Energy and momentum of photons in gravitational field

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We study a problem of energy and momentum of photons redshifted in a gravitational field. Based on the Einstein-Maxwell's equations for electromagnetic waves in curved spacetimes, we derive formulas for the speed, energy and momentum of photons in gravitational fields. The formulas are further specified for the Schwarzschild metric describing a local gravitational field around a massive body. It is shown that energy of photons is conserved in inertial (free-falling) as well as in non-inertial coordinate systems, and no energy is exchanged between photons and the gravitational field. The Planck energy-frequency relation valid in Special Relativity is modified to be applicable also to General Relativity. According to the new Planck relation, the photon energy depends not only on the frequency of photons but also on their speed. In a free-falling system, the photon energy is conserved, because no frequency shift and no change of the photon speed is detected. In non-inertial systems, the photon energy is also conserved, because the frequency shift due to gravity is compensated by the change of the photon speed.

I. INTRODUCTION

In general relativity (GR), gravitational field affects geometry of rays of electromagnetic waves or photons and changes their frequency f. This effect is called the gravitational redshift and it was first described by Einstein in 1907[7,8]. The gravitational redshift belongs to basic classical tests of GR and its experimental evidence for the Earth's gravity was first reported by Pound and Rebka[22] and Pound and Snider[23]. The authors detected the frequency shift of gamma-ray photons from ^{57}Fe at different altitudes. Since the effect was tiny, they utilized the Mössbauer effect to produce a narrow resonance line to improve the accuracy of the measurement. The later experiments measured, e.g., spectral lines in the Sun's gravitational field, the redshift of light coming from galaxies in clusters, and the change in rate of atomic clocks or optical lattice clocks in ground-based measurements or in space measurements with the clocks transported on aircrafts, rockets or satellites[3,5,6,26,28,29].

Since the gravitational redshift predicts a change of frequency of photons, it should affect also their energy. According to the Planck relation, the change of the energy of photons ΔE is related to the frequency change Δf as[4]

$$\Delta E = h \Delta f \, , \tag{1}$$

where h is the Planck constant. The energy change of photons is interpreted as a loss (or gain) of energy due to the interaction of photons with the gravitational field. If the photon propagates against the gravitational acceleration, it spends work and its energy decreases. If the photon propagates in the direction of the gravitational acceleration, its energy increases. The energy change ΔE of photons is calculated from a difference of the gravitational potential $\Delta \phi$ between two observers. For the gravitational field of the Earth, this difference is expressed as

$$\frac{\Delta E}{E} = \frac{\Delta f}{f} = \frac{\Delta \phi}{c^2} = \frac{gz}{c^2} \, , \tag{2}$$

where g is the gravitational acceleration, and z is the difference in height between the two observers.

The above-described explanation of the gravitational redshift as the effect of the energy change of photons due to gravity is, however, intuitive and very simplistic, because it interprets the GR effect in terms of the Newton gravity developed for massive particles. When examining this explanation in detail, we find that the problem is more involved than commonly treated. The idea of the gravitational redshift as a transformation of the potential gravitational energy into the kinetic energy and vice versa cannot work for photons for several reasons: (1) No gravitational potential energy of photons is defined in GR, and (2) no energy transfer between free propagating photons and gravity is assumed in GR. The free photons are massless and thus they cannot be coupled with gravity in the Einstein-Maxwell's equations. (3) If free photons couple with gravity (and with spacetime), rays could not be defined as the null geodesics of the spacetime. However, the idea of the null geodesics as paths of free photons propagating in a gravitational field is basic principle and one of pillars of GR.

Hence, a correct interpretation of the gravitational redshift is possible only within GR: the shift in the photon frequency should be considered as an effect of the time dilation of the Riemann spacetime curved by gravity. Since the time rate defined by the time-time component g_{tt} of the metric tensor $g_{\alpha\beta}$ is different for the emitter e and receiver r, the photon frequency must also vary

$$\frac{f(e)}{f(r)} = \sqrt{\frac{g_{tt}(r)}{g_{tt}(e)}} \, . \tag{3}$$

Nevertheless, the problem of energy of photons in gravity is still not fully clear and poses open questions: (1) In GR, the zero divergence of the stress-energy tensor ensures that the energy of any physical system in gravity must be conserved. If photons propagating in the gravitational field are redshifted, does it mean that they lose energy or not? If so, where does the energy go or how to understand the energy conservation in GR? (2) How does the energy of photons behave when studied in different coordinate systems? Is there any difference when the energy is evaluated in inertial (free-falling) systems and in non-inertial systems? (3) The photons change their frequency due to the deformation of the spacetime, but they also change their coordinate speed (in non-inertial systems). How does the change of the speed of photons affect the photon energy?

Since the energy of photons is related to the momentum of photons by formula $E = pc$, we can also ask questions about the behaviour of the photon momentum in a gravitational field. First of all, we would like to know, whether the photon momentum is conserved in GR or not. Also, how the photon momentum depends on the coordinate system, in which is evaluated. Note that the behaviour of the photon momentum is unclear not only for the vacuum with a gravitational field, but also for dielectric media. The problem is known as the so-called Abraham-Minkowski controversy, where two alternative theories predicting different formulas for the photon momentum exist[1,2,17,18,21].

In this paper, we try to address the above-posed questions. We study the problem of energy and momentum of photons in gravity by the Einstein-Maxwell's equations of the electromagnetic waves propagating in the curved spacetime. Based on covariant coordinate transformations between the Minkowski space and the Riemann space, we derive formulas for the speed, energy and momentum of photons in gravitational fields. The formulas are further specified for the Schwarzschild metric describing a local gravitational field around a massive body. It is shown that the energy of photons is conserved and no energy is exchanged between photons and the gravitational field. Finally, the Planck energy-frequency relation for photons is modified to be valid in GR.

II. ELECTROMAGNETIC WAVES IN THE MINKOWSKI SPACE

A. Maxwell's equations

Considering spacetime with coordinates $x^\alpha = (ct, x, y, z)$, we can introduce the electromagnetic 4-potential A^α[19]

$$A^\alpha = \left(\frac{\phi}{c}, \mathbf{A}\right), \ \alpha = 0, 1, 2, 3\,, \tag{4}$$

the electromagnetic (Faraday) tensor $F^{\alpha\beta}$

$$F^{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha\,, \tag{5}$$

and the electromagnetic stress-energy tensor

$$T^{\alpha\beta} = \frac{1}{4\pi}\left(F^{\alpha\mu}F^{\beta}{}_{\mu} - \frac{1}{4}\eta^{\alpha\beta}F_{\mu\nu}F^{\mu\nu}\right)\,, \tag{6}$$

where ϕ is the scalar potential, $\mathbf{A}$ is the vector potential, and c is the speed of light. The covariant and contravariant derivatives ∂_α and ∂^α are defined as

$$\partial_\alpha = \partial/\partial x^\alpha\,, \text{ and } \partial^\alpha = \eta^{\alpha\beta}\partial/\partial x^\beta\,, \tag{7}$$

where $\eta^{\alpha\beta}$ is the contravariant metric tensor of the Minkowski spacetime with the sign convention $(-,+,+,+)$.

Subsequently, the Maxwell's equations for electromagnetic waves in vacuum with no charges can be expressed in the several following alternative forms[16,19]

$$\partial_\alpha F^{\alpha\beta} = 0\,, \tag{8}$$

$$\Box A^\alpha = \partial_\beta\,\partial^\beta A^\alpha = 0\,, \tag{9}$$

$$\partial_\alpha T^{\alpha\beta} = 0\,, \tag{10}$$

where $\Box A^\alpha$ is the wave operator (d'Alambertian), and A^α should satisfy also the Lorentz gauge condition

$$\partial_\alpha A^\alpha = 0\,. \tag{11}$$

As seen from Eq. (10), the Maxwell's equations imply the energy-momentum conservation law for electromagnetic waves in vacuum.

B. Lagrangian formulation of Maxwell's equations

Using the Lagrangian density $\mathcal{L}$ of the electromagnetic field in the form

$$T^{\alpha\beta} = -\frac{1}{4\mu_0}F^{\alpha\beta}F_{\alpha\beta}\,, \tag{12}$$

the Maxwell's equations (8-10) can alternatively be obtained using the Lagrange equations

$$\partial_\alpha\left[\frac{\partial\mathcal{L}}{\partial A^\beta{}_{,\alpha}}\right] - \frac{\partial\mathcal{L}}{\partial A^\beta} = 0\,, \tag{13}$$

Introducing the momentum of the electromagnetic field $\mathbf{P}$ as

$$P_i = \frac{\partial\mathcal{L}}{\partial\left(A^i/\partial x^0\right)}\,, i = 1, 2, 3\,, \tag{14}$$

and taking into account that $\partial\mathcal{L}/\partial A^i$ is zero in vacuum, the Lagrange equations (13) imply for $\alpha = 0$

$$\frac{\partial P_i}{\partial x^0} = 0\,, i = 1, 2, 3\,, \tag{15}$$

which is the momentum conservation law for the electromagnetic field. Taking into account equations for the electric and magnetic fields $\mathbf{E}$ and $\mathbf{H}$

$$\mathbf{E} = -\nabla\phi - \partial\mathbf{A}/\partial t\,, \tag{16}$$

$$\mathbf{E} = \nabla \times \mathbf{A}\,, \tag{17}$$

we finally get for the momentum $\mathbf{P}$

$$\mathbf{P} = \frac{1}{c^2}\mathbf{E} \times \mathbf{H}\,, \tag{18}$$

where c is the speed of light in vacuum with no gravity (vacuum undisturbed by gravity).

III. ELECTROMAGNETIC WAVES IN THE RIEMANN SPACE

The above equations are valid for the vacuum free of gravitational field. If an electromagnetic wave (light) propagates in gravity, the Maxwell's equations must be modified. In order the Maxwell's equations to be valid even for spacetimes curved by gravity, the metric tensor of the Minkowski space $\eta^{\alpha\beta}$ must be substituted by the metric tensor of the Riemann space $g^{\alpha\beta}$, when calculating quantities A^α, $F^{\alpha\beta}$ and $T^{\alpha\beta}$ and their covariant derivatives[20,27]. The generalization of the equations for the Riemann space is not difficult, but it should be done with care, in order the equations and the involved physical quantities to be transformed properly.

The most of changes in properties of the electromagnetic waves in the Riemann space are connected to essentially different view on the speed of electromagnetic waves in GR with respect to Special Relativity (SR). While, the speed of light is constant in SR, it is generally varying in GR. Since all coordinate systems, which are in rest with respect to the gravitational field, are non-inertial, they are affected by gravity. The gravity affects not only geometry of rays but also the speed of waves propagating along the rays[9]. A famous example is the Schwarzschild solution for the gravitational field around a massive body, when the speed of light goes to zero for photons approaching the singularity[27]. Nevertheless, the basic laws implied from the Maxwell's equations, such as the energy-momentum conservation law for the electromagnetic waves (see Eq. 10) are valid even in the vacuum distorted by gravity.

A. Speed of light in curved spacetime

Let us assume the Riemann space described by metric tensor $g_{\alpha\beta}$. Since any symmetric tensor can be diagonalized using a coordinate transformation, we can write the metric tensor with no loss of generality in the following form

$$-c^2 d\tau^2 = -g_{tt} c^2 dt^2 + g_{ii} dx^i dx^i\,, \tag{19}$$

where g_{tt} and g_{ii} describe the time dilation and space deformation due to the gravity. The propagation of the electromagnetic waves is described by the equation of the null geodesics, $c^2 d\tau^2 = 0$. Hence,

$$g_{tt} c^2 dt^2 = g_{ii} dx^i dx^i\,, \tag{20}$$

and the contravariant (coordinate-dependent) speed of light c_g^i along the x^i-axis reads

$$c_g^i = \frac{\sqrt{dx^i dx^i}}{dt} = \sqrt{\frac{g_{tt}}{g_{ii}}}\,c \quad \text{(no summation over } i\text{)}\,. \tag{21}$$

In order to express the physical (proper) speed of light, which is coordinate invariant, we have to express the speed of light in the orthonormal coordinate basis[15]. Hence, the i-th component of the proper speed of light is

$$c_{g(i)} = \sqrt{g_{ii}}\,c_g^i = \sqrt{g_{tt}}\,c \quad \text{(no summation over } i\text{)}\,, \tag{22}$$

where $X_{(i)}$ denotes the i-th physical component of vector $\mathbf{X}$. Since, the proper light speed has the same magnitude in all directions, we can simply write

$$c_g = \sqrt{g_{tt}}\,c\,. \tag{23}$$

The proper speed of light c_g is a quantity measured in the coordinate system that is at rest with respect to the sources of the gravitational field. The system is not inertial or free-falling, hence it is affected by gravity. This causes that the speed of light is not constant but varying, being dependent on the distance from the observer to the source of gravity.

B. Four-potential in curved spacetime

The electromagnetic 4-potential A^α is transformed from the Minkowski space to the Riemann space as any other vector according to the following equation

$$A_g^\alpha = A^\beta \frac{\partial x^\alpha}{\partial y^\beta}\,, \tag{24}$$

where A_g^α is the electromagnetic 4-potential in the Riemann space distorted by gravity, and x^α and y^β are the coordinates of the Minkowski and Riemann spaces, respectively. Taking into account that Eq. (19) implies

$$\frac{\partial x^0}{\partial y^0} = \sqrt{g^{tt}}\,,\ \frac{\partial x^i}{\partial y^i} = \sqrt{g^{ii}} \quad \text{(no summation over } i\text{)}\,, \tag{25}$$

we get for the 4-potential A_g^α in the Riemann space

$$A_g^0 = A^0/\sqrt{g_{tt}}\,,\ A_g^i = A^i/\sqrt{g_{ii}}\,, \tag{26}$$

hence

$$A_g^\alpha = \left(\frac{\phi}{\sqrt{g_{tt}}\,c}\,, \mathbf{A}_g\right) = \left(\frac{\phi}{c_g}\,, \mathbf{A}_g\right)\,. \tag{27}$$

Obviously, the magnitude of the 4-potential A_g^α in the Riemann space is Lorentz invariant

$$\begin{aligned} A_g^\alpha A_{g\alpha} &= A_g^0 A_{g0} + A_g^i A_{gi} \\ &= \frac{\sqrt{g_{tt}}}{\sqrt{g_{tt}}}\frac{\phi^2}{c^2} + A^i A_i = A^\alpha A_\alpha\,. \end{aligned} \tag{28}$$

C. Four-momentum in curved spacetime

The 3-momentum of light in the Riemann space $\mathbf{P}_g$ can be expressed using Eq. (14), where we apply the coordinate transformation from the Minkowski to the Riemann space

$$\begin{aligned} P_{gi} &= \frac{\partial \mathcal{L}}{\partial\left(\partial A_g^i/\partial x^0\right)} = \frac{\partial \mathcal{L}}{\partial\left(\partial A^i/\partial y^0\right)}\frac{\partial x^0}{\partial y^0}\,\frac{\partial y^i}{\partial x^i} \\ &= P_i \frac{\partial x^0}{\partial y^0}\,\frac{\partial y^i}{\partial x^i}\ (\text{no summation over } i)\,, \end{aligned} \tag{29}$$

hence

$$P_{gi} = P_i \frac{\sqrt{g_{ii}}}{\sqrt{g_{tt}}}\ ,\ P_g^i = \frac{P_i}{\sqrt{g_{tt}g_{ii}}}\ , \tag{30}$$

and the physical magnitude of the 3-momentum in the Riemann space is

$$P_g = \sqrt{P_{gi}P_g^i} = P/\sqrt{g_{tt}} = P\frac{c}{c_g}\ . \tag{31}$$

To complete the full 4-momentum, we calculate the P_{g0} component similarly as the P_{gi} components in Eq. (29)

$$P_{g0} = \frac{\partial \mathcal{L}}{\partial\left(\partial A_g^0/\partial x^0\right)} = \frac{\partial \mathcal{L}}{\partial(\partial A^0/\partial y^0)}\frac{\partial x^0}{\partial y^0}\,\frac{\partial y^0}{\partial x^0} = P_0\,, \tag{32}$$

$$P_g^0 = P_0/\ g_{tt}\ . \tag{33}$$

Taking into account that the 4-momentum of light in the Minkowski space is $P^\alpha = (E/c, \mathbf{P})$, the physical components $P_{g(\alpha)}$ of the 4-momentum in the Riemann space read

$$\begin{aligned} P_{g(\alpha)} &= \left(P_g^0\sqrt{g_{tt}}\,,\ P_g^i\sqrt{g_{ii}}\right) = \left(\frac{P_0}{\sqrt{g_{tt}}}\,,\ \frac{P_i}{\sqrt{g_{tt}}}\right) \\ &= \left(\frac{E}{c_g}\,, \mathbf{P}_g\right)\,, \end{aligned} \tag{34}$$

where

$$\mathbf{P}_g = \frac{c}{c_g}\mathbf{P}\ . \tag{35}$$

As expected, the magnitude of the 4-momentum of light in the Riemann space is zero similarly as in the Minkowski space

$$P_g^\alpha P_{g\alpha} = -\frac{E^2}{c_g^2} + P_g^2 = -\frac{E^2}{c^2} + P^2 = 0\ . \tag{36}$$

Finally, we arrive at the equation for the energy of light

$$E = Pc = P_g c_g\ , \tag{37}$$

implying that the energy is conserved in the Riemann space.

D. Planck relation for energy of photons

We proved in the above section that the energy of light is conserved in the Riemann space, even though the frequency and speed of light vary depending on the time-time term g_{tt} of the Riemann metric tensor $g_{\alpha\beta}$. This implies that the famous Planck relation for the photon energy in the Minkowski space should be modified to be valid in the Riemann space. First, we transform the photon frequency f and momentum $P = \frac{1}{c}hf$ in the Minkowski space into the photon frequency f_g and momentum P_g in the Riemann space

$$f_g = f\frac{c}{c_g}\ , \tag{38}$$

$$P_g = P\frac{c}{c_g} = \frac{1}{c}hf\frac{c}{c_g} = \kappa f_g\ , \tag{39}$$

where $\kappa = h/c$ is the Planck constant h normalized to the speed of light c in vacuum with no gravity. Second, we modify the standard Planck formula for the photon energy $E = Pc$ as follows

$$E = P_g c_g = \kappa f_g c_g\ , \tag{40}$$

where f_g and c_g are the frequency and the speed of light measured in the Riemann space. Obviously, the photon energy is invariant, $E_g = E$, because if the photon frequency is increased, the speed of photons is decreased, and vice versa.

IV. LIGHT SPEED, GRAVITATIONAL REDSHIFT AND PHOTON ENERGY IN THE SCHWARZSCHILD METRIC

The Schwarzschild metric describing the gravitational field of a body with mass M situated at the origin of coordinates is defined as follows

$$ds^2 = -\left(1-\frac{r_s}{r}\right)c^2dt^2 + \left(1-\frac{r_s}{r}\right)^{-1}dr^2 + r^2d\Omega^2\ , \tag{41}$$

$$d\Omega^2 = d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\ , \tag{42}$$

where $r_s = 2GM/c^2$ is the Schwarzschild radius, G is the gravitational constant, r and t are the contravariant coordinate distance and time, ϑ and φ are the spherical angles, and velocity c is the speed of light far from the source of gravity.

Using Eq. (41), the gravitational redshift at distance r expressed as the relative change of the photon frequency f_g with respect to the photon frequency f at $r \to \infty$ reads

$$\frac{f_g}{f} = \sqrt{\frac{g_{tt}(\infty)}{g_{tt}(r)}} = \left(1-\frac{r_s}{r}\right)^{-\frac{1}{2}}\ . \tag{43}$$

Obviously, not only the frequency of light but also the light speed in the gravitational field is distance dependent. Using Eq. (23), we get for the physical speed of light c_g

$$c_g = \sqrt{g_{tt}}\, c = \sqrt{1 - \frac{r_s}{r}}\, c \ . \quad (44)$$

It follows from Eq. (44) that the proper speed of light c_g becomes c for $r \to \infty$ and goes to zero for $r \to r_s$. Inserting Eq. (43) and Eq. (44) into Eq. (40), we get for the photon energy E_g

$$E_g = \kappa f_g c_g = \kappa f c = E\left(r \to \infty\right) \ . \quad (45)$$

In the weak gravity approximation

$$\frac{r_s}{r} = \frac{GM}{rc^2} \ll 1 \ , \quad (46)$$

the formulas read for the gravitational redshift

$$g_{tt} = 1 + \frac{2\phi}{c^2} \ , \ f_g = \left(1 - \frac{\phi}{c^2}\right) f \ , \ \frac{\Delta f}{f} = -\frac{\phi}{c^2} \ , \quad (47)$$

and for the light speed and the photon energy

$$c_g = \left(1 + \frac{\phi}{c^2}\right) c \, , \quad (48)$$

$$E_g = \kappa f_g c_g = \kappa f c = E \, , \quad (49)$$

where $\phi = -GM/r$ is the Newtonian gravitational potential, and $\Delta f = f_g - f$ is the frequency change between the frequency f_g observed at finite r (affected by gravity) and the frequency f observed at $r \to \infty$ (no gravity).

V. DISCUSSION

The standard intuitive interpretation of the gravitational redshift as an effect of transferring energy between photons and gravitational field is wrong. A rigorous application of the Maxwell's equations to curved spacetimes reveals that the energy of electromagnetic waves and photons propagating in vacuum distorted by a gravitational field is conserved. The energy of electromagnetic waves and photons changes only through their interaction with matter (massive particles) via their absorption, reflection or scattering. For example, the interaction of the electromagnetic waves with gravity is possible, when the waves are captured in some closed box. In this case, the waves interact with boundaries of the box and produce a non-zero radiation pressure, which can affect the spacetime geometry. Free photons in vacuum do not generate pressure and do not affect the spacetime geometry. Hence, the photon energy is invariant in vacuum with a gravitational field.

Consequently, the common idea that the energy of photons depends only on their frequency must be corrected, because the energy is conserved even for redshifted photons. The standard energy-frequency relation is valid only in SR, where the light speed is considered as a constant. In GR, the energy of photons depends not only on the frequency of photons but also on the speed of photons. The original Planck energy-frequency relation $E = hf$ must be modified to

$$E = \kappa f_g c_g \, , \quad (50)$$

where $\kappa = h/c$ is the Planck constant normalized to the speed of light in vacuum with no gravity, and f_g and c_g denote now the frequency and speed of photons measured in a non-inertial coordinate system experiencing gravity. If the frequency of photons is shifted due to the gravitational redshift, the speed of photons is also changed, and both the effects are compensated.

Nevertheless, the relation between the photon energy and the photon momentum $E = p_g c_g$ is still valid and the momentum of photons is expressed from Eq. (49) as $p_g = \kappa f_g$. So, the photon momentum is not invariant, but it is changing due to the presence of gravity. Considering the vacuum with a gravitational field as a kind of a dielectric medium, the photon momentum is transformed in the same way as in the Minkowski theory of dielectric media[1,2,17,18,21]

$$p_g = np \, , \ n = \frac{c}{c_g} = \frac{1}{\sqrt{g_{tt}}} \, , \quad (51)$$

where n is the refractive index of the vacuum with a gravitational field, and c_g and c are the speed of light in the vacuum with and without gravity, respectively. Note that simulating propagation of photons in curved spacetimes by considering the gravitational field as a kind of a dielectric medium was proposed by several authors[10–14,24,25,30]. This allows studying photon geodesics using the methods of geometrical optics in dielectric media.

VI. CONCLUSIONS

The intuitive idea that the gravitational redshift is an effect of the energy change of photons due to gravity is misleading. This idea is based on the Newton gravity developed for massive particles and it cannot be applied to photons for the following reasons: (1) No gravitational potential energy of photons is defined in GR, and (2) no energy transfer between free propagating photons and gravity is admissible in the Einstein-Maxwell's equations.

A rigorous application of the Maxwell's equations to curved spacetimes reveals that the energy of photons is invariant, when the photons propagate in vacuum distorted by gravity. Therefore, the original Planck energy-frequency relation $E = hf$ must be modified to $E = \kappa f c$, where κ is the Planck constant normalized to the speed of light in vacuum with no gravity, and f and c denote the frequency and speed of photons measured in vacuum distorted by gravity. Hence, the photon energy does not depend only on the frequency of photons but also on their speed. The frequency and speed of photons can be measured at any coordinate system. In a free-falling system, the photon energy is conserved, because no frequency shift and no change of the photon speed is detected. In non-inertial systems, the photon energy is also conserved, because the frequency shift due to gravity is compensated by the change of the photon speed.

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