Analysis of Factorial Function for Non-Negative Real Numbers

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Abstract: The real numbers between any two non-negative integers are positive and infinite. This paper analyzes the numerical values of factorial function between any two non-negative integers. This analysis can help to find and correct the numerical error computed by the gamma function.

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1. Introduction

The factorial function [1-3] to positive integer n, denoted by n!, is the product of all positive integers less than or equal to n. The gamma function [4-6] is a generalization of the factorial function to positive real values, introduced by the Swiss mathematician Leonhard Euler in the 18th century.

$$\Gamma(n+1) = n(n-1)! = n\Gamma(n).$$

$$\Gamma(1) = 0! = 1;\ \Gamma(2) = 1! = 1;\ \Gamma(3) = 2! = 2.$$

$$\text{Also,} \qquad \Gamma(r) = (r-1)! = \int_0^\infty t^{r-1} e^{-t} dt.$$

$$\Gamma\left(\frac{1}{2}\right) = \left(\frac{1}{2} - 1\right)! = \left(-\frac{1}{2}\right)! = \sqrt{\pi}.$$

2. Error in Calculation using Gamma Function

$$\text{It is understood that } \Gamma\left(\frac{1}{2}\right) = \left(-\frac{1}{2}\right)! = \sqrt{\pi} \approx 1.77245385091,$$

$$\text{but } \Gamma(3) > \Gamma\left(\tfrac{1}{2}\right) > \Gamma(2), \text{ which is a contradiction.}$$

$$\Gamma\left(\frac{3}{2}\right) = \left(\frac{3}{2} - 1\right)\left(\frac{3}{2} - 1 - 1\right)! = \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)! = \frac{\sqrt{\pi}}{2}.$$

So, $\Gamma\left(\frac{1}{2}\right)$ is not true and also *c*alculating the value of $\Gamma\left(\frac{3}{2}\right)$ using $\Gamma\left(\frac{1}{2}\right)$ can not be true. From these results, it is concluded that the calculated values using $\Gamma\left(\frac{1}{2}\right)$ are not true.

3. Analysis of Factorials for Positive Real Values

The value of factorial for any positive real number between $0! = 1$ and $1! = 1$ must be 1,

$$i.e. \frac{(0+1)}{2} = 0.5 \Longrightarrow \frac{(0! + 1!)}{2} = 1. \text{ Thus, } (0.5)! = 1.$$

$$\text{Also,} \qquad \frac{(0+0.5)}{2} = 0.25 \Longrightarrow \frac{(0! + (0.5)!)}{2} = 1. \text{ Thus, } (0.25)! = 1;$$

$$\frac{(0.5+1)}{2}=0.75 \Longrightarrow \frac{\left((0.5)!+1!\right)}{2}=1. \text{Thus, } (0.75)!=1; \text{and so on.}$$

Therefore, the numerical values of factorials for positive real numbers between the consecutive integers 0 and 1 must be 1.

Similarly, the numerical values of factorials for positive real numbers between the consecutive integers 4 and 5 must be the values between $4!$ and 51, i.e. the values of factorials for positive real numbers between 4 and 5 must bet 24 and 120 because of $4!=24$ and $5!=120$.

Let n be a positive integers, $n!=r$, and $(n+1)!=s$. Then, the numerical values of factorials for positive real numbers between the consecutive integers n and $(n+1)$ must be the values between r and s.

4. Conclusion

This article analyzes the numerical values of factorial function between any two non-negative integers.

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