Barrier Option Valuation

Barrier options are set up conditionally, so that within the life of the option, a barrier may or may not be reached. The barrier option is dependent on an "extreme" price, either high or low, attained by the underlying. If a barrier is reached, or touched, the constraints are triggered.

The two types of contracts, knock-in barrier options and knock-out barrier options, each respond differently when the barrier is "triggered". If a *knock-out* option's underlying touches the barrier, the option is eliminated and the holder receives a rebate. Conversely, a *knock-in* option touches the barrier to activate the option. If the *knock-in* option never reaches the barrier, the holder will receive a rebate.

For single barrier option, there are four possible variations to the barrier:

- Up and In
- Up and Out
- Down and In
- Down and Out

The option is considered knocked-in (knocked-out) if, on a barrier observation date, the underlying asset's closing price exceeds (for up-barriers) or falls below (for down-barriers) the barrier level corresponding to that observation date.

Once knocked-in (knocked-out), the option's status is no longer subject to further modification by subsequent barrier observation events. Payoff at maturity may take the form of a Call, Put or Note. The payoff is contingent on the option having been knocked in (in the case of knock-in option), or having *not* been knocked out (in the case of a knock-out option), on some barrier observation date prior to maturity.

For European barrier options, a vanilla option is equal to the sum of the premiums for a knock-in and its corresponding knockout (assuming no rebates and equivalent input parameters). Since at expiration, only one option can have a value.

The Double Barrier is a discretely-observed barrier option on a single asset. An initial stock price can be fixed at some future date. The option is said to be *triggered* when the stock price, relative to its initial price, exceeds the high barrier or falls below the low barrier on any barrier observation date.

The option may be Knock-In (where triggering enables the payoff) or Knock-Out (where triggering disables the payoff). For Knock-Out options, a rebate amount may be specified to be paid if the option becomes triggered. The payoff at maturity, if active, may be a relative-return call or put payoff on the deal Notional.

An average Barrier is a call or put option on the average of the asset level over a discrete set of observation dates, with a knock-in or knock-out barrier applied to the running average on a subset of those dates.

More precisely, the option is said to be *triggered* when the running average of the asset price exceeds the high barrier or falls below the low barrier on any barrier observation date. In the case of a *knock-in* barrier, the buyer of the option receives the call/put payoff at maturity if the option has been triggered, and receives zero otherwise. In the case of *knock-out* barrier, the buyer receives the call/put payoff at maturity if the option was not triggered during the lifetime of the deal, while the buyer receives a knock-out rebate (at the barrier date or at maturity) if the option is triggered.

The option is said to be *triggered* when the asset level exceeds the high barrier or falls below the low barrier at any time within a barrier observation interval (or on a barrier observation date).

The option may be Knock-In (where triggering enables the payoff) or Knock-Out (where triggering disables the payoff). For Knock-Out options, a rebate amount may be specified to be paid if the option

becomes triggered. The payoff at maturity, if active, may be a call, a put, a digital call, or a digital put payoff on the deal Notional.

There are some relationships among knock-in, knock-out, and vanilla options.

$$C_{UpIn} + C_{UpOut} = C_{Vanilla}$$

$$C_{DownIn} + C_{DownOut} = C_{Vanilla}$$

$$P_{DownIn} + P_{DownOut} = P_{Vanilla}$$

$$P_{UpIn} + P_{UpOut} = P_{Vanilla}$$

Reference:

https://finpricing.com/lib/IrCurveIntroduction.html