} I_{\{B_{S_I} > K\}}) = L_0[T_i, S_i] N(\rho \sigma_i^L \sqrt{T_i} + d_1),$$

$$E^{S_i}(L_{T_i} I_{\{B_{S_I} < K\}}) = L_0[T_i, S_i] N(-\rho \sigma_i^L \sqrt{T_i} - d_1),$$

where N denotes the cumulative distribution function of the normal distribution and

$$d_1 = \frac{\ln(\frac{B_0[T_i, S_i]}{K}) - 0.5 \sigma_i^B \sigma_i^B S_i}{\sigma_i^B \sqrt{S_i}}.$$

Libor Delta for each Libor curvebuild instrument perturb the market rate by 1bp, rebuild the Libor and BMA curves, then re-price the digital Libor KO deal (see https://finpricing.com/knowledge.html)..

The i -th Libor Delta is defined as LiborDeltai = V (the i -th Libor instrument rate + 1bp) - V (nominal)

BMA Ratio Delta for each BMA curve build market ratio perturbs the market ratio by one point, rebuild the BMA curve, and then re-price the digital Libor KO deal. The i -th BMA Ratio Delta is defined as

BMARatioDeltai = V (the i -th BMA Ratio rate + 1%) - V (nominal)

Libor Vega Perturb all Libor vols by 1 point then re-price.

LiborVega = V (all Libor vols + 1%) - V (nominal)