Solving Triangular Fuzzy Linear Fractional Programming Problem

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ABSTRACT
In this paper we propose to solve linear fractional programming problem, where all the decision parameters are trapezoidal fuzzy numbers. Our approach and computational procedures may be efficient and simple to implement for calculation in a trapezoidal fuzzy environment for all fields of engineering and science where impreciseness occur. Here we are formulating the given objective function into three objective bounds with constraints which can be solved further by Simplex method. A numerical example is presented to illustrate the proposed approach.

Keywords: *Fuzzy numbers, fractional programming*

INTRODUCTION

To figure out a linear fractional programming model that represents the real-world choice circumstances, different variables in regards to the real framework should be reflected in the objective function and constraints. Specialists assign several parameters with values to the objective functions and constraints. In practice, it is usual to consider that the possible values of these parameters are fuzzy numerical data, which can be represented as fuzzy numbers. There are certain decision situations that necessitate consideration of uncertainties in working environment best captured by fuzzy set theory. In previous studies, several methods have been suggested for solving linear fractional programming problems. Zadeh [1] is most popular for proposing fuzzy Mathematics, comprising of fuzzy mathematical concepts.

Zimmermann [2] was the first to involve the fuzzy concepts in mathematical programming proposing a fuzzy way to deal with linear programming with multiple objective function. Chakraborty and Gupta[3] comprised of the modeling and optimization of a multi objective linear programming problem in fuzzy environment in which some goals are fractional and some are linear. Pramy and Islam [4] presented a modified method to find the efficient solutions of multi-objective linear fractional programming (MOLFP) problem. Loganathan and Ganesan [5] proposed a method of solving the fully fuzzy linear fractional programming problems Das et al. [6] developed numerical model for the solution of single-objective linear fractional programming problem.

Arya, Singh, Kumari and Obaidat [7] proposed an algorithm for solving fully fuzzy multi-objective linear fractional optimization problem with the help of the ranking function and the weighted approach. Stanojevic et al.[8] suggested a method for solving the fuzzy linear fractional programming problems with equality constraints occurring in real-life situations, using the concept of crisp linear programming and ranking function. Dasa, Edalatpanah [9] proposed a method to solve fractional linear programming problem under fuzzy environment based on ranking and decomposition methods. Stanojević et al. [10] provided empirical solutions to a special class of full fuzzy linear fractional programming problem.

PRELIMINARIES

Definition 2.1 A fuzzy set A is defined on universal set of real numbers R, is said to be a fuzzy number if its membership function satisfies the following conditions:

1. $\mu_A : R \to [0,1]$ is continuous
2. $\mu_A(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$
3. $\mu_A(x)$ is strictly increasing on [a, b] and strictly decreasing on [c, d]
4. $\mu_A(x)$ = 1 for all $x \in [b, c]$ where a < b < c < d

Definition 2.2 (Triangular Fuzzy Number) A number $\bar{X} = (l, m, n)$ is said to be a triangular fuzzy number if its membership function is defined as

$$\mu_A(x) = \frac{x-a}{b-a}, a \le x \le b$$
$$= 1, \quad , b \le x \le c$$
$$= \frac{x-d}{c-d}, c \le x \le d$$

Definition 2.3 (Generalized Trapezoidal Fuzzy Numbers) A fuzzy set A is defined on universal set of real numbers R, is said to be a generalized fuzzy number if its membership function satisfies the following conditions

$\mu_A : R \to [0,1]$ is continuous
$\mu_A(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$
$\mu_A(x)$ is strictly increasing on [a, b] and strictly decreasing on [c, d]
$\mu_A(x)$ = ω for all $x \in [b, c]$ where $0 < \omega \le 1$

Definition 2.4 A number $A = (a, b, c, d, \omega)$ is said to be a generalized trapezoidal fuzzy number if its membership function is defined as

$$\mu_A(x) = \frac{\omega(x-a)}{b-a}, a \le x \le b$$
$$= \omega, \quad b \le x \le c$$
$$= \frac{\omega(x-d)}{c-d}, c \le x \le d$$
$$= 0, \quad otherwise$$

Definition 2.5 Let the two triangular fuzzy numbers are $\bar{X} = (l, m, n), \bar{Y} = (p, q, r)$ said to be equal if and only if $l = p$, $m = q$ and $n = r$

Definition 2.6 Let the two triangular fuzzy numbers are $\bar{X} = (l, m, n)$, $\bar{Y} = (p, q, r)$ then the arithmetic operations will be

$$\bar{X} + \bar{Y} = (l,m,n) + (p,q,r) = (l+p, m+q, n+r)$$

$$\bar{X} - \bar{Y} = (l,m,n) - (p,q,r) = (l-p, m-q, n-r)$$

$$\bar{X} \otimes \bar{Y} = (lp, mq, nr) \qquad for \qquad l > 0$$

$$\frac{\bar{X}}{\bar{Y}} = \left(\frac{l}{r}, \frac{m}{q}, \frac{n}{p}\right)$$

FUZZY LINEAR FRACTIONAL PROGRAMMING PROBLEM

The fuzzy linear fractional programming problem can be written as

$$Max \frac{\sum \bar{X}_j x_j + \beta_j}{\sum \bar{Y}_j x_j + \gamma_j} = \frac{N(x)}{D(x)}$$

s.t. $\sum a_{ij} x_j \le b_i \quad i = 1,2,...m, \qquad j = 1,2,...,n$

$x_j \ge 0$ $\qquad \ldots(1)$

For some values of x_j, $\sum \bar{Y}_j x_j + \gamma_j = 0$ Now, to avoid this condition, we assume that either $\sum a_{ij} x_j \le b_i, x_j \ge 0 \Rightarrow \sum \bar{Y}_j x_j + \gamma_j > 0$ or $\sum a_{ij} x_j \le b_i, x_j \ge 0 \Rightarrow \sum \bar{Y}_j x_j + \gamma_j < 0$

Assume that it satisfies the condition $\sum a_{ij} x_j \le b_i, x_j \ge 0 \Rightarrow \sum \bar{Y}_j x_j + \gamma_j > 0$

Let us assume that $X_j = (l_j, m_j, n_j), Y_j = (p_j, q_j, r_j), \beta_j = (\beta_1, \beta_2, \beta_3), \gamma_j = (\gamma_1, \gamma_2, \gamma_3)$ are triangular fuzzy numbers, therefore problem (1) can be rewritten as

$$Max \frac{\sum (l_j, m_j, n_j) x_j + (\beta_{j1}, \beta_{j2}, \beta_{j3})}{\sum (p_j, q_j, r_j) x_j + (\gamma_{j1}, \gamma_{j2}, \gamma_{j3})}$$

s.t. $\sum (s_{ij}, t_{ij}, v_{ij}) x_j \le (\sigma_i, \delta_i, \eta_i) \qquad i = 1,2,...m, \qquad j = 1,2,...,n$

$x_j \ge 0$ $\qquad \ldots(2)$

To find the optimized solution of the given problem, we transform the objective function and constraints as follows

$$Max \frac{\sum l_j x_j + \beta_1, \sum (m_j - l_j) x_j + (\beta_2 - \beta_1), \sum (n_j + l_j) x_j + (\beta_3 + \beta_1)}{\sum p_j x_j + \gamma_1, \sum (q_j - p_j) x_j + (\gamma_2 - \gamma_1), \sum (r_j + p_j) x_j + (\gamma_3 + \gamma_1)}$$

s.t. $\sum s_{ij} x_j \le \sigma_i$

$\sum (t_{ij} - s_{ij}) x_j \le \delta_i - \sigma_i$

$\sum (v_{ij} + s_{ij}) x_j \le \eta_i + \sigma_i$

$x_j \ge 0 \quad i = 1,2,...m, \qquad j = 1,2,...,n$

Assume that no point *(z, 0)* with $z \geq 0$ is feasible for the given problem

$$Max\sum l_j z_j + \beta_1 t, \sum (m_j - l_j) z_j + (\beta_2 - \beta_1) t, \sum (n_j + l_j) z_j + (\beta_3 + \beta_1) t$$
$$\text{s.t. } \sum s_{ij} z_j - \sigma_i t = 0$$
$$\sum (t_{ij} - s_{ij}) z_j - (\delta_i - \sigma_i) t = 0$$
$$\sum (v_{ij} + s_{ij}) z_j - (\eta_i + \sigma_i) = 0$$
$$\sum p_j z_j + \gamma_1 t, \sum (q_j - p_j) z_j + (\gamma_2 - \gamma_1) t, \sum (r_j + p_j) z_j + (\gamma_3 + \gamma_1) t = 1$$
$$z_j \geq 0, t \geq 0 \quad i = 1,2,...m, \quad j = 1,2,...,n \qquad \ldots(3)$$

Above problem can be converted into three crisp bounds
Lower Bound

$$Max\sum l_j z_j + \beta_1 t$$
$$\text{s.t. } \sum s_{ij} z_j - \sigma_i t = 0$$
$$\sum (t_{ij} - s_{ij}) z_j - (\delta_i - \sigma_i) t = 0$$
$$\sum (v_{ij} + s_{ij}) z_j - (\eta_i + \sigma_i) = 0$$
$$\sum p_j z_j + \gamma_1 t, \sum (q_j - p_j) z_j + (\gamma_2 - \gamma_1) t, \sum (r_j + p_j) z_j + (\gamma_3 + \gamma_1) t = 1$$
$$z_j \geq 0, t \geq 0 \quad i = 1,2,...m, \quad j = 1,2,...,n \qquad \ldots(4)$$

Middle Bound

$$Max\sum (m_j - l_j) z_j + (\beta_2 - \beta_1) t$$
$$\text{s.t. } \sum s_{ij} z_j - \sigma_i t = 0$$
$$\sum (t_{ij} - s_{ij}) z_j - (\delta_i - \sigma_i) t = 0$$
$$\sum (v_{ij} + s_{ij}) z_j - (\eta_i + \sigma_i) = 0$$
$$\sum p_j z_j + \gamma_1 t, \sum (q_j - p_j) z_j + (\gamma_2 - \gamma_1) t, \sum (r_j + p_j) z_j + (\gamma_3 + \gamma_1) t = 1$$
$$z_j \geq 0, t \geq 0 \quad i = 1,2,...m, \quad j = 1,2,...,n \qquad \ldots(5)$$

Upper Bound

$$Max\sum (n_j + l_j) z_j + (\beta_3 + \beta_1) t$$
$$\text{s.t. } \sum s_{ij} z_j - \sigma_i t = 0$$
$$\sum (t_{ij} - s_{ij}) z_j - (\delta_i - \sigma_i) t = 0$$
$$\sum (v_{ij} + s_{ij}) z_j - (\eta_i + \sigma_i) = 0$$
$$\sum p_j z_j + \gamma_1 t, \sum (q_j - p_j) z_j + (\gamma_2 - \gamma_1) t, \sum (r_j + p_j) z_j + (\gamma_3 + \gamma_1) t = 1$$
$$z_j \geq 0, t \geq 0 \quad i = 1,2,...m, \quad j = 1,2,...,n \qquad \ldots(6)$$

The solution values of *z* and *t* provide the optimal value of the objective function.

NUMERICAL EXAMPLE

$$Max\frac{(1,4,6)\otimes x_1+(1,2,3)\otimes x_2+(3,4,5)}{(4,5,6)\otimes x_1+(2,3,4)\otimes x_2+(1,2,5)}$$
$$s.t.(1,2,3)\otimes x_1+(2,4,6)\otimes x_2\leq(11,12,13)$$
$$(2,4,6)\otimes x_1+(5,6,7)\otimes x_2\leq(9,11,15)$$
$$x_1,x_2\geq 0$$

$$Max(1,4,6)\otimes z_1+(1,2,3)\otimes z_2+(3,4,5)t$$
$$s.t.(1,2,3)\otimes z_1+(2,4,6)\otimes z_2-(11,12,13)t=0$$
$$(2,4,6)\otimes z_1+(5,6,7)\otimes z_2-(9,11,15)t=0$$
$$(4,5,6)\otimes z_1+(2,3,4)\otimes z_2+(1,2,5)t=1$$
$$z_1,z_2,t\geq 0$$

Transform this into a system of fuzzy linear inequalities in parametric form
$$Max\quad z_1+z_2+3t, 3z_1+z_2+t, 7z_1+4z_2+8t$$
$s.t.$
$$z_1+2z_2-11t=0$$
$$z_1+2z_2-t=0$$
$$4z_1+8z_2-24t=0$$
$$2z_1+5z_2-9t=0$$
$$2z_1+z_2-2t=0$$
$$8z_1+12z_2-24t=0$$
$$4z_1+2z_2+t=1$$
$$z_1+z_2+t=1$$
$$10z_1+6z_2+6t=1$$
$$z_1,z_2,t\geq 0$$

To find the optimal solution of the above problem, it can be converted into the following crisp lower bound, middle bound and upper bound problem.

Lower Bound
$$Max\quad z_1+z_2+3t$$
$s.t.$
$$z_1+2z_2-11t=0$$
$$z_1+2z_2-t=0$$
$$4z_1+8z_2-24t=0$$
$$2z_1+5z_2-9t=0$$
$$2z_1+z_2-2t=0$$
$$8z_1+12z_2-24t=0$$
$$4z_1+2z_2+t=1$$

$$z_1 + z_2 + t = 1$$
$$10z_1 + 6z_2 + 6t = 1$$
$$z_1, z_2, t \geq 0$$

Middle Bound

$$Max \quad 3z_1 + z_2 + t$$
$$s.t.$$
$$z_1 + 2z_2 - 11t = 0$$
$$z_1 + 2z_2 - t = 0$$
$$4z_1 + 8z_2 - 24t = 0$$
$$2z_1 + 5z_2 - 9t = 0$$
$$2z_1 + z_2 - 2t = 0$$
$$8z_1 + 12z_2 - 24t = 0$$
$$4z_1 + 2z_2 + t = 1$$
$$z_1 + z_2 + t = 1$$
$$10z_1 + 6z_2 + 6t = 1$$
$$z_1, z_2, t \geq 0$$

Upper Bound

$$Max \quad 7z_1 + 4z_2 + 8t$$
$$s.t.$$
$$z_1 + 2z_2 - 11t = 0$$
$$z_1 + 2z_2 - t = 0$$
$$4z_1 + 8z_2 - 24t = 0$$
$$2z_1 + 5z_2 - 9t = 0$$
$$2z_1 + z_2 - 2t = 0$$
$$8z_1 + 12z_2 - 24t = 0$$
$$4z_1 + 2z_2 + t = 1$$
$$z_1 + z_2 + t = 1$$
$$10z_1 + 6z_2 + 6t = 1$$
$$z_1, z_2, t \geq 0$$

The above transformed LPP can be solved by classical methods.

CONCLUSION

In this paper we propose to solve fractional programming problem, where all the decison variables are trapezoidal fuzzy numbers. Our methodology and computational strategies might be proficient and easy to carry out for estimation in a trapezoidal fuzzy variable for all fields of designing and science where uncertainty happen.

REFERENCES

1. Zadeh, L. A. (1965). Zadeh, fuzzy sets. *Inform Control*, *8*, 338-353.
2. Zimmermann, H. J. (1978). Fuzzy programming and linear programming with several objective functions. *Fuzzy sets and systems*, *1*(1), 45-55.
3. Chakraborty, M., & Gupta, S. (2002). Fuzzy mathematical programming for multi objective linear fractional programming problem. *Fuzzy sets and systems*, *125*(3), 335-342.
4. Pramy, F. A., & Islam, M. A. (2017). Determining efficient solutions of multi-objective linear fractional programming problems and application. *Open Journal of Optimization*, *6*(04), 164.
5. Loganathan, T., & Ganesan, K. (2019, November). A solution approach to fully fuzzy linear fractional programming problems. In *Journal of Physics: Conference Series* (Vol. 1377, No. 1, p. 012040). IOP Publishing.
6. Das, S. K., Edalatpanah, S. A., & Mandal, T. (2018). A proposed model for solving fuzzy linear fractional programming problem: Numerical Point of View. *Journal of computational science*, *25*, 367-375.
7. Arya, R., Singh, P., Kumari, S., & Obaidat, M. S. (2020). An approach for solving fully fuzzy multi-objective linear fractional optimization problems. *Soft Computing*, *24*, 9105-9119.
8. Stanojević, B., Dzitac, S., & Dzitac, I. (2020). Fuzzy numbers and fractional programming in making decisions. *International Journal of Information Technology & Decision Making*, *19*(04), 1123-1147.
9. Das, S. K., & Edalatpanah, S. A. (2020). New insight on solving fuzzy linear fractional programming in material aspects. *Fuzzy optimization and Modelling*, *1*, 1-7.
10. Stanojević, B., & Stanojević, M. (2022). Empirical (α, β)-acceptable optimal values to full fuzzy linear fractional programming problems. *Procedia Computer Science*, *199*, 34-39.