TO‘RINCHI TARTIBLI INTEGRO-DIFFERENSIAL TENGLAMA UCHUN TO‘G‘RI VA TESKARI MASALA

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Annotatsiya: *Ushbu ishda to‘rtinchi tartibli integro-differensial tenglama uchun bir teskari masala bayon qilingan va tadqiq etilgan.*

Kalit so‘zlar: *to‘rinchi tartibli integro-differensial tenglama, Riman-Liuvill ma'nosida γ (kasr) tartibli integral, chegaraviy shartli, teskari masala.*

ПРЯМАЯ И ОБРАТНАЯ ЗАДАЧА ДЛЯ ИНТЕГРО-ДИФФЕРЕНЦИАЛЬНОГО УРАВНЕНИЯ ЧЕТВЕРТОГО ПОРЯДКА

Аннотация: *В этой работе была сформулирована и исследована обратная задача для Интегро-дифференциального уравнения четвертого порядка.*

Ключевые слова: *Интегро-дифференциальное уравнение четвертого порядка, Интеграл Римана-Лювилля в смысле (дроби), граничное условие, обратная задача.*

A DIRECT AND INVERSE PROBLEM FOR AN INTEGRO-DIFFERENTIAL EQUATION OF THE FOURTH ORDER

Annotation: *In this paper, the inverse problem for a fourth-order Integro-differential equation was formulated and investigated.*

Keywords: *Integro-differential equation of the fourth order, Riemann-Liouville integral in the sense of (fractions), boundary condition, inverse problem.*

So`ngi vaqtlarda noma`lum manbali differensial tengalamalar bilan shug`illanishga bo`lgan qiziqish ortib bormoqda. Bunga sabab ko`plab issiqlik taqalish va diffuziya jarayonlarini matematik modelini tuzish noma'lum manbali differensial tenglama uchun qo`yiladigan masalalarga keltiriladi. Bunday differensial tenglamalar uchun teskari masalalar ko`plab tadqiqotchilar tomonidan o`rganilgan (masalan, ushbu $[1]-[7]$ ishlarga qaralsin). Ammo yuqori tartibli tenglamalar uchun teskari masalalar kam o`rganilgan. Shu sababdan biz ushbu ishda to`rtinchi tartibli integro-differensial tenglama uchun bir teskari masalani bir qiymatli yechilishini ko`rsatamiz.

(0, 1) oraliqda ushbu

$$y^{(4)}(x)-\lambda I_{0x}^{\gamma}y(x)=f(x) \qquad (1)$$

to`rtinchi tartibli integro-differensial tenglamani qaraylik, bu yerda $y(x)-$ noma'lum funksiya; $f(x)-$ berilgan uzluksiz funksiya; λ,γ - o‘zgarmas haqiqiy sonlar bo'lib; $I_{0x}^{\gamma}y(x)$ - Riman-Liuvill ma'nosida γ (kasr) tartibli integral,

$$I_{0x}^{\gamma}y(x)=\frac{1}{\Gamma(\gamma)}\int_0^x (x-t)^{\gamma-1}y(t)dt,$$

A masala. Shunday $y(x)$ funksiya topilsinki, u quyidagi xossalarga ega bo'lsin:

1) (0, 1) oraliqda (1) tenglamani qanoatlantirsin;

2) $C^3[0,1]\cap C^3(0,4)$ sinfga tegishli bo‘lsin;

3) $x=0$, $x=1$ nuqtalarda esa

$$y(0)=A_1,\ y'(0)=A_2,\ y(1)=B_1,\ y'(1)=B_2 \qquad (2)$$

chegaraviy shartlarni qanoatlantisin, bu yerda A_1,A_2,B_1,B_2- berilgan o‘zgarmas haqiqiy sonlar.

(1)tenglamani

$$y(0)=A_1,\ y'(0)=A_2,\ y''(0)=A_3,\ y'''(0)=A_4 \qquad (3)$$

boshlang‘ich shartlarni qanoatlantiruvchi yechimini

$$y(x)=A_1E_{\beta,1}(\lambda x^{\beta})+A_2xE_{\beta,2}(\lambda x^{\beta})+A_3x^2E_{\beta,3}(\lambda x^{\beta})+$$

$$+A_4x^3E_{\beta,4}(\lambda x^{\beta})+\int_0^x (x-z)^3E_{\beta,4}\left[\lambda(x-z)^{\beta}\right]f(z)dz$$

ko‘rinishda yozib olishimiz mumkin $[7]$, bu yerda A_3va A_4 no’malum sonlar, B_1 va B_2 berilgan o‘zgarmas haqiqiy sonlar.

A_3,A_4 ni $y(1)=B_1$, $y'(1)=B_2$ chegaraviy shartdan foydalanib,

$$A_3=-\frac{A_1\left[E_{\beta,1}(\lambda)E_{\beta,3}(\lambda)-E_{\beta,\beta}(\lambda)E_{\beta,4}(\lambda)\right]+A_2\left[E_{\beta,2}(\lambda)E_{\beta,3}(\lambda)-E_{\beta,1}(\lambda)E_{\beta,4}(\lambda)\right]}{E_{\beta,3}(\lambda)E_{\beta,3}(\lambda)-E_{\beta,2}(\lambda)E_{\beta,4}(\lambda)}-$$

$$-\frac{E_{\beta,3}(\lambda)\int_0^1(1-z)^3E_{\beta,4}\left[\lambda(1-z)^{\beta}\right]f(z)dz-E_{\beta,4}(\lambda)\int_0^1(1-z)^2E_{\beta,3}\left[\lambda(1-z)^{\beta}\right]f(z)dz}{E_{\beta,3}(\lambda)E_{\beta,3}(\lambda)-E_{\beta,2}(\lambda)E_{\beta,4}(\lambda)}+$$

$$+\frac{B_1E_{\beta,3}(\lambda)-B_2E_{\beta,4}(\lambda)}{E_{\beta,3}(\lambda)E_{\beta,3}(\lambda)-E_{\beta,2}(\lambda)E_{\beta,4}(\lambda)}$$

$$A_4=-\frac{A_1\left[E_{\beta,1}(\lambda)E_{\beta,2}(\lambda)-E_{\beta,\beta}(\lambda)E_{\beta,3}(\lambda)\right]+A_2\left[E_{\beta,2}(\lambda)E_{\beta,2}(\lambda)-E_{\beta,1}(\lambda)E_{\beta,3}(\lambda)\right]}{E_{\beta,4}(\lambda)E_{\beta,2}(\lambda)-E_{\beta,3}(\lambda)E_{\beta,3}(\lambda)}-$$

$$-\frac{E_{\beta,2}(\lambda)\int_0^1(1-z)^3E_{\beta,4}\left[\lambda(1-z)^{\beta}\right]f(z)dz-E_{\beta,3}(\lambda)\int_0^1(1-z)^2E_{\beta,3}\left[\lambda(1-z)^{\beta}\right]f(z)dz}{E_{\beta,4}(\lambda)E_{\beta,2}(\lambda)-E_{\beta,3}(\lambda)E_{\beta,3}(\lambda)}+$$

$$+\frac{B_1E_{\beta,2}(\lambda)-B_2E_{\beta,3}(\lambda)}{E_{\beta,3}(\lambda)E_{\beta,3}(\lambda)-E_{\beta,2}(\lambda)E_{\beta,4}(\lambda)}$$

ko‘rinishda topamiz.

Topilgan A_3,A_4 ni (3) ga qo‘yib, (1) masala yechimini

$$y(x)=x^2E_{\beta,3}\left(\lambda x^\beta\right)\left[\frac{A_1\left[E_{\beta,1}(\lambda)E_{\beta,3}(\lambda)-E_{\beta,\beta}(\lambda)E_{\beta,4}(\lambda)\right]+A_2\left[E_{\beta,2}(\lambda)E_{\beta,3}(\lambda)-E_{\beta,1}(\lambda)E_{\beta,4}(\lambda)\right]}{E_{\beta,4}(\lambda)E_{\beta,2}(\lambda)-E_{\beta,3}(\lambda)E_{\beta,3}(\lambda)}\right]-$$

$$-x^3E_{\beta,4}\left(\lambda x^\beta\right)\left[\frac{A_1\left[E_{\beta,1}(\lambda)E_{\beta,2}(\lambda)-E_{\beta,\beta}(\lambda)E_{\beta,3}(\lambda)\right]+A_2\left[E_{\beta,2}(\lambda)E_{\beta,2}(\lambda)-E_{\beta,1}(\lambda)E_{\beta,3}(\lambda)\right]}{E_{\beta,4}(\lambda)E_{\beta,2}(\lambda)-E_{\beta,3}(\lambda)E_{\beta,3}(\lambda)}\right]+$$

$$+B_1\left[\frac{-x^2E_{\beta,3}\left(\lambda x^\beta\right)E_{\beta,3}(\lambda)+x^3E_{\beta,4}\left(\lambda x^\beta\right)E_{\beta,1}(\lambda)}{E_{\beta,4}(\lambda)E_{\beta,2}(\lambda)-E_{\beta,3}(\lambda)E_{\beta,3}(\lambda)}\right]+A_1E_{\beta,1}\left(\lambda x^\beta\right)+A_2xE_{\beta,2}\left(\lambda x^\beta\right)+$$

$$+B_2\left[\frac{x^2E_{\beta,3}\left(\lambda x^\beta\right)E_{\beta,4}(\lambda)-x^3E_{\beta,4}\left(\lambda x^\beta\right)E_{\beta,3}(\lambda)}{E_{\beta,4}(\lambda)E_{\beta,2}(\lambda)-E_{\beta,3}(\lambda)E_{\beta,3}(\lambda)}\right]+\int_0^x(x-z)^3E_{\beta,4}\left[\lambda(x-z)^\beta\right]f(z)dz+$$

$$+\int_0^1(1-z)^3E_{\beta,4}\left[\lambda(1-z)^\beta\right]f(z)dz\left[\frac{x^2E_{\beta,3}\left(\lambda x^\beta\right)E_{\beta,3}(\lambda)-x^3E_{\beta,4}\left(\lambda x^\beta\right)E_{\beta,2}(\lambda)}{E_{\beta,4}(\lambda)E_{\beta,2}(\lambda)-E_{\beta,3}(\lambda)E_{\beta,3}(\lambda)}\right]+$$

$$+\int_0^1(1-z)^2E_{\beta,3}\left[\lambda(1-z)^\beta\right]f(z)dz\left[\frac{-x^2E_{\beta,3}\left(\lambda x^\beta\right)E_{\beta,4}(\lambda)+x^3E_{\beta,4}\left(\lambda x^\beta\right)E_{\beta,3}(\lambda)}{E_{\beta,4}(\lambda)E_{\beta,2}(\lambda)-E_{\beta,3}(\lambda)E_{\beta,3}(\lambda)}\right] \quad (4)$$

ko‘rinishda topamiz.

1-teorema. Agar $E_{\beta,4}(\lambda)E_{\beta,2}(\lambda)-E_{\beta,3}(\lambda)E_{\beta,3}(\lambda)\neq 0,\ \ f(x)\in C[0,1]$ bo‘lsa u holda A masala yagona yechimga ega bo‘ladi va u (4) formula bilan aniqlanadi.

1-izoh. Agar $E_{\beta,4}(\lambda)E_{\beta,2}(\lambda)=E_{\beta,3}(\lambda)E_{\beta,3}(\lambda)$ bo`lsa, u holda A masala yechimga ega bo`lmaydi.

$$y^{(4)}(x)-\lambda I_{0x}^{\gamma}y(x)=kf(x) \quad (5)$$

T masala. Shunday $y(x)$ funksiya va k son topilsinki, u quyidagi xossalarga ega bo‘lsin:

1) (0, 1) oraliqda (5) tenglamani qanoatlantirsin;

2) $C^3[0,1]\cap C^3(0,4)$ sinfga tegishli bo‘lsin;

3) $x=0$, $x=1$ nuqtalarda esa (2) chegraviy shartni va

$$y''(1)=b \quad (6)$$

nolokal shartni qanoatlantisin, bu yerda $A_1, A_2, B_1, B_2, b-$ o'zgarmas haqiqiy sonlar bo‘lib.

k sonni vaqtincha ma’lum deb, T masalaning yechimini (4) formuladan foydalanib,

$$y(x)=-kx^3E_{\beta,4}\left(\lambda x^\beta\right)\left[\frac{E_{\beta,2}(\lambda)\int_0^1(1-z)^3E_{\beta,4}\left[\lambda(1-z)^\beta\right]f(z)dz-E_{\beta,3}(\lambda)\int_0^1(1-z)^2E_{\beta,3}\left[\lambda(1-z)^\beta\right]f(z)dz}{E_{\beta,4}(\lambda)E_{\beta,2}(\lambda)-E_{\beta,3}(\lambda)E_{\beta,3}(\lambda)}\right]-$$

$$-x^3E_{\beta,4}\left(\lambda x^\beta\right)\left[\frac{A_1\left\{E_{\beta,1}(\lambda)E_{\beta,2}(\lambda)-E_{\beta,\beta}(\lambda)E_{\beta,3}(\lambda)\right\}+A_2\left\{E_{\beta,2}(\lambda)E_{\beta,2}(\lambda)-E_{\beta,1}(\lambda)E_{\beta,3}(\lambda)\right\}}{E_{\beta,4}(\lambda)E_{\beta,2}(\lambda)-E_{\beta,3}(\lambda)E_{\beta,3}(\lambda)}\right]+$$

$$+x^3E_{\beta,4}\left(\lambda x^\beta\right)\left[\frac{B_1E_{\beta,1}(\lambda)-B_2E_{\beta,3}(\lambda)}{E_{\beta,4}(\lambda)E_{\beta,2}(\lambda)-E_{\beta,3}(\lambda)E_{\beta,3}(\lambda)}\right]-x^2E_{\beta,3}\left(\lambda x^\beta\right)\left[\frac{B_1E_{\beta,3}(\lambda)-B_2E_{\beta,4}(\lambda)}{E_{\beta,4}(\lambda)E_{\beta,2}(\lambda)-E_{\beta,3}(\lambda)E_{\beta,3}(\lambda)}\right]+$$

$$+kx^2E_{\beta,3}\left(\lambda x^\beta\right)\left[\frac{E_{\beta,3}(\lambda)\int_0^1(1-z)^3E_{\beta,4}\left[\lambda(1-z)^\beta\right]f(z)dz-E_{\beta,4}(\lambda)\int_0^1(1-z)^2E_{\beta,3}\left[\lambda(1-z)^\beta\right]f(z)dz}{E_{\beta,4}(\lambda)E_{\beta,2}(\lambda)-E_{\beta,3}(\lambda)E_{\beta,3}(\lambda)}\right]+$$

$$+x^2E_{\beta,3}\left(\lambda x^\beta\right)\left[\frac{A_1\left\{E_{\beta,1}(\lambda)E_{\beta,3}(\lambda)-E_{\beta,\beta}(\lambda)E_{\beta,4}(\lambda)\right\}+A_2\left\{E_{\beta,2}(\lambda)E_{\beta,3}(\lambda)-E_{\beta,1}(\lambda)E_{\beta,4}(\lambda)\right\}}{E_{\beta,4}(\lambda)E_{\beta,2}(\lambda)-E_{\beta,3}(\lambda)E_{\beta,3}(\lambda)}\right]+$$

$$+A_1E_{\beta,1}\left(\lambda x^\beta\right)+A_2xE_{\beta,2}\left(\lambda x^\beta\right)+k\int_0^x(x-z)^3E_{\beta,4}\left[\lambda(x-z)^\beta\right]f(z)dz \qquad (7)$$

ko‘rinishda yozib olamiz.

$$M=\frac{E_{\beta,2}(\lambda)\int_0^1(1-z)^3E_{\beta,4}\left[\lambda(1-z)^\beta\right]f(z)dz-E_{\beta,3}(\lambda)\int_0^1(1-z)^2E_{\beta,3}\left[\lambda(1-z)^\beta\right]f(z)dz}{E_{\beta,4}(\lambda)E_{\beta,2}(\lambda)-E_{\beta,3}(\lambda)E_{\beta,3}(\lambda)}$$

$$M_1=\frac{A_1\left\{E_{\beta,1}(\lambda)E_{\beta,2}(\lambda)-E_{\beta,\beta}(\lambda)E_{\beta,3}(\lambda)\right\}+A_2\left\{E_{\beta,2}(\lambda)E_{\beta,2}(\lambda)-E_{\beta,1}(\lambda)E_{\beta,3}(\lambda)\right\}}{E_{\beta,4}(\lambda)E_{\beta,2}(\lambda)-E_{\beta,3}(\lambda)E_{\beta,3}(\lambda)}$$

$$M_2=\frac{B_1E_{\beta,1}(\lambda)-B_2E_{\beta,3}(\lambda)}{E_{\beta,4}(\lambda)E_{\beta,2}(\lambda)-E_{\beta,3}(\lambda)E_{\beta,3}(\lambda)},\quad N_2=\frac{B_1E_{\beta,3}(\lambda)-B_2E_{\beta,4}(\lambda)}{E_{\beta,4}(\lambda)E_{\beta,2}(\lambda)-E_{\beta,3}(\lambda)E_{\beta,3}(\lambda)}$$

$$N_1=\frac{A_1\left\{E_{\beta,1}(\lambda)E_{\beta,3}(\lambda)-E_{\beta,\beta}(\lambda)E_{\beta,4}(\lambda)\right\}+A_2\left\{E_{\beta,2}(\lambda)E_{\beta,3}(\lambda)-E_{\beta,1}(\lambda)E_{\beta,4}(\lambda)\right\}}{E_{\beta,4}(\lambda)E_{\beta,2}(\lambda)-E_{\beta,3}(\lambda)E_{\beta,3}(\lambda)}$$

$$N=\frac{E_{\beta,3}(\lambda)\int_0^1(1-z)^3E_{\beta,4}\left[\lambda(1-z)^\beta\right]f(z)dz-E_{\beta,4}(\lambda)\int_0^1(1-z)^2E_{\beta,3}\left[\lambda(1-z)^\beta\right]f(z)dz}{E_{\beta,4}(\lambda)E_{\beta,2}(\lambda)-E_{\beta,3}(\lambda)E_{\beta,3}(\lambda)}$$

belgilashlarni kiritib, $y''(x)$ ni

$$y''(x)=-MkxE_{\beta,2}\left(\lambda x^\beta\right)-(M_1-M_2)xE_{\beta,2}\left(\lambda x^\beta\right)+kNE_{\beta,1}\left(\lambda x^\beta\right)+(N_1-N_2)E_{\beta,1}\left(\lambda x^\beta\right)+$$

$$+A_1\lambda x^{\beta-2}E_{\beta,\beta-1}\left(\lambda x^\beta\right)+A_2\lambda x^{\beta-1}E_{\beta,\beta}\left(\lambda x^\beta\right)+k\int_0^x(x-z)E_{\beta,2}\left[\lambda(x-z)^\beta\right]f(z)dz$$

ko‘rinishda aniqlaymiz. Topilgan $y''(x)$ ni (6) shartga bo‘ysuntirib, k ni

$$k=\frac{b+(M_1-M_2)E_{\beta,2}(\lambda)-(N_1-N_2)E_{\beta,1}(\lambda)-A_1\lambda E_{\beta,\beta-1}(\lambda)+A_2\lambda E_{\beta,\beta}(\lambda)}{\left(-M+N+\int_0^1(1-z)E_{\beta,2}\left[\lambda(1-z)^\beta\right]f(z)dz\right)} \qquad (8)$$

ko‘rinishda topamiz.

(8) formulani (7) formulaga qo‘yib, $y(x)$ funksiyani

$$y(x)=-Mx^3E_{\beta,4}\left(\lambda x^\beta\right)\frac{b+\left(M_1-M_2\right)E_{\beta,2}(\lambda)-\left(N_1-N_2\right)E_{\beta,1}(\lambda)-A_1\lambda E_{\beta,\beta-1}(\lambda)+A_2\lambda E_{\beta,\beta}(\lambda)}{\left(-M+N+\int\limits_0^1(1-z)E_{\beta,2}\left[\lambda(1-z)^\beta\right]f(z)dz\right)}+$$

$$+Nx^2E_{\beta,3}\left(\lambda x^\beta\right)\frac{b+\left(M_1-M_2\right)E_{\beta,2}(\lambda)-\left(N_1-N_2\right)E_{\beta,1}(\lambda)-A_1\lambda E_{\beta,\beta-1}(\lambda)+A_2\lambda E_{\beta,\beta}(\lambda)}{\left(-M+N+\int\limits_0^1(1-z)E_{\beta,2}\left[\lambda(1-z)^\beta\right]f(z)dz\right)}+$$

$$+\int\limits_0^x(x-z)^3E_{\beta,4}\left[\lambda(x-z)^\beta\right]f(z)dz\frac{b+\left(M_1-M_2\right)E_{\beta,2}(\lambda)-\left(N_1-N_2\right)E_{\beta,1}(\lambda)-A_1\lambda E_{\beta,\beta-1}(\lambda)+A_2\lambda E_{\beta,\beta}(\lambda)}{\left(-M+N+\int\limits_0^1(1-z)E_{\beta,2}\left[\lambda(1-z)^\beta\right]f(z)dz\right)}-$$

$$-\left(M_1-M_2\right)x^3E_{\beta,4}\left(\lambda x^\beta\right)+\left(N_1-N_2\right)x^2E_{\beta,3}\left(\lambda x^\beta\right)+A_1E_{\beta,1}\left(\lambda x^\beta\right)+A_2xE_{\beta,2}\left(\lambda x^\beta\right)\tag{9}$$

ko'rinishda aniqlaymiz.

2-teorema. Agar $-M+N+\int\limits_0^1(1-z)E_{\beta,2}\left[\lambda(1-z)^\beta\right]f(z)dz\neq 0$, $f(x)\in C(0,1)$ bo'lsa u holda T masala yagona yechimga ega bo'ladi va u (8) va (9) formulalar bilan aniqlanadi.

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