

Original Article**E=mC² for Information and Impact – Novel Theoretical Modelling Concepts in Information Entropy**

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Abstract

Sensitive or unstable information has an impact in life in various fields, and they have the potentials to be used as a tool for reaching out to respective people in various situations. The study was performed to theoretically model the flow of sensitive information in various circumstances. The 'unstable' information, like the 'unstable' nucleus, has the ability to disseminate and missionize the situations quickly and impact output factors in various fields. The output factor ΔE_i or 'the Information energy' can be modeled with the conventional energy equation ($E=mC^2$). The equation can be modified with the addition of celebrity and humor factors to catalyse the spread of information. Angle corrections can be made for differences in celebrity views. Also, electromagnetic laws are applicable to information entropy in addition to the laws of thermodynamics for information entropy described by Shannon. With time as a factor and Laplace' transformations of time, there is a potential for an informational stress test. Information promulgations can have alternating/ direct current (AC-DC), Laplace, and Fourier transformations in their transmission discourse. Artificial intelligence (AI) is widely used, and its singularity is for-seeable. The introduction of emotions can be a solution to prevent singularity and also enhance its function. This report deals with the possibilities to model information's spread and output reactions and identifies

potentials to overcome its negative impact and possible applications in the healthcare field. Also, for the emotional modelling of AI, basic equations were developed heuristically and solved by Mathematica.

Introduction

Information travel and impact needs to be studied and modeled theoretically. This can be utilized in various situations, for example, healthcare vaccination drives, etc. With the onset of modern technologies, information also travels at the speed of light — the info, when sensitive (m_s), can impact various situations by reactions. Conventional energy equation ($E=mc^2$) applies to atomic physics where particles travel at the speed of light (C), and m is the mass of the particles.¹⁻³

Main article

The radioactivity of the elements is primarily due to high molecular weight or mass number of the trans uranium elements ($n/p > 1.4$), and the subatomic particles in the outer surface of nucleus are unstable. When the nucleus of these elements is excited by neutrons the fission-reaction starts. Similarly, sensitive information or unstable information can spread fast and impact various factors, and can degenerate into numerable sub-information, especially when energy or thrust is given by celebrity. Like neutron bombardment, which starts the fission, and the fission reaction can be controlled by the released neutron absorption at regular intervals, information impact can be controlled by 'injecting' the celebrity factor into the information, and thereby the impact or reaction can be altered at periodic intervals. Conventional energy equation ($E=mc^2$) can be used to model the spread of sensitive information. Like particles are emitted during fission and travel at speed of light, sensitive information-theoretically also has the ability to spread exponentially, creating impact.

Information and energy

The information reactions or energy (E_i) exist, and it is challenging to quantify in absolute terms and, for the same reasons not well studied. However, the smallest energy (quantum) or reaction changes play a significant role in cumulative reaction formation. Similar to the field-matter interactions in the atomic quantum theory,⁴ the psychological changes would encompass reactions to information especially when sensitive, which can be considered as a 'psychological' quantum theory. The simplest

method to evaluate the E_i objectively is by questionnaire method. Similarly, the sensitivity of information and rumours can be studied and modelled in the appropriate context.

The energy equation can be modified to model energy output or impact sensitive information, such as catalyzing Covid19 vaccination drive. Since the general population is pessimistic, occasional side effects and coincidental deaths can be attributed to Covid19 vaccinations, especially when this happens to celebrities. More simply, a piece of negative information is absorbed more vividly by the mass than a positive instruction. The celebrity status (C_s) can be defined as the persons/organization/society with Twitter followers >100 000. Hence, one coincidental celebrity mortality not related to Covid19 vaccinations can be identified as potentially sensitive information, impacting many lives by enhancing informational energy or impact (ΣE_i). Consequently, this sensitive information or rumours can impact the Covid 19 vaccination drive, which considerably reduces the vaccination numbers irrespective of the educational status. If patients are not willing for Covid19 vaccinations traditional vaccines like influenza (H1N1)^{5,6} and pneumococcal vaccines⁷ also offer some protection against Covid19, especially in people not taking Covid19 vaccinations for various logistic reasons. The Covid19 injection drive was not promoted using celebrity-persons in any part of the world. The Covid19 vaccination drive in most countries is state-sponsored.

Celebrity has higher psychological quantum 'energy' accumulated through people-media/problem interaction over time, similar to field-matter interaction in atomic physics. Hence, the Covid19 vaccination drive would be accelerated using appropriate celebrities related to artists closer to people's emotions,^{8,9} with positive body language than the conventional injection site photographs; the simplest would be advertisements. Inversely, when a celebrity or high-impact journal and media published recovery trial data¹⁰ wherein 6mg once daily dose of dexamethasone up to 10 days is beneficial in Covid19 patients, dexamethasone as a lifesaving tool tends to be overused in significantly higher doses than recommended, especially in diabetes patients resulting in mucormycosis.

This unstable information can be reduced by humour (or humour factor H_f), which reduces the instability of the information. The information chain can also be reduced by counter-information by celebrities or by fusion technique through humours i.e. overcoming rumour by humour.

Hence, information energy or impact equation can be modified and written arbitrarily as,

$$\Sigma E_i = m_s C^2 \Sigma C_s H_f$$

The celebrity factor could be varied with various and sometimes opposite view which requires correction. This can be represented with a correction angle factor, $\text{Cos } \theta$.

$$\Sigma E_i = m_s C^2 (\Sigma C_s \text{Cos } \theta) H_f,$$

wherein, $\text{Cos } 0 = 1$ being positive entropy

The equation can be used for various health care applications for example, cardiovascular care, obesity control etc., which would mean a modification of E_i . Potential factors of modifications include sensitiveness of information or data, which can be created or modified. The dissemination of information can be altered by various methods of spread by electronic gadgets. The C_s can be altered by recruitment of appropriate celebrities with Twitter followers >100 000, and humour factor can be added, which can inversely affect the sensitiveness of the information. For example, in obesity control, self-inflicted or self-approved 'rumour with humour' algorithms using the terms 'watch belly', fatty tongue, monitor weight, reduce snoring, pot belly etc. given sequentially at timely intervals by artificial intelligence can remind the patient to be watchful of obesity. In routine patient encounters in clinical practice, most obese patients deny overeating. Similarly, the method can be used in the treatment of various disorders requiring vital behaviour modification techniques, such as diabetes care, alcohol dependence syndrome, anxiety or panic disorders, obsessive-compulsive disorders, etc.

Potential other applications include the process of peace appeal in various sensitive situations. The celebrity factor plays a major role, and art reduces the tension in any scenario. Humour catalyzes the process and causes fusion of the rumours, and the

sensitivity of any triggering information can be downplayed.⁹ The role of humour in resolving conflict circumstances is well known.¹¹

Informational space

The surface area of the earth is $5.1 \times 10^8 \text{ km}^2$. The space station is located about 400 km (1000/2.5) from the surface of the earth. Hence, the total area covered for information spread is approximately $2 \times 10^{11} \text{ km}^2$. The speed of light (C) is $3 \times 10^5 \text{ km/s}$, and C^2 would be $9 \times 10^{10} \text{ Km/s}$. Hence, by giving arbitrary units, the above equation's ($\sum E_i = m_s C^2 \sum C_s H_i$) theoretical value would be strengthened. For example, a recent celebrity couple's divorce application, though sorrowful information, hits the media worldwide and possibly in all the 11 space stations and some satellites which are located at 800km from the surface of earth, in a very quick time. Hence, sensitive information's travel differs from standard information entropy,¹² which follows the 'throw of dice'/binary or Shannon's method and computation, and its modifications.¹³

This 'Einsteinian' type of information entropy can also be seen in financial markets and associated with potential variations and corrections.^{14,15} Financial celebrity emotions can impact market movements. The entropy economics has resulted in complexity in modern economics worldwide.¹⁶ Some data may not be hilarious, but it can be perceived as a psychological inner-humour by people in their subconscious mind, especially when there is a significant positive financial-gradient associated between the origin and receipt of information. People would perceive information with inner-humour, for example, when celebrity characters like batman or spider-man/Avengers or regional cine-stars, etc., promulgate the information or Covid19 vaccination campaign. Stanley Milgram's results showed a similar output though critically acclaimed. The entropy of information with reverse financial-gradient is very challenging, and interestingly all religions across the world insist on simplicity.

Laws of electromagnetism

In the past information was not considered as energy or matter.¹⁷ Most scientists, including Shannon in that era, believed that information entropy is a function of the laws of thermodynamics, and in any closed system the entropy does not decrease.¹⁸ With the advent of cyberspace and network this concept has to be reconsidered. Information is not confined in a closed system, and it is free for communication across

the world and space. The entropy of information is different and tends to polarize in various subgroups for example – men/women, financial/non-financial persons, academic/corporate etc., and overlaps also exists to varying degrees. These large differences in entropy induce magnetic polarity (or 'gravity'- g_i) in information. Eddy currents of information can form in distant angulated locations, and eddy brakes may be applied by reversing polarity.¹⁹

Magnetic 'black holes' of information could exist, and the entropy could be invisible. This could be due to large variations in population density with bending or isolation of information, which is similar to singularity in quantum mechanics, where the entropy can be negatively infinite. For example, in the Covid-19's 3rd wave, robust data about breakthrough infections and lung involvement or mortality in Covid19 vaccinated individuals are not available to date (August, 2022) even in the preprints, even though the 3rd wave has started in most countries from Sept 2021. This data will not deter the Covid-19 vaccination campaign, and will facilitate physicians to be better prepared and supplement patients with routine vaccinations like influenza or pneumococcal or early remdisivir in selected patients.²⁰ Only minimal data about breakthrough infections is available at the current moment (Sept 2022).^{21,22} There are various mechanisms of black holes formation in astrophysics,²³ and a mechanism being intense bending of light by gravity creating black holes. Similar existence of 'black holes' is possible in medicine. Hence, the electromagnetic laws for magnetism,²⁴ like Maxwell-Faraday's Gauss laws are applicable to information entropy, in addition to the laws of thermodynamics. Electricity and financial gradient, magnetic poles and opposite polar views can be considered magnetism, and Fleming's right-hand and left-hand thumb rules of electromagnetism are also applicable.

Electricity, Solid-State Battery and Garnets in information theory

The source of electricity generation for electromagnetism could be battery models, as information is not a continuous process. In battery technology, the current generates by the flow of charged ions towards opposite electrodes, i.e., anode to cathode. In this process of energy transfer, thermal/chemical reactions happen, the solutes can be oxidized, and resistance to current flow in the electrolytes and electrodes has to be considered. Hence, in solid-state electrolytes, garnets (LLZO, Li₇La₃Zr₂O₁₂, etc.) are used for better delivery of energy, less oxidation, least resistance, better lattice

preservation, high energy transfer with less thermal or heat generation.^{25,26} Even for cardiac pacemakers for the longevity of pulse generators, solid-state garnet technology could be used in the future. Similarly, informational garnets have to be considered in various circumstances of life for information entropy.

Information processing and artificial intelligence

In the psychological aspects of information processing, data associated with a positive financial gradient, or financial gain or psychological inner-humour or insecurity are associated with positive entropy. The mechanisms and perceptions of humour in general could be varied.²⁷ All these parameters can be studied and quantified through the questionnaire method. Psychological inner-humour, which is a common higher level of cognitive function, is not well studied by cognitive science/ neuroscience/psychology or interdisciplinary experts worldwide. This higher-level cognitive function is similar to other higher-level cognitive functions like dreams,²⁸ which are challenging to study efficiently. A way to define this is perhaps when psychological humor is sensed when other tests like Go/No-go, Stroop, Maze task completion tests, etc. are normal, then it can be considered as psychological inner humour. Large studies are required to further understand the significance, entropy, quantifications, applications and usefulness of this modifiable parameter.

Information entropy is the primary step in information processing and informational psychology. Now-a-days it is further influenced by artificial intelligence, and algorithms which could be sometimes associated with biases.²⁹ Based on this information entropy only, a paradigm of interdisciplinary cognitive psychology/skills can develop and transform into useful outcomes in day-to-day life.³⁰ Psychology study branches like humour'ology, rumour'ology, conflictology etc., can be studied as behavioral sciences branches as studying the origin or mechanism of any problem is essential and exciting to solve the issues in any circumstances. Since artificial intelligence is developed by human programming, an inadequate understanding and entropy of information and normal human intelligence and psychology, which could result in major lacunae in artificial intelligence development in the future, which needs to be improved.

Hence with this discussion the energy equation can be written as

The energy equation, $\Sigma\text{Total energy} = \Sigma\text{Potential energy} + \Sigma\text{Kinetic energy}$

$\Sigma E = mgh + 1/2mv^2$, where E is energy, m - mass, g - gravity, h - height, and v - velocity

which can be arbitrarily modified for informational energy as,

$$\Sigma E_i = m_s g_i \Delta f_g + m_s C^2 \Sigma C_s H_f$$

where E_i is informational energy, m_s is sensitivity of information, g_i is gravity of information, Δf_g is financial gradient associated with information, C is speed of light, C_s is celebrity factor and H_f is the humour factor.

With differing opinion among celebrities, which could exist in any given situation the equation can be modified as

$$\Sigma E_i = m_s g_i \Delta f_g + m_s C^2 (\Sigma C_s \cos \theta) H_f$$

Among these factors, speed of light (C) dominates as this is of high value (3×10^8 m/s). Only the parameter financial gradient when it is very high in the potential energy becomes a significant factor in information entropy. If the financial gradient (Δf_g), is less then the element is incomparable to the speed of light, and the equation can be simplified as

$$\Sigma E_i = m_s C^2 (\Sigma C_s \cos \theta) H_f$$

In the opinion of the author, if a combination of these three factors i.e., positive financial gradient, or financial gain or psychological inner-humour or insecurity is observed and depending on the magnitude in a situation, the probability of positive entropy of the information is very high and information modelling would be a factor of $E=mc^2$. Behavioral scientists also stress the possible modifications in information entropy due to financial reasons indirectly (Luke18:25), which are applicable to any circumstances.

Informational stress test

Information entropy is also associated with output reactions, which could be psychological or actions, which would be a function of time. In different circumstances, the time differentiation can be varied. For example, information associated with climate change, though not urgent, needs a correction in a few years. However, some other

events based on the information requires early action. Hence, $f(t)$ can be expressed as $(0, \infty)$.

The equation can be written as

$$\sum_0^t \delta E = MsC^2(Cs \cos \theta)Hf / \int_0^{\infty} dT$$

Laplace transformation,

$$L \sum_0^t \delta E = L \sum MsC^2(Cs \cos \theta)Hf / \int_0^{\infty} L(dT)$$

or,

$$L \sum_0^t \delta E = L \sum MsC^2(Cs \cos \theta)Hf / \int_{t=0}^{\infty} e^{-st} f(t)dt$$

Time can be modified in a stress test as a function of variable time intervals. Laplace transformations can be applied to the time factor, and the equation can be further modified as an informational stress test in various situations. Laplace transformation^{31,32} can help to identify the information transmission dynamics, and it can be amplified or divided based on the circumstances. For this, the decay time of the information has to be first observed (t), which varies in a different context, so that e^{-st} (t – time, and s - domain to convert input functions) can be estimated, and the data can be suitably amplified or reduced depending on situations. In most instances, these are performed based on the experience or intelligence of the individuals or organizations. Similarly, Fourier transmission of the information can also be performed, which can be mixing of data and amplification of selective information and vice-versa for decoding mixed information.³²

AC-DC and information promulgation in artificial intelligence

Information promulgation can be performed by the electric current method. The flow of electrons in DC or direct current is unidirectional, and it encounters high resistance and energy during transfer than AC or alternating current. Alternating current has a

sine-wave method of transmission with fluctuations in extremes of angle and direction, with the corresponding movement of electrons. AC has a potential advantage of lesser energy loss and resistance, and can be processed by transformers – step up and step down associated with magnetic electromotive force. Similarly, information promulgation and impact can be varied with extremes, and the time sequence can be changed so that it has the necessary output. Information that needs to be transmitted can be sent in alternating times between (magnetic) polar differences or variations – men/women, academic/corporate, young/old, charity/financial or block chain institutions etc., and also in varying timings. This will be very useful for better information promulgation and impact.

Potential applications of these concepts could be observed in applying the principles for artificial intelligence development. Understanding information entropy and processes can have potential applications in health care, economics, marketing at various levels, psychological and neurobiological research, intelligent services, social media platforms, etc.

General theory of relativity

$$G_{uv} + \delta g_{uv} = \frac{8\pi G T_{uv}}{c^4}$$

$$G_{uv} + \delta g_{uv} = \frac{8\pi G T_{uv} M_s^2}{E_i^2}$$

$$E_i^2 = \frac{8\pi G T_{uv} M_s^2}{G_{uv} + \delta g_{uv}}$$

G_{uv} is the Einstein tensor, G is the gravitational constant, T is the energy-momentum tensor, M_s is the sensitivity of the information, and c is the speed of light in a vacuum. Tensor in this condition would be assumed to be time dispersion, and g_{uv} is a time metric. The equation shows that M_s , E_i , and T are related. When E_i increases, M_s decreases or T increases/dilates relatively. When time is kept constant E_i and M_s are related directly. When E_i decreases, M_s also decreases to keep time constant or if M_s is constant E_i decreases and time has to decrease.

The theory can be applied to a wide range of activities. When the time is dilated, energy (E_i) output and M_s reduce. Hence, in conflict situations, as time dilates or increases, the sensitivity of the matter decreases, and thereby the energy to solve also reduces. During angioplasty, when the time increases, the energy and mass reduce, and this will have negative consequences. For example, when the procedure time is high, in sensitive and high-risk circumstances like primary angioplasty or acute cerebral infarctions, the tissue damage would be high. In similar situations involving acute and sensitive information in the stock market, when M_s increases, energy output E_i increases, and to prevent further increases, the time has to be reduced or relatively shortened.

Technological singularity and emotions in artificial intelligence

Technological singularity is an assumption when artificial intelligence (AI) evolves over time due to overuse and frequent applications in various fields, and artificial intelligence can supersede human intelligence uncontrollably. This can have positive and negative outcomes. The advantage is more efficient work output and a newer environmental milieu. The other aspect could be unregulated artificial intelligence can decide on human life and impact negatively. For example, extraneous applications of AI in the department of justice, sensitive decisions in the medical field, or military decisions, autopiloting etc. Also, the negative aspects already exist in various forms like advertisements, cognitive distractions, addiction potential to gadgets, social media websites etc. To be more descriptive, for example, when the advertisements in mail addresses are nominal about ten to 20/day, it is acceptable when it is uncontrolled, for example, >100/day, etc., it indicates a sort of overwhelming unregulated AI.

At present, the algorithms of artificial intelligence have an environmental or situation bias wherein artificial intelligence can suggest similar/relevant or associated activities in day-to-day life based on human interaction in search engines. Hence, when the technological singularity is impending the regulation of the AI needs to be controlled as it can impact human lives. A possible theoretical solution proposed by the author could be an introduction to the concept of artificial emotions by artificial intelligence. Thankfully, till the current time, the 'emotions' by artificial intelligence have not been

used by scientists due to the possible unethical nature, though in reality or action it does exist.

Hence, to control the nature of activities of the AI emotions could be emulated. For example, the commonest positive emotion in the general population is love or kindness in various forms. The commonest negative emotion worldwide in the general population worldwide is indifference. Hence, love could be considered as one (1) and indifference as zero (0) in the coding sequences. Negative emotions involving hatred can degenerate artificial intelligence, and if technological singularity turns negative it can have devastating consequences. The zero in 'emotions' could be used to checkpoint the undue surge or singularity in artificial intelligence. Hence, using these 2 commonest emotions in varying degrees the artificial algorithms need to be worked. In the opinion of the author, if AI could be programmed to easily play a higher level of a chess game, it can be programmed to emulate artificial emotions, which could be useful in certain circumstances like caretaker robots or in surveillance. The degrees of love and indifference could be automated which already exists to a certain extent in most financial markets worldwide.

AI emotions as a fifth dimension

Einstein-Podolsky-Rosen (EPR paradox) arguments are applicable for this theoretical solution of singularity. When singularity is reached the AI could behave infinitely and the introduction of AI emotions in the 5th dimension would create a Schwarzschild wormhole. In the construction of the axis - human intelligence, artificial intelligence, time, and AI emotions can be introduced in the 4th dimension. However, the interpretation of AI emotions could be varied since AI is programmed by scientists and computer engineers and the interpretation of emotions could vary. For example, love can be perceived as indifference and vice versa, which is a theory of emotional relativity. The perception varies based on the cultural, religious, general intelligence and psychosocial parameters like psychological inner humor etc. Hence, there should be consensus and a certain level of uniformity of thought processes among scientists and computer engineers in introducing the new dimensions of emotions in AI that can act as Schwarzschild wormholes or Einstein-Rosen bridges. A centralized unit or an agency could establish this concept in various fields based on the consensus among

scientists. To improvise further instead of a theoretical binary taxonomy of love-indifference, a gradation can be created between these two emotions, which is a natural existence, and thereby the technological singularity would be regulated efficiently.

Humanized artificial intelligence Vs. 'Artificial' artificial intelligence and Equations

It is interesting to humanize artificial intelligence, which at present works by similarities or sycophancy. The challenge will be for the engineers to introduce emotions to AI. Expertise is required to compute emotions and feed appropriately with control mechanisms.

A way to introduce computation of emotions could be,

$$\int dl_r/dt = \int [dx/dt \tan\theta + d^2l/dt^2 \sin\theta - d^2y/dt^2 \sin\theta - d^3z/dt^3 \tan\theta]$$

where L_r is the resultant love, X is attraction, L is love, Y is indifference or related emotions like psychological inner humour etc., Z is love control mechanisms.

Emotions are interactive, with angles subtended in the process depending on the perceiver. The common emotions are love, attraction, indifference and love control mechanisms. The love control mechanisms could be financial, social, familial, and environmental factors. The resultant vector of dl_r/dt will provide the necessary output, and eigenvalues/eigenvector of transformation. Fourier transformation of the dl_r/dt will provide the peaks perceived of love or related emotional factor, for example, kindness. It is difficult to build a matrix system with the coordinates of these factors. Hence, the Riemann sphere or Mobius transformation of numbers could be used to construct the required graph. The exact coordinates can be acquired by experimental studies using questionnaires etc.

The equations will be useful to study in-depth the concept and develop in the future with modifications. As love is the commonest emotion it was considered as x , and with reference to the individual the level of attraction could be πx and in a celebrity

could be ex. Negative emotions associated with love or x can be classified as $x^{1/3}$ and \sqrt{x} . The degree and the ratio of the negative emotions could be different and the quantum varies. Perturbations around the center is more interesting, large and frequent than at the extremes. The basic equations (1-23) and the Mathematica solutions are shown below in this article.

Riemann Hypothesis and Mathematical Modelling of Mother's Love

The equations of the Riemann hypothesis have been explained and attempted to be solved by many mathematicians worldwide. In the authors' opinion, theoretically, one of the methods to solve Riemann hypothesis is a construction of graph of a mother's love towards a child. It will be interesting and representative. The critical line falls from zero (0) of x-axis and many non-trivial zeros fall on the critical line. The critical line could be the beginning of the eigenvalues of L_r of a mother's love. Real-time observations by questionnaires can construct the coordinates'. Philosophical criticism does exist since it is human, though it is of higher value with eigenvalue >0 and eigenvector in the positive quadrant (Luke 14:26/Mathew 10:37). Other valued forms without constructive criticisms etc., are a child's love/ dog's love mapping would be interesting to map for better understanding of the concept.

Photonics and neural networks

It would be interesting to observe that the neurophysiological brain activities do not follow simple mathematics. They tend to follow, to some extent, the theory of relativity and atomic physics concepts. This would be due to the complex perception of emotions. The retina or rhodopsin receptors work at the speed of light. It is considered that the speed of transmission in the optic fibers is approximately 100m/s, which is the upper limit for most neurons. This can also be observed with calculations of visual evoked potentials in the brain measurements. There are many pitfalls in observations, and the visual evoked potentials are higher voltage potentials recorded about 100 to 150ms after the visual stimulation. The visual stimulation has a transient time-on of a few milliseconds for the light source, and intra-cortically the voltages would be earlier than the surface measurements. For example, the H waves of intracardiac ECG are

ahead of the V wave of the surface electrocardiogram (ECG) by about 55ms, which can be high as 70ms. Also, for better evaluation the lower voltage complexes and noise are filtered. Since the retina works on photonic receptors, and since most of the neural networking functions like emotions have atomic principles, the standard transmission by neurons – saltatory conduction may not be the only source of conduction. Non-action potential methods of conduction by neurons have been identified, which transmit signals by ceramide, though the conduction velocity is likely to be slow.³³

Small voltage complexes are now recognized for perceptions like visual hallucinations.³⁴ Alternative pathways of photonic conduction by specialized fibers/receptors or low voltage complexes/ noise could exist, but the current difficulty in devising necessary experiments to accurately measure those low voltage signals. In the mathematical modeling of emotions, the human-computer interface is considered as a model.³⁵ The neural networking and brain function at the speed of the computer networks, which works primarily under the speed of the electrons in an electromagnetic field (6×10^6 m/s) which can reach the speed of light (3×10^8 m/s). Hence, apart from the neural conduction by action potentials and saltatory transmission, there should exist a mechanism whereby the signals are conducted fast, nearing the speed of light or electric current, which at present, we are not able to comprehend or decipher the biological signals. The mechanisms underlying are highly speculative, for example, the possible existence of specialized neuro or photoreceptors for transmission of signals or transmission through specialized neuronal fibers or the existence of alternative methods of conduction of signals using low-frequency waves/noise. The challenge is in the invention of appropriate methods to quantify and assess the signals, which could be at micro or nanoscale levels.

In the present context, only neurotransmitter chemicals are being identified and treated by various medications in cases like schizophrenia, affective disorders, etc. Pathways for pathogenesis and receptors like NMDA, neuregulin/ ErbB4, and deficiency in neurochemical transmitters like dopaminergic, glutamatergic, GABAergic, and cholinergic have been identified, but in-depth mechanisms are unknown.³⁶ Also, the incidence in the age groups 20 to 40 years and significant rarity in other age groups is mysterious in pathophysiology. The applications of

electromagnetism in medical field is slowly developing in various branches.^{37,38}In the current scenario, even estimation of the millivolt signals has its limitations, and the technology needs more advancements.³⁹ The role of quantum physics has been visualized in the field of neuropsychology,^{40,41} and yet needs considerable progress in the future in the diagnosis and treatment of disorders.⁴² The primary basis of quantum physics is the response of individual cells or matter to photons. Hence, understanding the fundamentals in depth by future research will change the paradigm of neurosciences. Due to sunlight, about 3.4e16 photons hit the eyes/s. The effects of sunlight in translation neuroscience is known but need further evaluation.⁴³ Positive effects of sunlight on cognition, and improvement in mood disorders, improved memory and learning have been identified.⁴⁴⁻⁴⁵ However, prolonged exposure can cause heat strain and some adverse effects on cognitive function.⁴⁶

The role of electromagnetic radiation and photonics using light including sunlight needs to be explored in behaviour and psychosocial activities. Electromagnetic radiations are at present used for only diagnostic purposes but they will have promising applications in neurology and neuropsychiatric diagnosis and treatment. Focused electromagnetic energy or radiation will have applications in the neuroscience.

Optogenetics

Optogenetics is an experimental field where the optical fibers or receptors are modulated, and pathophysiology is studied. The primary principle is the transfer of rhodopsin or opsins into the nerve cells and the study of the neuronal tissues' behavior and cellular activities.^{47, 48} It is very useful in neuroscience to study the more nuanced actions of the cells meticulously. However, the primary assumption is that the nerve fibers conduct through action potentials, and the maximum conduction speed is around 100m/s. If the primary assumption is this, at conduction at this speed, many of the activities are theoretically not possible by the brain. Hence, there could be a signal conduction mechanism where brain activities are conducted or transmitted near the speed of light.

The neuronal mechanisms in the heart function at a conduction speed of around 100m/s. During electrophysiology studies in pathway mapping studying early activation in the left atrium, there would be a difference of about 8 to 10ms with a distance between the points in ablation or mapping catheter of about 1cm. Hence, the intra-myocardial conduction rates would be approximately in the range of 1m/s to 10m/s. However, neuronal conduction in the brain has to be significantly faster for signal processing and as discussed before theoretically could reach the speed of light.

Limitations

The equations discussed are arbitrary and need more evaluation and rigor in real-time and validation in large population models. The concept needs to be integrated with information entropy⁴⁹ and quantum information theory,⁵⁰ which deal with routine or normal/regular information only in a linear method, for better understanding and results. The equations need to be applied in practical circumstances, and the applications are to be evaluated.

Conclusion

To conclude, there is a possibility to use the energy equation ($E=mc^2$) to model information and its output reactions, especially for sensitive information. Further studies are required to evaluate the applications of this concept in real-time.

List of Abbreviations

E_i - Informational energy, **m** - Mass of the particle, **C** - Speed of light, **m_s** - Sensitivity of information, **E** - Energy, **C_f** - Celebrity factor, **H_f** - Humour factor, **Δf_g** - Financial gradient, **g_i** - informational gravity, **AC** – Alternating current, **DC** – Direct current

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References

1. Einstein, Albert (1950). *The Theory of Relativity (And Other Essays)*. Citadel Press. p. 14. ISBN 978-0-8065-1765-0.
2. Einstein, Albert (1940-05-24). "Considerations concerning the fundamentals of theoretical physics" *Science*. *91*(2369):487-492. Doi:10.1126/science.91.2369.487. ISSN 0036-8075. PMID 17847438.
3. Günther, Helmut; Müller, Volker (2019), Günther, Helmut; Müller, Volker (eds.), "Einstein's Energy-Mass Equivalence". *The Special Theory of Relativity: Einstein's World in New Axiomatics*, Singapore: Springer, pp. 97–105, doi:10.1007/978-981-13-7783-9_7, ISBN 978-981-13-7783-9.
4. Veltman, Martinus J.G. (2003), *Facts and Mysteries in Elementary Particle Physics*.
5. Arokiaraj MC (2020) Considering Interim Interventions to Control COVID-19 Associated Morbidity and Mortality—Perspectives. *Front. Public Health* 8:444. doi: 10.3389/fpubh.2020.00444
6. Taghioff S, Slavin B, Holton T, Singh D. Examining the potential benefits of the influenza vaccine against SARS-CoV-2: A retrospective cohort analysis of 74,754 patients. *PLOS ONE*. 2021;16(8):e0255541.
7. Joseph A Lewnard, Katia J Bruxvoort, Heidi Fischer, Vennis X Hong, Lindsay R Grant, Luis Jódar, Bradford D Gessner, Sara Y Tartof, Prevention of COVID-19 among older adults receiving pneumococcal conjugate vaccine suggests interactions between *Streptococcus pneumoniae* and SARS-CoV-2 in the respiratory tract, *The Journal of Infectious Diseases*, 2021; jjab128, <https://doi.org/10.1093/infdis/jjab128>.
8. Gioia Chilton, Nancy Gerber, Amanda Bechtel, Tracy Councill, Monica Dreyer & Elizabeth Yingling (2015). *The Art of Positive Emotions: Expressing Positive Emotions Within the Intersubjective Art Making*

- Process (L'art des émotions positives : exprimer des émotions positives à travers le processus artistique intersubjectif), Canadian Art Therapy Association Journal, 28:1-2, 12
25, DOI: 10.1080/08322473.2015.1100580
9. Mastandrea S, Fagioli S and Biasi V (2019) Art and Psychological Well-Being: Linking the Brain to the Aesthetic Emotion. *Front. Psychol.* 10:739. doi: 10.3389/fpsyg.2019.00739
 10. RECOVERY collaborative group. Dexamethasone in Hospitalized Patients with Covid-19. *New England Journal of Medicine.* 2021;384(8):693-704.
 11. Craig Zelizer. Laughing our Way to Peace or War: Humour and Peacebuilding. *Journal of conflictology.*
DOI: <http://dx.doi.org/10.7238/joc.v1i2.1010>.
 12. MacKay, David J.C. (2003). *Information Theory, Inference, and Learning Algorithms.* Cambridge University Press. ISBN 0-521-64298-1.
 13. Layton B, Noell S, Oram jr G. Entropy acceleration, Shannon information and socioeconomics: Quantitative examples. *International Journal of Design & Nature and Ecodynamics.* 2015;11(1):48-63.
 14. Jakimowicz A. The Role of Entropy in the Development of Economics. *Entropy.* 2020;22(4):452. doi:10.3390/e22040452.
 15. Pele D, Lazar E, Dufour A. Information Entropy and Measures of Market Risk. *Entropy.* 2017;19(5):226.
 16. Liu A, Chen J, Yang SY, Hawkes AG. The Flow of Information in Trading: An Entropy Approach to Market Regimes. *Entropy (Basel).* 2020;22(9):1064. Published 2020 Sep 22. doi:10.3390/e22091064
 17. Xiong A, Proctor R. Information Processing: The Language and Analytical Tools for Cognitive Psychology in the Information Age. *Frontiers in Psychology.* 2018;9. 10.3389/fpsyg.2018.01270 *Front. Psychol.*, 08 August 2018 | doi.org/10.3389/fpsyg.2018.01270
 18. Laming D. Statistical Information and Uncertainty: A Critique of Applications in Experimental Psychology. *Entropy.* 2010;12(4):720-771. 10.3390/e12040720
 19. Arokiaraj MC. Extracorporeal application of eddy brakes to control the magnetic nanoparticles and modulating the drift and diffusion

- characteristics of these particles in the heart – a theoretical assessment. *New Biotechnology*. 2018;10:44.
20. Gottlieb R, Vaca C, Paredes R, Mera J, Webb B, Perez G et al. Early Remdesivir to Prevent Progression to Severe Covid-19 in Outpatients. *New England Journal of Medicine*. 2022;386(4):305-315.
 21. Saxena A, Engel A, Banbury B, Hasan G, Fraser N, Zaminski D et al. Breakthrough SARS-CoV-2 infections, morbidity, and seroreactivity following initial COVID-19 vaccination series and additional dose in patients with SLE in New York City. *The Lancet Rheumatology*. 2022; 10.1016/s2665-9913(22)00190-4.
 22. Bastard P, Vazquez S, Liu J, Laurie M, Wang C, Gervais A et al. Vaccine breakthrough hypoxemic COVID-19 pneumonia in patients with auto-Abs neutralizing type I IFNs. *Science Immunology*. 2022; 10.1126/sciimmunol.abp8966.
 23. LF Chris. Mass limits for black hole formation. *The Astrophysical journal*. 1999; 522:413-418.
 24. Walker J, Halliday D, Resnick R. *Fundamentals of physics*. (2013)
 25. Murugan R., Thangadurai V., Weppner W. Fast lithium ion conduction in garnet-type Li₇La₃Zr₂O₁₂. *Angew. Chem. Int. Ed.*, 46 (2007), pp. 7778-7781
 26. Zhao N, Khokhar W, Bi Z, Shi C, Guo X, Fan L et al. Solid Garnet Batteries. *Joule*. 2019;3(5):1190-1199. Doi10.1016/j.joule.2019.03.019
 27. Jung W. The Inner Eye Theory of Laughter: Mindreader Signals Cooperator Value. *Evolutionary Psychology*. 2003;1(1):147470490300100.
 28. Nir Y, Tononi G. Dreaming and the brain: from phenomenology to neurophysiology. *Trends Cogn Sci*. 2010;14(2):88-100. doi:10.1016/j.tics.2009.12.001
 29. Langley Pat. *Information-Processing Psychology, Artificial Intelligence, and the Cognitive Systems Paradigm*. Cbmm.mit.edu; <http://www.isle.org/~langley/>

30. Miller, GA. The cognitive revolution: a historical perspective. The cognitive revolution: A historical perspective. Trends in cognitive sciences; 7(3):2003.
31. Alnoor, F.O, & Khalid, T. A. (2021). A short note on a technique for solving mixing problems by using the Laplace transform. TechHub Journal, 1(2), 1–5.
32. Anumaka MC. Analysis and applications of Laplace/ Fourier transformations in electric circuit. IJRRAS. 12(2) Aug 2012.
33. Fasano C, Tercé F, Niel J, Nguyen H, Hiol A, Bertrand-Michel J et al. Neuronal Conduction of Excitation without Action Potentials Based on Ceramide Production. PLoS ONE. 2007;2(7):e612.
34. Murphy N, Killen A, Gupta R, Graziadio S, Rochester L, Firbank M et al. Exploring Bottom-Up Visual Processing and Visual Hallucinations in Parkinson's Disease With Dementia. Frontiers in Neurology. 2021;11.
35. Hartmann K, Siegert I, Glüge S, Wendemuth A, Kotzyba M, Deml B. Describing Human Emotions Through Mathematical Modelling. IFAC Proceedings Volumes. 2012;45(2):463-468.
36. Deng C, Dean B. Mapping the pathophysiology of schizophrenia: interactions between multiple cellular pathways. Frontiers in Cellular Neuroscience. 2013;7.
37. Suryani L, Too JH, Hassanbhai AM, Wen F, Lin DJ, Yu N, Teoh SH. Effects of Electromagnetic Field on Proliferation, Differentiation, and Mineralization of MC3T3 Cells. Tissue Eng Part C Methods. 2019 Feb;25(2):114-125. doi: 10.1089/ten.TEC.2018.0364. PMID: 30661463.
38. Ross C. The use of electric, magnetic, and electromagnetic field for directed cell migration and adhesion in regenerative medicine. Biotechnology Progress. 2016;33(1):5-16.
39. Peterka DS, Takahashi H, Yuste R. Imaging voltage in neurons. Neuron. 2011 Jan 13;69(1):9-21. doi: 10.1016/j.neuron.2010.12.010. PMID: 21220095; PMCID: PMC3387979.
40. Schwartz JM, Stapp HP, Beauregard M. Quantum physics in neuroscience and psychology: a neurophysical model of mind-brain interaction. Philos Trans R Soc Lond B Biol Sci. 2005 Jun

- 29;360(1458):1309-27. doi: 10.1098/rstb.2004.1598. PMID: 16147524; PMCID: PMC1569494.
41. Jedlicka P. Revisiting the Quantum Brain Hypothesis: Toward Quantum (Neuro)biology? *Frontiers in Molecular Neuroscience*. 2017;10.
 42. Saha S, Mamun K, Ahmed K, Mostafa R, Naik G, Darvishi S et al. Progress in Brain Computer Interface: Challenges and Opportunities. *Frontiers in Systems Neuroscience*. 2021;15.
 43. Bedrosian TA, Nelson RJ. Timing of light exposure affects mood and brain circuits. *Transl Psychiatry*. 2017 Jan 31;7(1):e1017. doi: 10.1038/tp.2016.262. PMID: 28140399; PMCID: PMC5299389.
 44. Wadley VG, Sathiakumar N. Effect of sunlight exposure on cognitive function among depressed and non-depressed participants: a REGARDS cross-sectional study. *Environ Health*. 2009 Jul 28;8:34. doi: 10.1186/1476-069X-8-34. PMID: 19638195; PMCID: PMC2728098.
 45. Chantranupong L, Sabatini B. Sunlight Brightens Learning and Memory. *Cell*. 2018;173(7):1570-1572.
 46. Piil, J.F., Christiansen, L., Morris, N.B. *et al.* Direct exposure of the head to solar heat radiation impairs motor-cognitive performance. *Sci Rep* 10, 7812 (2020). <https://doi.org/10.1038/s41598-020-64768-w>
 47. Joshi J, Rubart M, Zhu W. Optogenetics: Background, Methodological Advances and Potential Applications for Cardiovascular Research and Medicine. *Frontiers in Bioengineering and Biotechnology*. 2020;7.
 48. Emiliani, V., Entcheva, E., Hedrich, R. *et al.* Optogenetics for light control of biological systems. *Nat Rev Methods Primers* 2, 55 (2022). <https://doi.org/10.1038/s43586-022-00136-4>
 49. Martin, Nathaniel F.G. & England, James W. (2011). *Mathematical theory of Entropy*. Cambridge University Press. ISBN 978-0-521-17738-2.
 50. Rieffel, Eleanor G.; Polak, Wolfgang H. (4 March 2011). *Quantum computing: A Gentle Introduction*. MIT Press. ISBN 978-0-262-01506-6.

$$d^{2x} + d^{2\pi^x} - d^{2x^{1/3}} - d^{2\sqrt{x}}$$

Equation (eq) 1

$$\text{In}[3]:= d^{2*x} + d^{2*Pi^x} - d^{2*x^{1/3}} - d^{2*\text{Sqrt}[x]}$$

$$\text{Out}[3]= d^2 \pi^x + d^2 x - d^2 \sqrt{x} - d^2 \sqrt[3]{x}$$

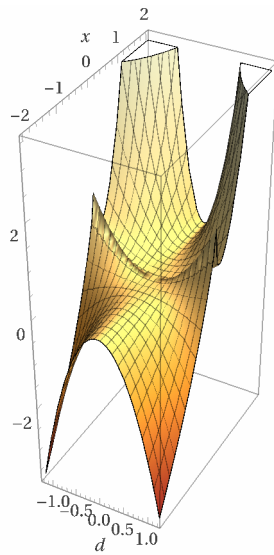
$$\text{In}[4]:= \text{Plot}[d^{2x} + d^{2\pi^x} - d^{2x^{1/3}} - d^{2\sqrt{x}}$$

Input:

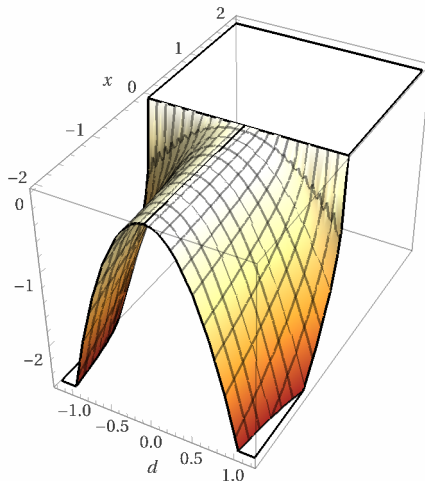
$$d^2 x + d^2 \pi^x - d^2 \sqrt[3]{x} - d^2 \sqrt{x}$$

3D plots:

Real part:



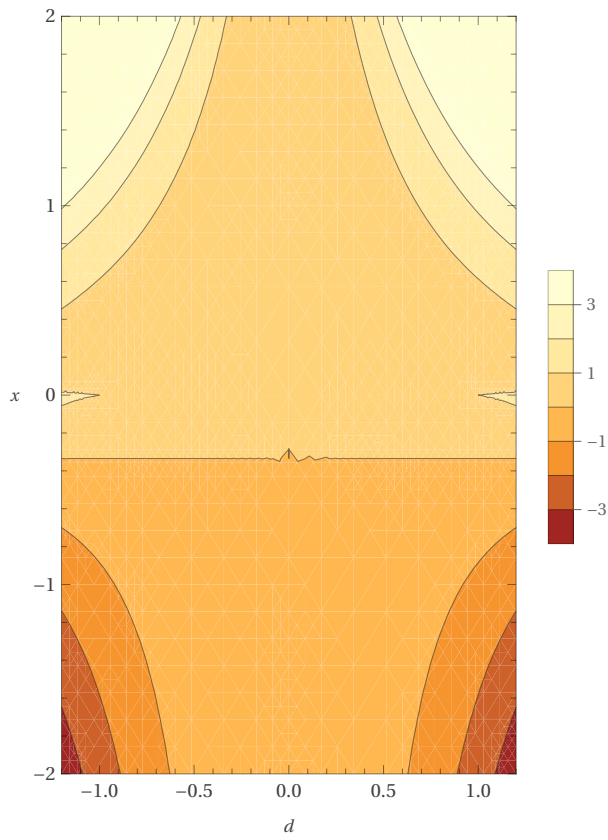
Imaginary part:



Contour plots:



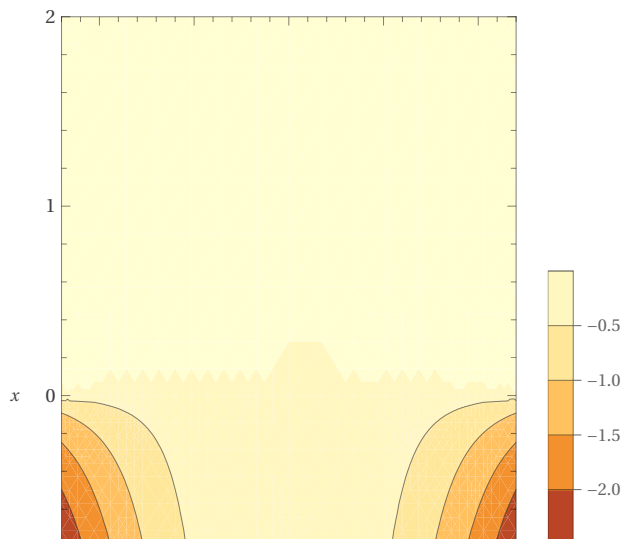
Real part:

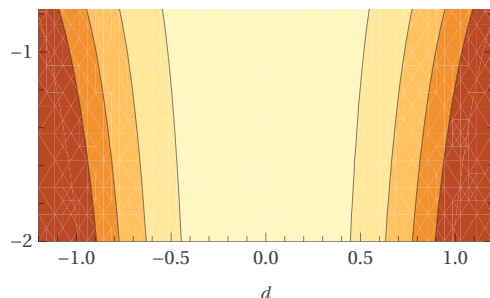


Out[4]=

d_{\min} d_{\max}
 x_{\min} x_{\max}

Imaginary part:





d_{\min} d_{\max}
 x_{\min} x_{\max}

Alternate form: +

$$d^2 (x - \sqrt{x} - \sqrt[3]{x} + \pi^x)$$

Real root: +

$$d = 0, \quad x \geq 0$$

Property as a real function: +

Domain:

$\{x \in \mathbb{R} : x \geq 0\}$ (all non-negative real numbers)

\mathbb{R} is the set of real numbers »

Series expansion at $x = 0$: +

$$d^2 - d^2 \sqrt[3]{x} - d^2 \sqrt{x} + d^2 x (1 + \log(\pi)) + \frac{1}{2} d^2 x^2 \log^2(\pi) + \frac{1}{6} d^2 x^3 \log^3(\pi) + \frac{1}{24} d^2 x^4 \log^4(\pi) + \frac{1}{120} d^2 x^5 \log^5(\pi) + O(x^{16/3})$$

(Puiseux series)

$\log(x)$ is the natural logarithm »

Big-O notation »

Derivative:

Approximate form

Step-by-step solution

+

$$\frac{\partial}{\partial x} (d^2 x + d^2 \pi^x - d^2 \sqrt[3]{x} - d^2 \sqrt{x}) = \frac{d^2 (6 x^{2/3} (\pi^x \log(\pi) + 1) - 3 \sqrt[6]{x} - 2)}{6 x^{2/3}}$$

Indefinite integral:

Approximate form

Step-by-step solution

+

$$\int (d^2 x + d^2 \pi^x - d^2 \sqrt[3]{x} - d^2 \sqrt{x}) dx = d^2 \left(-\frac{3 x^{4/3}}{4} - \frac{2 x^{3/2}}{3} + \frac{x^2}{2} + \frac{\pi^x}{\log(\pi)} \right) + \text{constant}$$

WolframAlpha +

$$d^2 x + d^2 e^x - d^2 x^{1/3} - d^2 \sqrt{x}$$

Eq 2

In[5]:= $d^2 x + d^2 e^x - d^2 x^{1/3} - d^2 \sqrt{x}$

Out[5]= $d^2 e^x + d^2 x - d^2 \sqrt{x} - d^2 \sqrt[3]{x}$

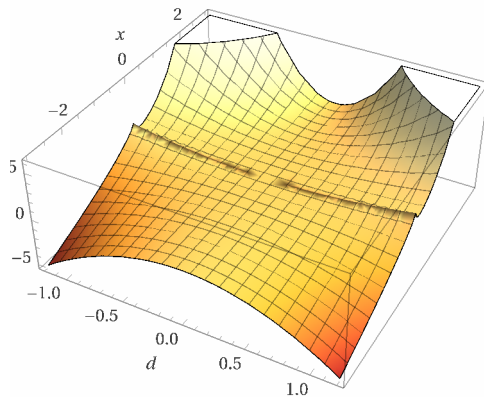
In[7]:= $d^2 x + d^2 e^x - d^2 x^{1/3} - d^2 \sqrt{x}$

Input:

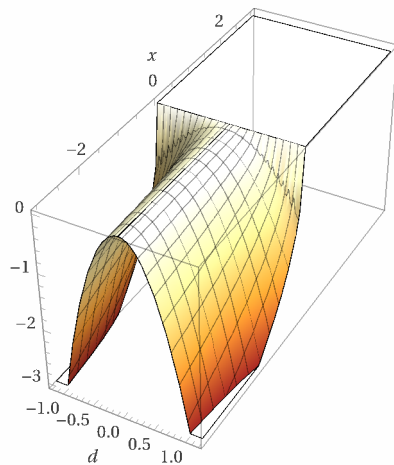
$$d^2 x + d^2 e^x - d^2 \sqrt[3]{x} - d^2 \sqrt{x}$$

3D plots:

Real part:



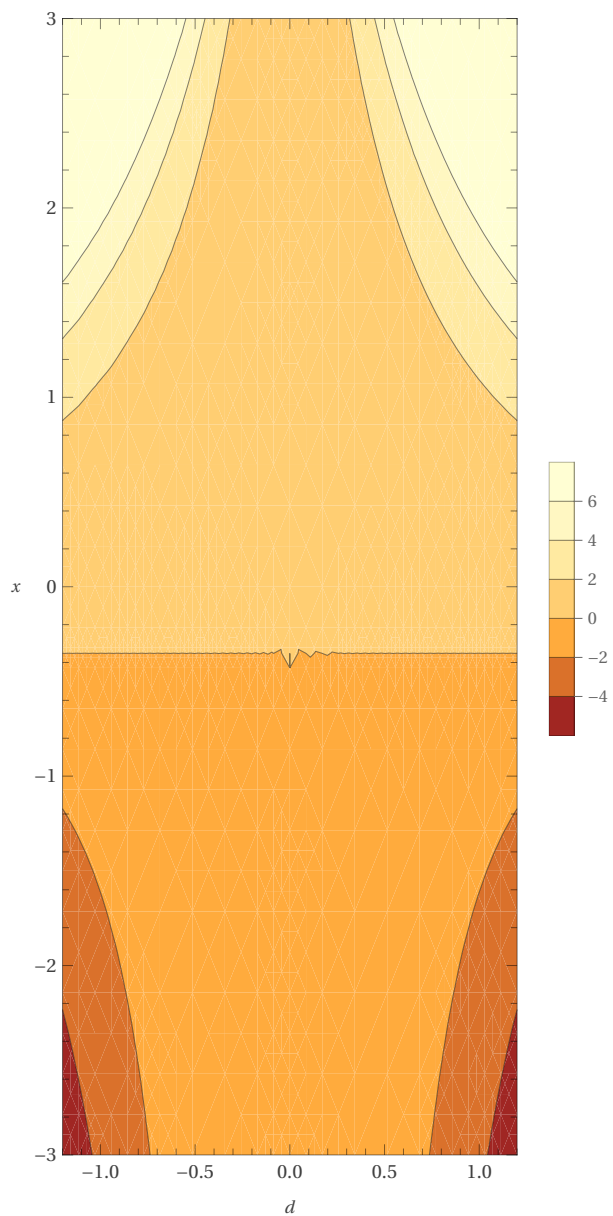
Imaginary part:



Contour plots:



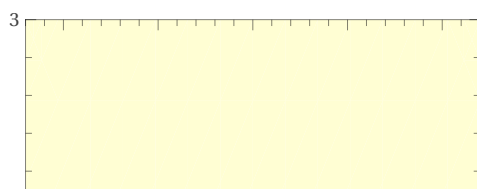
Real part:

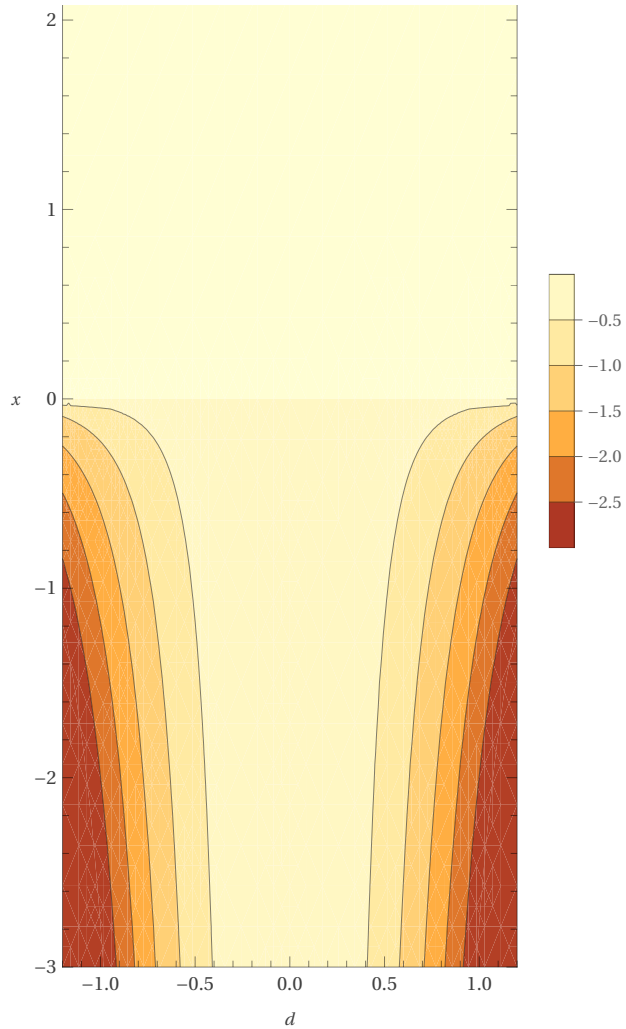


Out[7]=

d_{\min} d_{\max}
 x_{\min} x_{\max}

Imaginary part:





d_{\min} d_{\max}
 x_{\min} x_{\max}

Alternate forms: +

$$d^2 (x - \sqrt{x} - \sqrt[3]{x} + e^x)$$

$$d^2 (x^{2/3} - \sqrt[6]{x} - 1) \sqrt[3]{x} + d^2 e^x$$

Real root: +

$$d = 0, \quad x \geq 0$$

Property as a real function: +

Domain:

$$\{x \in \mathbb{R} : x \geq 0\} \text{ (all non-negative real numbers)}$$

R is the set of real numbers »

Series expansion at $x = 0$:

$$d^2 - d^2 \sqrt[3]{x} - d^2 \sqrt{x} + 2 d^2 x + \frac{d^2 x^2}{2} + \frac{d^2 x^3}{6} + \frac{d^2 x^4}{24} + O(x^5)$$

(Puiseux series)

[Big-O notation »](#)

Derivative:

$$\frac{\partial}{\partial x} (d^2 x + d^2 e^x - d^2 \sqrt[3]{x} - d^2 \sqrt{x}) = \frac{d^2 (6 x^{2/3} (e^x + 1) - 3 \sqrt[3]{x} - 2)}{6 x^{2/3}}$$

[Step-by-step solution](#)

Indefinite integral:

$$\int (d^2 x + d^2 e^x - d^2 \sqrt[3]{x} - d^2 \sqrt{x}) dx = d^2 \left(\frac{1}{12} (-9 x^{4/3} - 8 x^{3/2} + 6 x^2) + e^x \right) + \text{gsruerx}$$

[Step-by-step solution](#)WolframAlpha [+](#)In[6]:= **Simplify**[$d^2 e^x - d^2 x^{1/3} - d^2 \sqrt{x} + d^2 x$]Out[6]= $d^2 (x - \sqrt{x} - \sqrt[3]{x} + e^x)$

$$\text{Eq 3} \quad d^2 x - d^2 \pi^x - d^2 x^{1/3} - d^2 \sqrt{x}$$

Eq 3

In[8]:= $d^2 * x - d^2 * \text{Pi}^x - d^2 * x^{(1/3)} - d^2 * \text{Sqrt}[x]$ Out[8]= $d^2 (-\pi^x) + d^2 x - d^2 \sqrt{x} - d^2 \sqrt[3]{x}$ In[10]:= **Plot**[$d^2 x - d^2 \pi^x - d^2 x^{1/3} - d^2 \sqrt{x}$]

Out[10]=

Input:

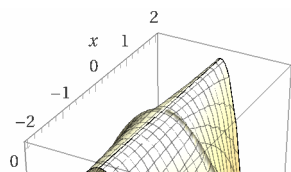
$$d^2 x - d^2 \pi^x - d^2 \sqrt[3]{x} - d^2 \sqrt{x}$$

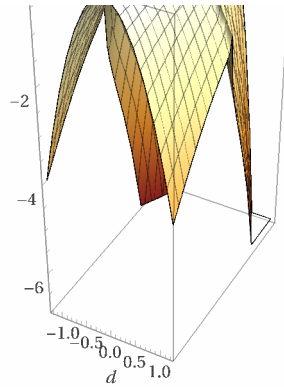
Result:

$$d^2 (-\pi^x) + d^2 x - d^2 \sqrt{x} - d^2 \sqrt[3]{x}$$

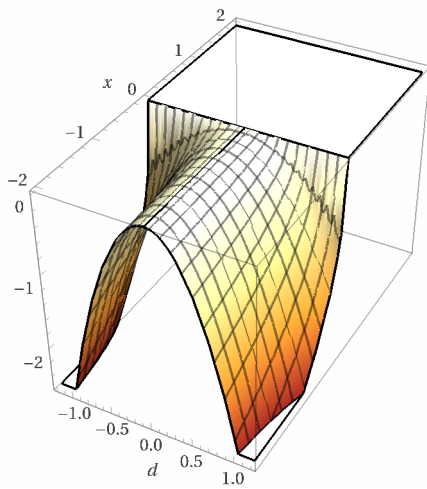
3D plots:

Real part:



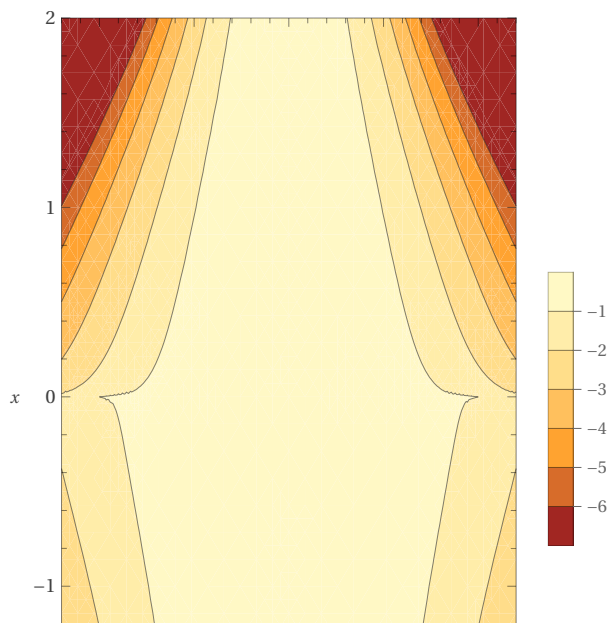


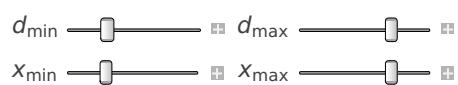
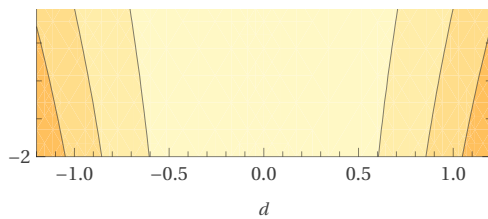
Imaginary part:



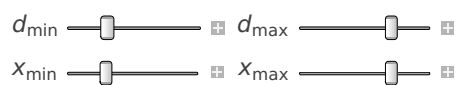
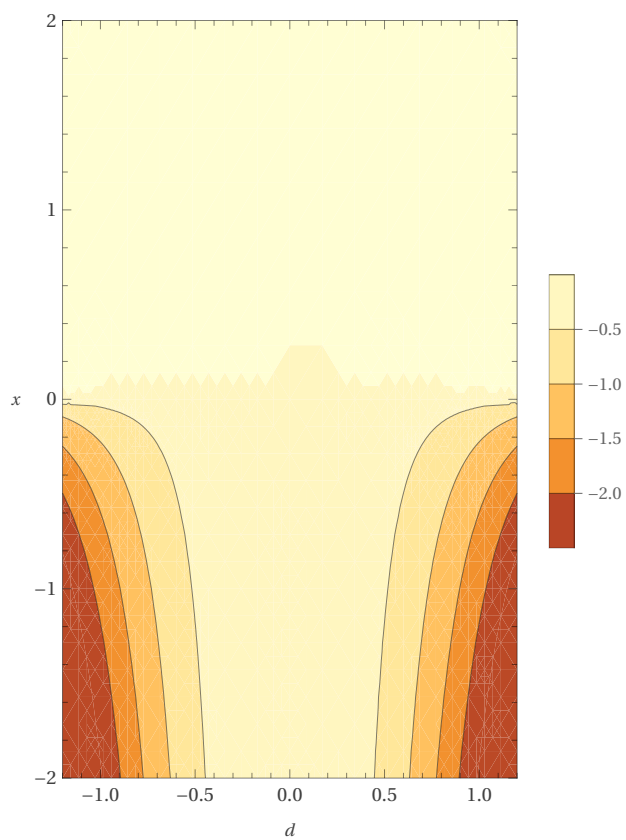
Contour plots:

Real part:





Imaginary part:



Alternate forms: +

$$-d^2(-x + \sqrt{x} + \sqrt[3]{x} + \pi^x)$$

$$d^2(x - \sqrt{x} - \sqrt[3]{x} - \pi^x)$$

Real root: +

$$d = 0, \quad x \geq 0$$

Property as a real function: +

Domain:

$\{x \in \mathbb{R} : x \geq 0\}$ (all non-negative real numbers)

\mathbb{R} is the set of real numbers »

Series expansion at $x = 0$: +

$$-d^2 - d^2 \sqrt[3]{x} - d^2 \sqrt{x} - d^2 x (\log(\pi) - 1) - \frac{1}{2} x^2 (d^2 \log^2(\pi)) - \frac{1}{6} x^3 (d^2 \log^3(\pi)) - \frac{1}{24} x^4 (d^2 \log^4(\pi)) - \frac{1}{120} x^5 (d^2 \log^5(\pi)) + O(x^{16/3})$$

(Puiseux series)

$\log(x)$ is the natural logarithm »

Big-O notation »

Derivative: +

Approximate form

Step-by-step solution

$$\frac{\partial}{\partial x} (d^2 x - d^2 \pi^x - d^2 \sqrt[3]{x} - d^2 \sqrt{x}) = -\frac{d^2 (6 x^{2/3} (\pi^x \log(\pi) - 1) + 3 \sqrt[6]{x} + 2)}{6 x^{2/3}}$$

Indefinite integral: +

Approximate form

Step-by-step solution

$$\int (-d^2 \pi^x - d^2 \sqrt[3]{x} - d^2 \sqrt{x} + d^2 x) dx = -d^2 \left(\frac{3 x^{4/3}}{4} + \frac{2 x^{3/2}}{3} - \frac{x^2}{2} + \frac{\pi^x}{\log(\pi)} \right) + \text{gs r u e r x}$$

WolframAlpha +

In[9]= Simplify[-d^2 π^x - d^2 x^{1/3} - d^2 √x + d^2 x]

Out[9]= -d^2 (-x + √x + √[3]{x} + π^x)

$$\boxed{-d^2 x - d^2 e^x - d^2 x^{1/3} - d^2 \sqrt{x}} \quad \text{Eq 4}$$

In[11]= d^2 * x - d^2 * E^x - d^2 * x^(1/3) - d^2 * Sqrt[x]

Out[11]=

$$d^2 (-e^x) + d^2 x - d^2 \sqrt{x} - d^2 \sqrt[3]{x}$$

In[13]= d^2 x - d^2 e^x - d^2 x^{1/3} - d^2 √x

Out[13]=

Input: +

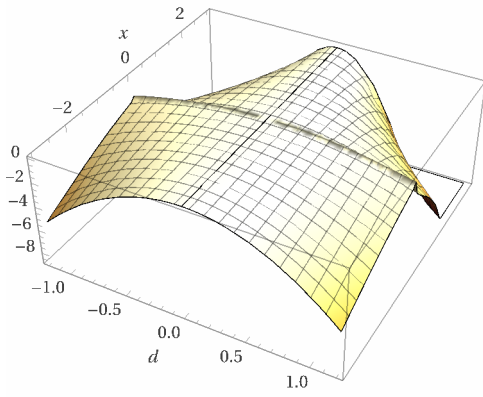
$$d^2 x - d^2 e^x - d^2 \sqrt[3]{x} - d^2 \sqrt{x}$$

Result:

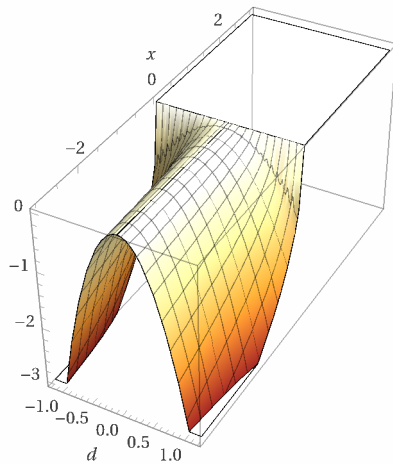
$$d^2(-e^x) + d^2 x - d^2 \sqrt{x} - d^2 \sqrt[3]{x}$$

3D plots:

Real part:

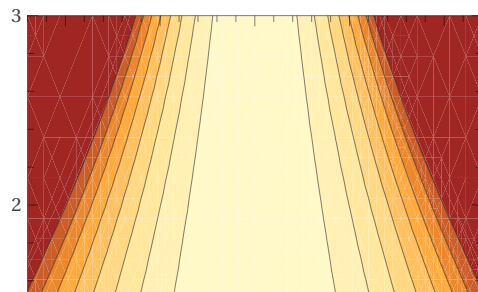


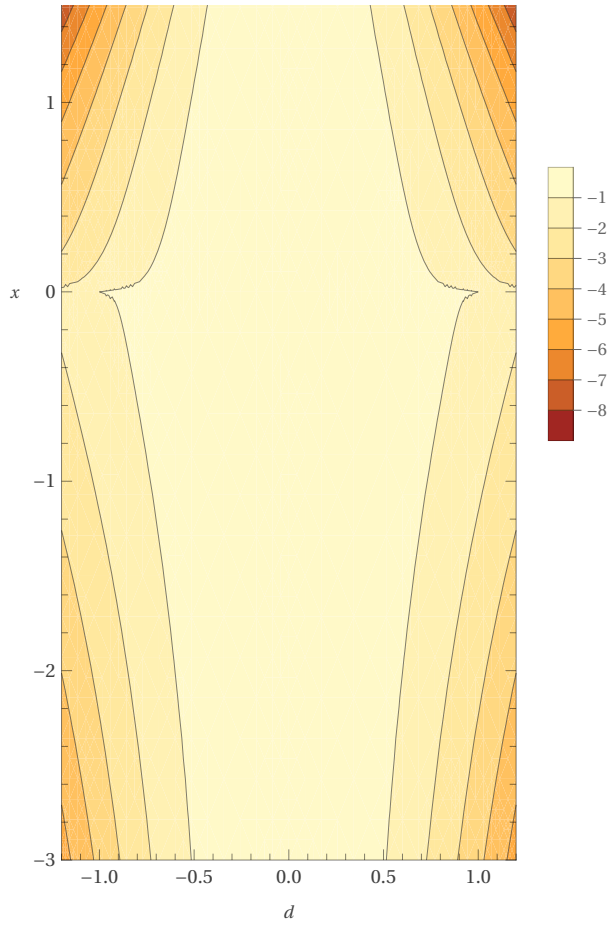
Imaginary part:



Contour plots:

Real part:

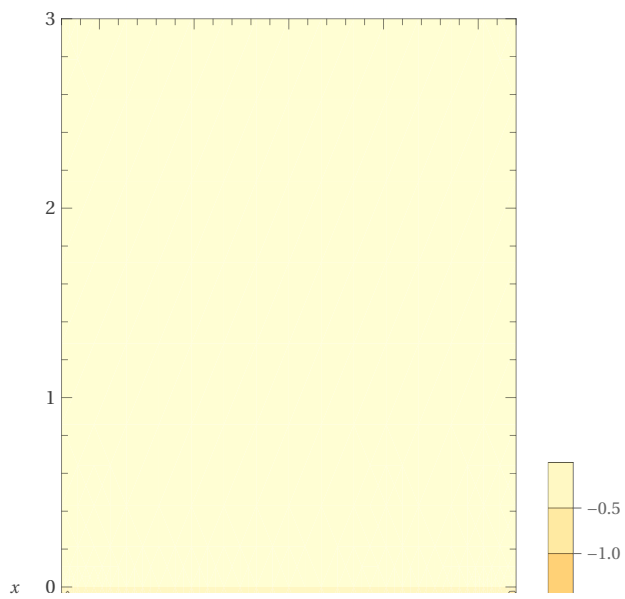


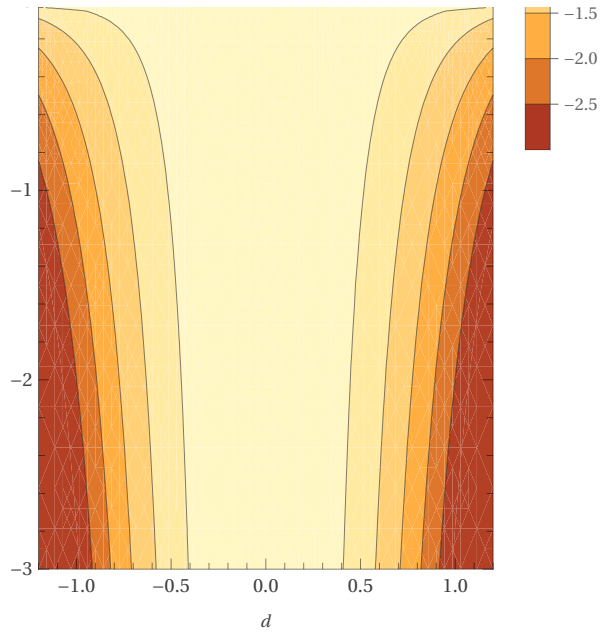


d_{\min} d_{\max}

x_{\min} x_{\max}

Imaginary part:





d_{\min} d_{\max}
 x_{\min} x_{\max}

Alternate forms: +

$$-d^2(-x + \sqrt{x} + \sqrt[3]{x} + e^x)$$

$$d^2(x - \sqrt{x} - \sqrt[3]{x} - e^x)$$

$$d^2(x^{2/3} - \sqrt[6]{x} - 1)\sqrt[3]{x} - d^2 e^x$$

Real root: +

$$d = 0, \quad x \geq 0$$

Property as a real function: +

Domain:

$\{x \in \mathbb{R} : x \geq 0\}$ (all non-negative real numbers)

R is the set of real numbers »

Series expansion at $x = 0$: +

$$-d^2 - d^2 \sqrt[3]{x} - d^2 \sqrt{x} - \frac{d^2 x^2}{2} - \frac{d^2 x^3}{6} - \frac{d^2 x^4}{24} + O(x^5)$$

(Puiseux series)

Big-O notation »

Derivative:

Step-by-step solution +

$$d \cdot \dots \quad d^2(6x^{2/3}(e^x - 1) + 3\sqrt[3]{x} + 2)$$

$$\frac{d^2}{dx} (d^2 x - d^2 e^x - d^2 \sqrt[3]{x} - d^2 \sqrt{x}) = -\frac{6x^{2/3}}{6x^{2/3}}$$

Indefinite integral:

[Step-by-step solution](#) 

$$\int (-d^2 e^x - d^2 \sqrt[3]{x} - d^2 \sqrt{x} + d^2 x) dx = -\frac{1}{12} d^2 (9x^{4/3} + 8x^{3/2} - 6x^2 + 12e^x) + \text{constant}$$

WolframAlpha In[12]:= **Simplify**[-d² e^x - d² x^{1/3} - d² √x + d² x]

Out[12]=

$$-d^2 (-x + \sqrt{x} + \sqrt[3]{x} + e^x)$$


$$\boxed{d^2 x / dt^2 - d^2 e^x / dt^2 - d^2 x^{1/3} / dt^2 - d^2 \sqrt{x} / dt^2}$$

Eq 5

In[14]:= D[x, {t, 2}] - d² * (E^x / D[t² (2 - d² * (x^{1/3}) / dt²) - d² * Sqrt[x], t]²)

Out[14]=

$$\frac{d^2 e^x t^{d^2 \sqrt[3]{x} (2/dt^2) + 2d^2 \sqrt{x} - 2}}{(d^2 \sqrt[3]{x} (-1/dt^2) - d^2 \sqrt{x} + 2)^2}$$

In[15]:=  **d² x / dt² - d² e^x / dt² - d² x^{1/3} / dt² - d² √x / dt²**

Out[15]=

Input interpretation: 

$$x''(t) - d^2 \times \frac{e^x}{\left(\frac{\partial t^{2-d^2 \times \sqrt[3]{x}} / dt^2 \text{ (per metric deciton squared)} - d^2 \sqrt{x}}{\partial t} \right)^2}$$

Result: 

$$x''(t) - \frac{d^2 e^x t^{d^2 \sqrt[3]{x} 2/dt^2 \text{ (per metric decitons squared)} + 2d^2 \sqrt{x} - 2}}{(d^2 \sqrt[3]{x} - 1/dt^2 \text{ (per metric deciton squared)} - d^2 \sqrt{x} + 2)^2}$$

WolframAlpha 

$$\{d^2x/d^2t\}-\{d^2e^x/d^2t\}-\{d^2x^{1/3}/d^2t\}-\{d^2\sqrt{x}/d^2t\} \quad \text{Eq 5}$$

In[16]:= $d^2*(x/d^2)*t - d^2*(E^x/d^2)*t - d^2*(x^{1/3}/d^2)*t - d^2*(Sqrt[x]/d^2)*t$

Out[16]= $t(-e^x) + tx - t\sqrt{x} - t\sqrt[3]{x}$

In[18]:= **⚡** $\{d^2x/d^2t\}-\{d^2e^x/d^2t\}-\{d^2x^{1/3}/d^2t\}-\{d^2\sqrt{x}/d^2t\}$

Out[18]=

Input: +

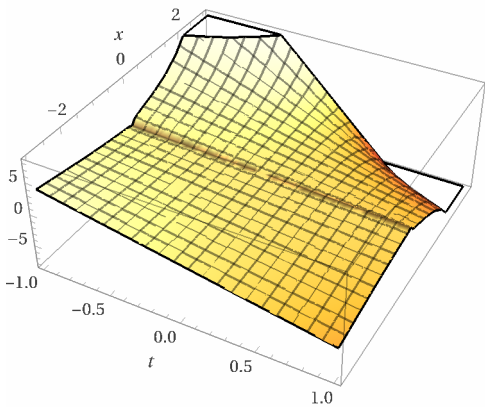
$$d^2 \times \frac{x}{d^2} t - d^2 \times \frac{e^x}{d^2} t - d^2 \times \frac{\sqrt[3]{x}}{d^2} t - d^2 \times \frac{\sqrt{x}}{d^2} t$$

Result: +

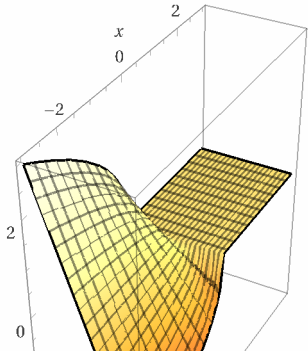
$$t(-e^x) + tx - t\sqrt{x} - t\sqrt[3]{x}$$

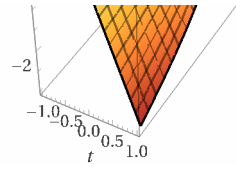
3D plots: +

Real part:



Imaginary part:

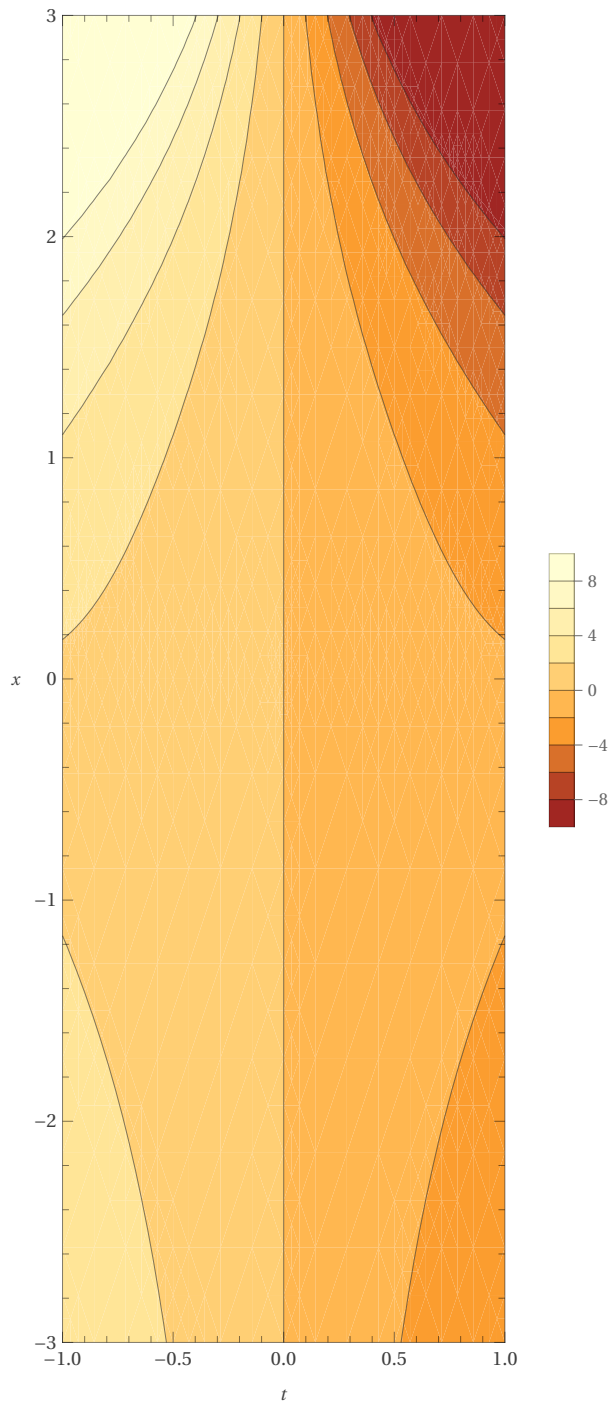




Contour plots:

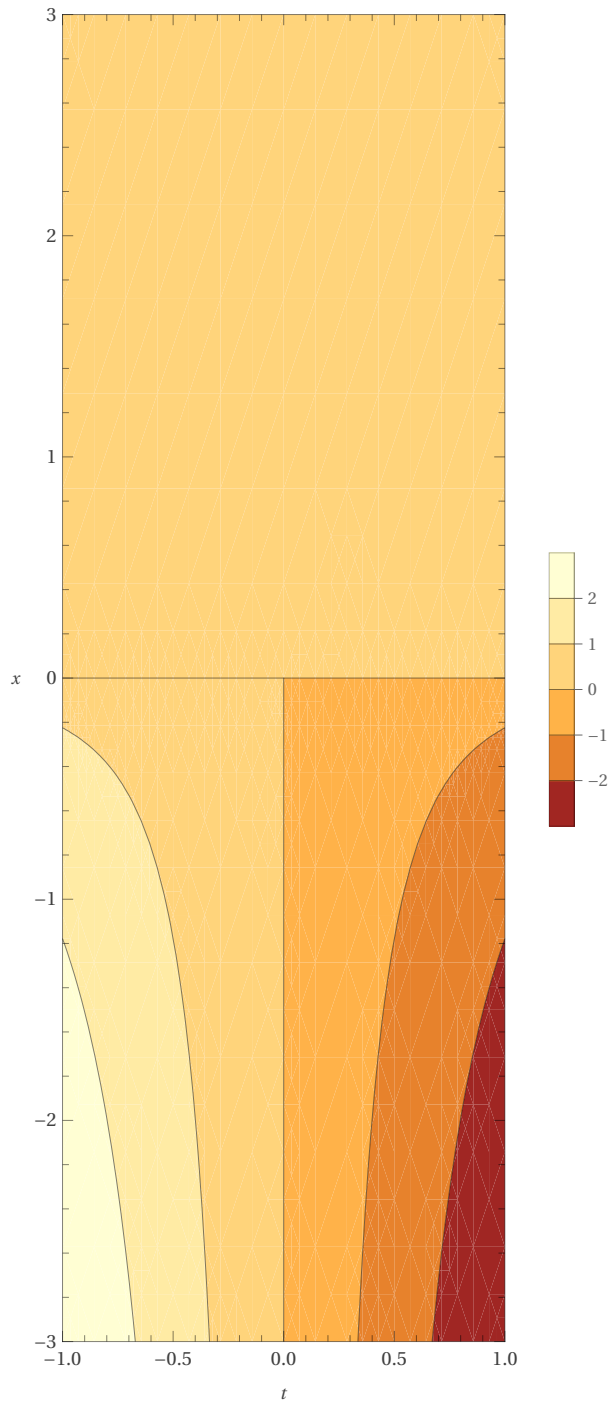


Real part:



t_{\min} t_{\max}
 x_{\min} x_{\max}

Imaginary part:



t_{\min} t_{\max}
 x_{\min} x_{\max}

Alternate forms: +

$$-t(-x + \sqrt{x} + \sqrt[3]{x} + e^x)$$

$$t(x - \sqrt{x} - \sqrt[3]{x} - e^x)$$

$$t(x^{2/3} - \sqrt[6]{x} - 1) \sqrt[3]{x} - t e^x$$

Real root: +

$$t = 0, \quad x \geq 0$$

Series expansion at $x = 0$: +

$$-t - t \sqrt[3]{x} - t \sqrt{x} - \frac{t x^2}{2} - \frac{t x^3}{6} - \frac{t x^4}{24} + O(x^5)$$

(Puiseux series)

[Big-O notation »](#)Derivative: Step-by-step solution +

$$\frac{\partial}{\partial x} \left(\frac{d^2 x t}{d^2} - \frac{d^2 e^x t}{d^2} - \frac{d^2 \sqrt[3]{x} t}{d^2} - \frac{d^2 \sqrt{x} t}{d^2} \right) = \frac{1}{6} t \left(-\frac{2}{x^{2/3}} - 6 e^x - \frac{3}{\sqrt{x}} + 6 \right)$$

Indefinite integral: Step-by-step solution +

$$\int (-e^x t - t \sqrt[3]{x} - t \sqrt{x} + t x) dx = -\frac{1}{12} t (9 x^{4/3} + 8 x^{3/2} - 6 x^2 + 12 e^x) + \text{constant}$$

WolframAlpha +

In[17]:= Simplify[-e^x t - t x^{1/3} - t \sqrt{x} + t x]

Out[17]=

$$-t(-x + \sqrt{x} + \sqrt[3]{x} + e^0)$$

$$\boxed{\{d^2 x / d^2 t\} - \{d^2 \pi^x / d^2 t\} - \{d^2 x^{1/3} / d^2 t\} - \{d^2 \sqrt{x} / d^2 t\}} \quad \text{Eq 6}$$

In[1]=

$$d^2 * (x / d^2) * t - d^2 * (\pi^x / d^2) * t - d^2 * (x^{1/3} / d^2) * t - d^2 * (\text{Sqrt}[x] / d^2) * t$$

Out[1]= $t(-\pi^0) + t x - t \sqrt{x} - t \sqrt[3]{x}$

In[2]=

$$\{d^2 x / d^2 t\} - \{d^2 \pi^x / d^2 t\} - \{d^2 x^{1/3} / d^2 t\} - \{d^2 \sqrt{x} / d^2 t\}$$

Input: +

$$x \quad \pi^x \quad \sqrt[3]{x} \quad \sqrt{x}$$

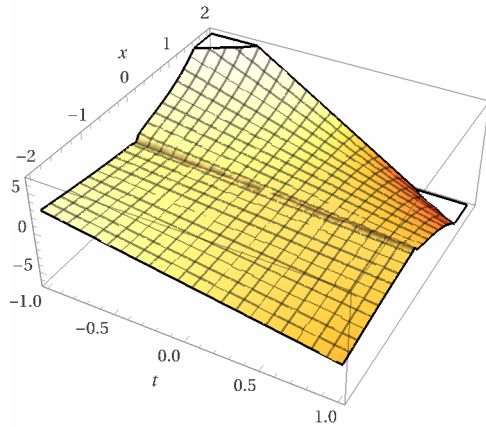
$$d^2 \times \frac{\sim}{d^2} t - d^2 \times \frac{\wedge}{d^2} t - d^2 \times \frac{\vee \sim}{d^2} t - d^2 \times \frac{\vee \sim}{d^2} t$$

Result:

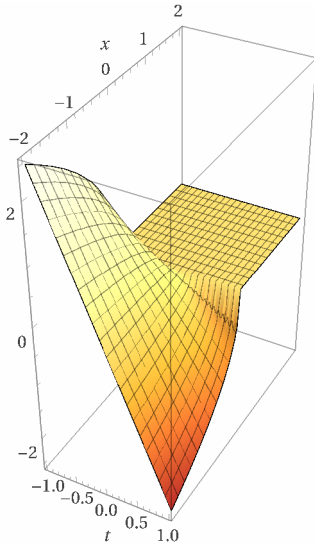
$$t(-\pi^x) + tx - t\sqrt{x} - t\sqrt[3]{x}$$

3D plots:

Real part:

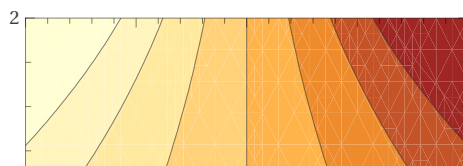


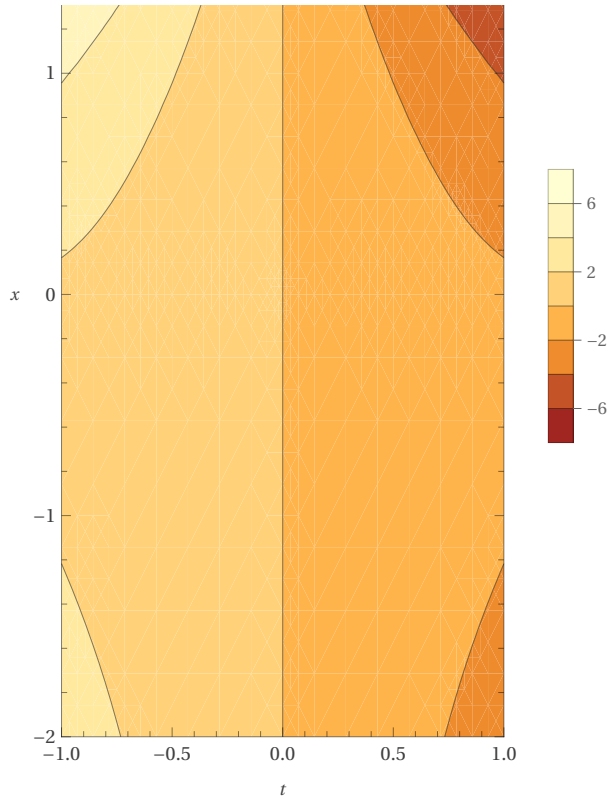
Imaginary part:



Contour plots:

Real part:

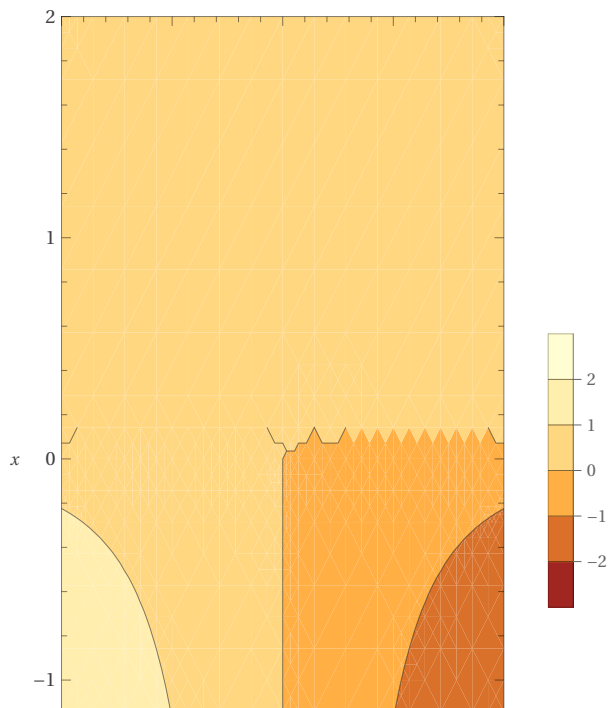


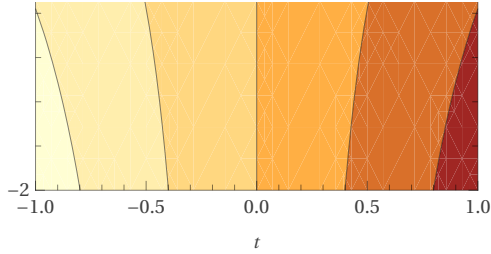


Out[2]=

t_{\min} t_{\max}
 x_{\min} x_{\max}

Imaginary part:





t_{\min} t_{\max}
 x_{\min} x_{\max}

Alternate forms: +

$$-t(-x + \sqrt{x} + \sqrt[3]{x} + \pi^x)$$

$$t(x - \sqrt{x} - \sqrt[3]{x} - \pi^x)$$

Real root: +

$$t = 0, \quad x \geq 0$$

Series expansion at $x = 0$: +

$$\begin{aligned}
 & -t - t \sqrt[3]{x} - t \sqrt{x} + x(t - t \log(\pi)) - \frac{1}{2} x^2 (t \log^2(\pi)) - \\
 & \frac{1}{6} x^3 (t \log^3(\pi)) - \frac{1}{24} x^4 (t \log^4(\pi)) - \frac{1}{120} x^5 (t \log^5(\pi)) + O(x^{16/3})
 \end{aligned}$$

(Puiseux series)

[log\(x\) is the natural logarithm »](#)

[Big-O notation »](#)

Derivative:

[Approximate form](#)

[Step-by-step solution](#) +

$$\frac{\partial}{\partial x} \left(\frac{d^2 x t}{d^2} - \frac{d^2 \pi^x t}{d^2} - \frac{d^2 \sqrt[3]{x} t}{d^2} - \frac{d^2 \sqrt{x} t}{d^2} \right) = \frac{1}{6} t \left(-\frac{2}{x^{2/3}} - \frac{3}{\sqrt{x}} - 6 \pi^x \log(\pi) + 6 \right)$$

Indefinite integral:

[Approximate form](#)

[Step-by-step solution](#) +

$$\int (-\pi^x t - t \sqrt[3]{x} - t \sqrt{x} + t x) dx = -t \left(\frac{3 x^{4/3}}{4} + \frac{2 x^{3/2}}{3} - \frac{x^2}{2} + \frac{\pi^x}{\log(\pi)} \right) + \text{gsrwerx}$$

WolframAlpha +

$$\{d^2 x / dt^2\} - \{d^2 \pi^x / dt^2\} - \{d^2 x^{1/3} / dt^2\} - \{d^2 \sqrt{x} / dt^2\}$$

Eq 7

$$\text{In[3]= } D[x, \{t, 2\}] - D[\pi^x, \{t, 2\}] - d^2 * (x^{1/3} / dt^2) - D[\text{Sqrt}[x], \{t, 2\}]$$

$$\text{Out[3]= } d^2 \sqrt[3]{x} (-1/dt^2)$$

$$\text{In[4]= } \{d^2 x/dt^2\} - \{d^2 \pi^x/dt^2\} - \{d^2 x^{1/3}/dt^2\} - \{d^2 \sqrt{x}/dt^2\}$$

Input interpretation: +

$$x''(t) - \frac{\partial^2 \pi^x}{\partial t^2} - d^2 \times \frac{\sqrt[3]{x}}{dt^6 \text{ (metric deciton squared)}} - \frac{\partial^2 \sqrt{x}}{\partial t^2}$$

Out[4]=

Result: +

$$d^2 \sqrt[3]{x} - 1/dt^6 \text{ (per metric deciton squared)} + x''(t)$$

WolframAlpha +

$$\{d^2 x/dt^2\} - \{d^2 e^x/dt^2\} - \{d^2 x^{1/3}/dt^2\} - \{d^2 \sqrt{x}/dt^2\}$$

Eq 8

$$\text{In[5]= } D[x, \{t, 2\}] - D[E^x, \{t, 2\}] - d^2 * (x^{1/3} / dt^2) - D[\text{Sqrt}[x], \{t, 2\}]$$

$$\text{Out[5]= } d^2 \sqrt[3]{x} (-1/dt^2)$$

$$\text{In[6]= } \{d^2 x/dt^2\} - \{d^2 e^x/dt^2\} - \{d^2 x^{1/3}/dt^2\} - \{d^2 \sqrt{x}/dt^2\}$$

Input interpretation: +

$$x''(t) - \frac{\partial^2 e^x}{\partial t^2} - d^2 \times \frac{\sqrt[3]{x}}{dt^2 \text{ (metric deciton squared)}} - \frac{\partial^2 \sqrt{x}}{\partial t^2}$$

Out[6]=

Result: +

$$d^2 \sqrt[3]{x} - 1/dt^2 \text{ (per metric deciton squared)} + x''(t)$$


WolframAlpha +

$$\{d^2 x/dt^2\} + \{d^2 e^x/dt^2\} - \{d^2 x^{1/3}/dt^2\} - \{d^2 \sqrt{x}/dt^2\}$$

Eq 9

$$\text{In[7]= } D[x, \{t, 2\}] + D[E^x, \{t, 2\}] - d^2 * (x^{1/3} / dt^2) - D[\text{Sqrt}[x], \{t, 2\}]$$

$$\text{Out[7]= } d^2 \sqrt[3]{x} (-1/dt^2)$$

In[10]:=  $\{d^2 x / dt^2\} + \{d^2 e^x / dt^2\} - \{d^2 x^{1/3} / dt^2\} - \{d^2 \sqrt{x} / dt^2\}$

Out[10]=

Input interpretation: +

$$x''(t) + \frac{\partial^2 e^x}{\partial t^2} - d^2 \times \frac{\sqrt[3]{x}}{dt^6 \text{ (metric deciton squared)}} - \frac{\partial^2 \sqrt{x}}{\partial t^2}$$

Result: +

$$d^2 \sqrt[3]{x} - 1/dt^6 \text{ (per metric deciton squared)} + x''(t)$$

WolframAlpha +

In[8]:= $d \left(d \ x / \text{Quantity}[-1, \frac{1}{\text{"MetricDecitons"}}] \right)$

Out[8]= $\frac{d^2 \left(-\frac{1}{3} / dt^2 \right)}{x^{2/3}}$

In[9]:= $\text{Numerator} \left[\frac{d \ \text{Quantity}[-, \frac{0BQOF@ B@FQLKP}{x /}]}{x /} \right]$

Out[9]= $d^2 \left(-\frac{1}{3} / dt^2 \right)$

In[11]:= $d_A \left(d \ \text{Quantity}[-\frac{1}{3}, \frac{1}{\text{"MetricDecitons"}}] \right)$

Out[11]=

$d \left(-\frac{2}{3} / dt^2 \right)$

In[12]:= $\text{Solve} \left[d \ \text{Quantity}[-\frac{2}{3}, \frac{1}{\text{"MetricDecitons"}^2}] == 0, d \right]$

Out[12]=

$\{ \{ d \rightarrow 0 \text{ kg} \} \}$

In[13]:= $\{ \{ d \rightarrow \text{Quantity}[0, \text{"Kilograms"}] \} \} /. \text{Rule} \rightarrow \text{Equal}$

Out[13]=

$(d = 0 \text{ kg})$

In[14]:= $\text{Flatten}[\{ \{ d == \text{Quantity}[0, \text{"Kilograms"}] \} \}]$

Out[14]=

$\{ d = 0 \text{ kg} \}$

$$\boxed{\{d^2x/d^2t\}+\{d^2e^x/d^2t\}-\{d^2x^{1/3}/d^2t\}-\{d^2\sqrt{x}/d^2t\}} \quad \text{Eq 10}$$

$$\text{In[1]:= } d^2*(x/d^2)*t + d^2*(E^x/d^2)*t - d^2*(x^{(1/3)}/d^2)*t - d^2*(\text{Sqrt}[x]/d^2)*t$$

$$\text{Out[1]= } t e^x + t x - t \sqrt{x} - t \sqrt[3]{x}$$

$$\text{In[3]= } \star \{d^2x/d^2t\} + \{d^2e^x/d^2t\} - \{d^2x^{1/3}/d^2t\} - \{d^2\sqrt{x}/d^2t\}$$

Input:

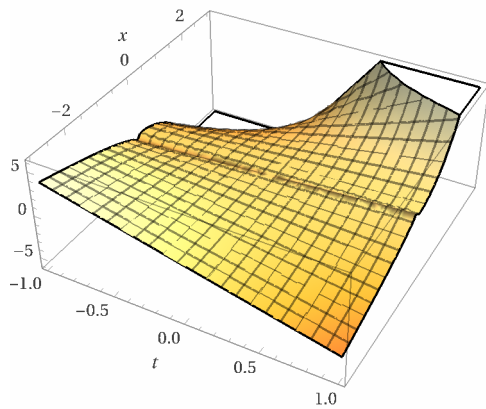
$$d^2 \times \frac{x}{d^2} t + d^2 \times \frac{e^x}{d^2} t - d^2 \times \frac{\sqrt[3]{x}}{d^2} t - d^2 \times \frac{\sqrt{x}}{d^2} t$$

Result:

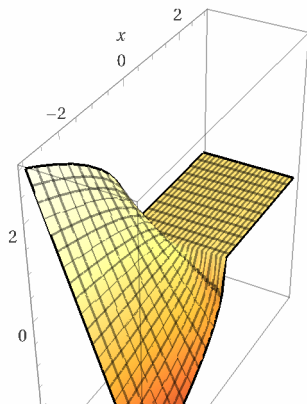
$$t e^x + t x - t \sqrt{x} - t \sqrt[3]{x}$$

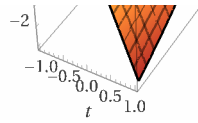
3D plots:

Real part:



Imaginary part:

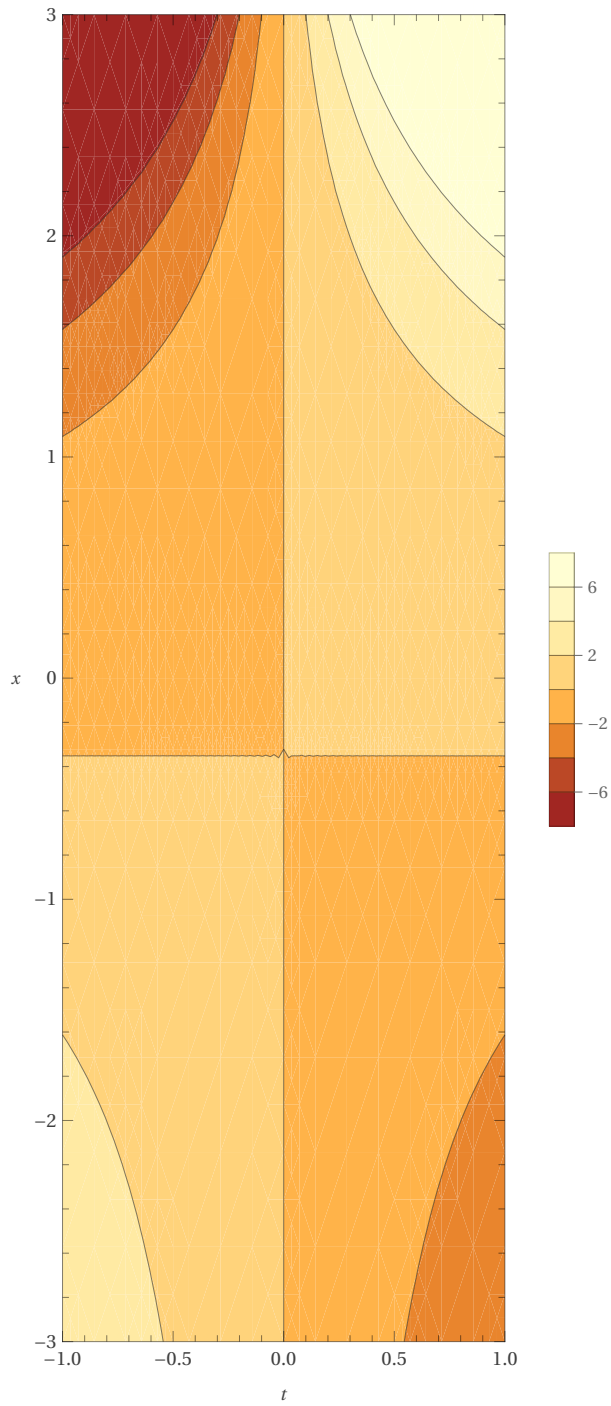




Contour plots:



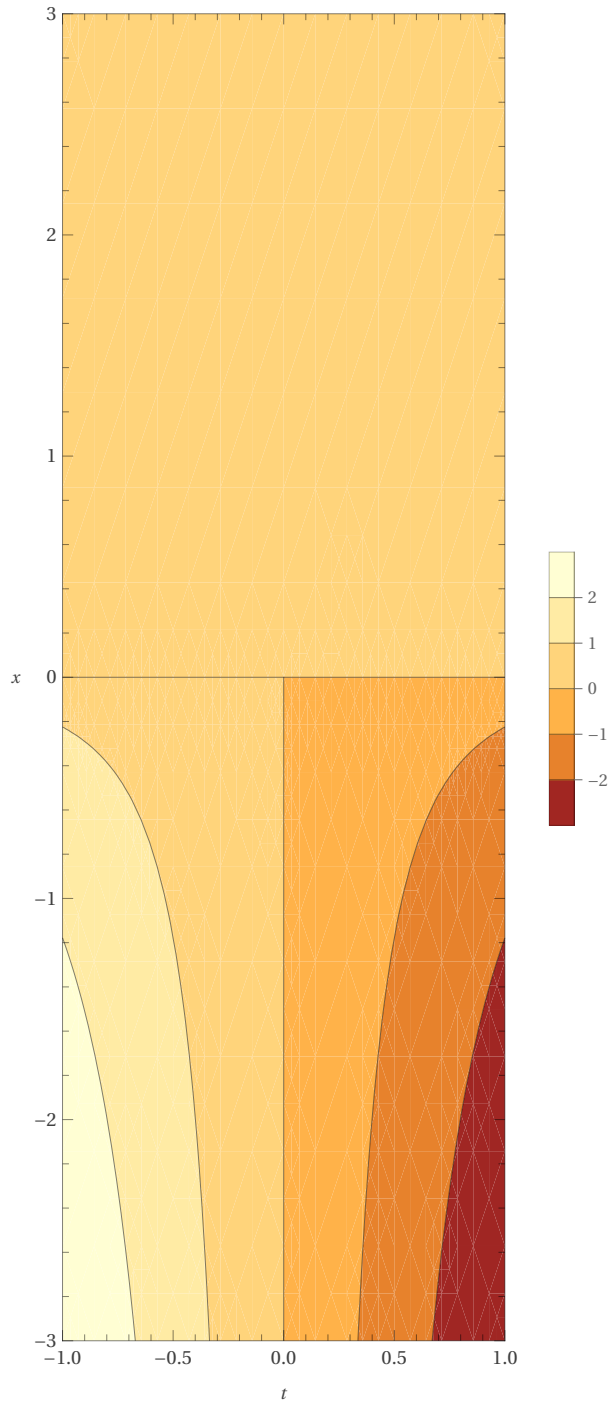
Real part:



Out[3]=

τ_{\min} τ_{\max}
 X_{\min} X_{\max}

Imaginary part:



t_{\min} t_{\max}
 X_{\min} X_{\max}

Alternate forms: +

$$t(x - \sqrt{x} - \sqrt[3]{x} + e^x)$$

$$t(x^{2/3} - \sqrt[6]{x} - 1) \sqrt[3]{x} + t e^x$$

Real root: +

$$t = 0, \quad x \geq 0$$

Series expansion at $x = 0$: +

$$t - t \sqrt[3]{x} - t \sqrt{x} + 2tx + \frac{tx^2}{2} + \frac{tx^3}{6} + \frac{tx^4}{24} + O(x^5)$$

(Puiseux series)

[Big-O notation »](#)Derivative: Step-by-step solution +

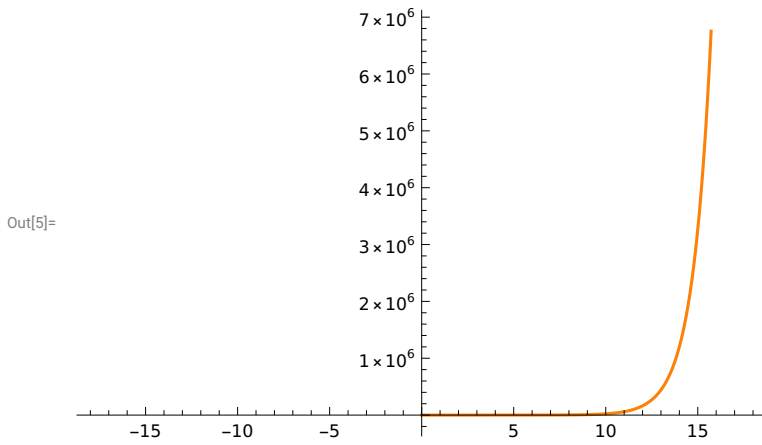
$$\frac{\partial}{\partial x} \left(\frac{d^2 x t}{d^2} + \frac{d^2 e^x t}{d^2} - \frac{d^2 \sqrt[3]{x} t}{d^2} - \frac{d^2 \sqrt{x} t}{d^2} \right) = \frac{1}{6} t \left(-\frac{2}{x^{2/3}} + 6e^x - \frac{3}{\sqrt{x}} + 6 \right)$$

Indefinite integral: Step-by-step solution +

$$\int (e^x t - t \sqrt[3]{x} - t \sqrt{x} + tx) dx = t \left(\frac{1}{12} (-9x^{4/3} - 8x^{3/2} + 6x^2) + e^x \right) + \text{gsrwerx}$$

WolframAlpha +In[2]:= `Simplify[$e^x t - t x^{1/3} - t \sqrt{x} + t x$]`Out[2]= $t(x - \sqrt{x} - \sqrt[3]{x} + e^x)$ In[4]:= `$\partial_t(t(e^x - x^{1/3} - \sqrt{x} + x))$` Out[4]= $x - \sqrt{x} - \sqrt[3]{x} + e^x$

In[5]:= Plot[e^x - x^{1/3} - √x + x, {x, -18., 18.}]



$$\{d^2x/d^2t\} + \{d^2\pi^x/d^2t\} - \{d^2x^{1/3}/d^2t\} - \{d^2\sqrt{x}/d^2t\} \quad \text{Eq 11}$$

Out[5]= $d^2*(x/d^2)*t + d^2*(\text{Pi}^x/d^2)*t - d^2*(x^{1/3}/d^2)*t - d^2*(\text{Sqrt}[x]/d^2)*t$

Out[6]= $t\pi^x + tx - t\sqrt{x} - t\sqrt[3]{x}$

In[7]:= $\{d^2x/d^2t\} + \{d^2\pi^x/d^2t\} - \{d^2x^{1/3}/d^2t\} - \{d^2\sqrt{x}/d^2t\}$

Input: +

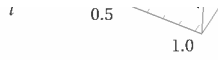
$$d^2 \times \frac{x}{d^2} t + d^2 \times \frac{\pi^x}{d^2} t - d^2 \times \frac{\sqrt[3]{x}}{d^2} t - d^2 \times \frac{\sqrt{x}}{d^2} t$$

Result: +

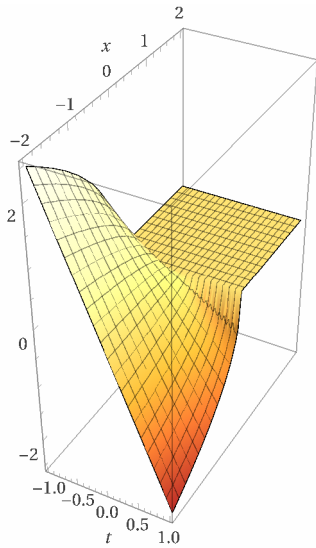
$$t\pi^x + tx - t\sqrt{x} - t\sqrt[3]{x}$$

3D plots: +

Real part:



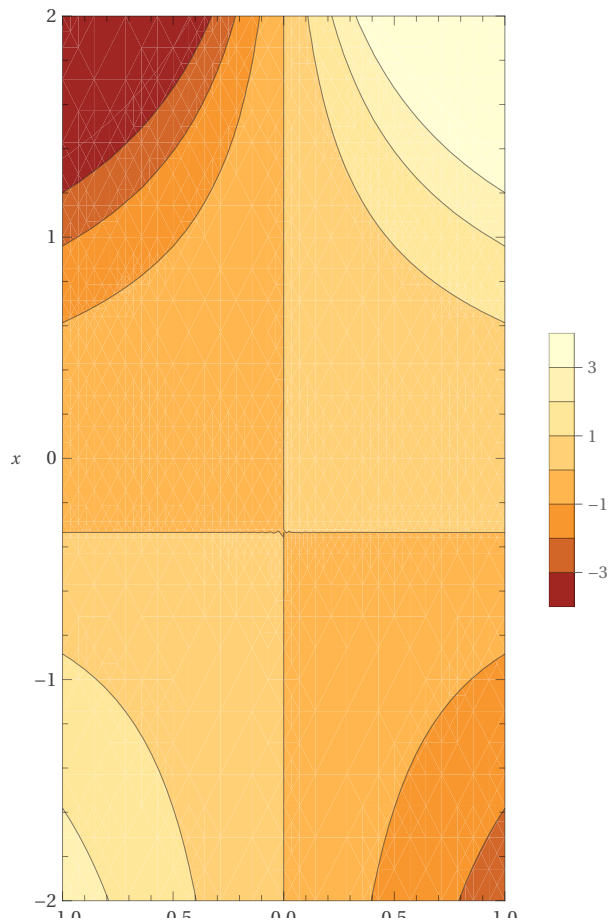
Imaginary part:



Contour plots:



Real part:

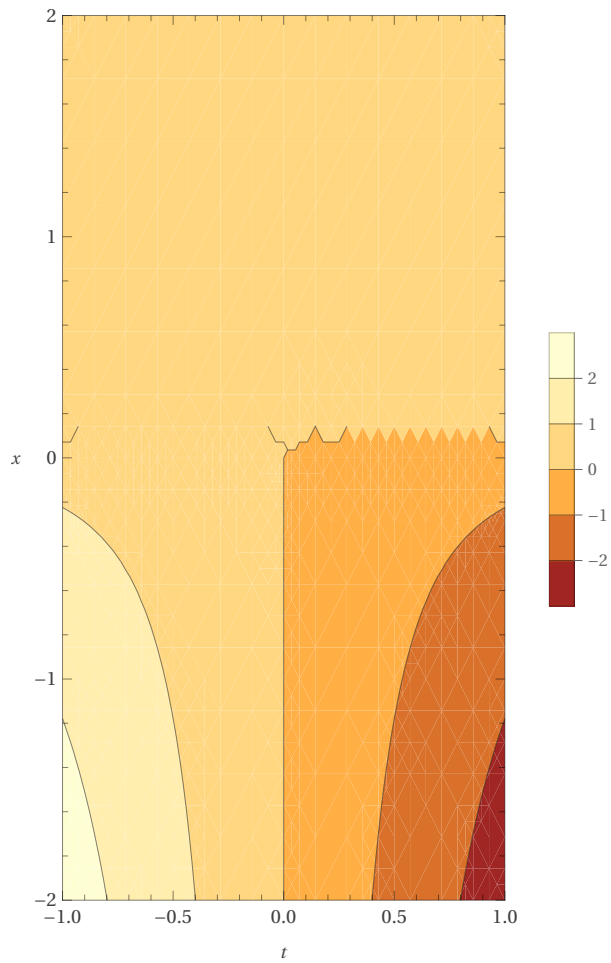


Out[7]=

-1.0 -0.5 0.0 0.5 1.0
 t

t_{\min} t_{\max}
 x_{\min} x_{\max}

Imaginary part:



t_{\min} t_{\max}
 x_{\min} x_{\max}

Alternate form: +

$$t(x - \sqrt{x} - \sqrt[3]{x} + \pi^x)$$

Real root: +

$$t = 0, \quad x \geq 0$$

Series expansion at $x = 0$: +

$$t - t \sqrt[3]{x} - t \sqrt{x} + t x(1 + \log(\pi)) + \frac{1}{2} t x^2 \log^2(\pi) + \frac{1}{6} t x^3 \log^3(\pi) + \frac{1}{24} t x^4 \log^4(\pi) + \frac{1}{120} t x^5 \log^5(\pi) + O(x^{16/3})$$

(Puiseux series)

log(x) is the natural logarithm »

Big-O notation »

Derivative:

Approximate form

Step-by-step solution +

$$\frac{\partial}{\partial x} \left(\frac{d^2 x t}{d^2} + \frac{d^2 \pi^x t}{d^2} - \frac{d^2 \sqrt[3]{x} t}{d^2} - \frac{d^2 \sqrt{x} t}{d^2} \right) = \frac{1}{6} t \left(-\frac{2}{x^{2/3}} - \frac{3}{\sqrt{x}} + 6 \pi^x \log(\pi) + 6 \right)$$

Indefinite integral:

Approximate form

Step-by-step solution +

$$\int (\pi^x t - t \sqrt[3]{x} - t \sqrt{x} + t x) dx = t \left(-\frac{3 x^{4/3}}{4} - \frac{2 x^{3/2}}{3} + \frac{x^2}{2} + \frac{\pi^x}{\log(\pi)} \right) + \text{gsrwerx}$$

WolframAlpha +

simplify{d^2x/d^2t}+{d^2π^x/d^2t}-{d^2x^1/3/d^2t}-{d^2√x/d^2t}

In[8]:= Simplify[d^2*(x/d^2)*t + d^2*(Pi^x/d^2)*t - d^2*(x^(1/3)/d^2)*t - d^2*(Sqrt[x]/d^2)*t]

Out[8]= $t(x - \sqrt{x} - \sqrt[3]{x} + \pi^x)$

In[11]:= **simplify{d^2x/d^2t}+{d^2π^x/d^2t}-{d^2x^1/3/d^2t}-{d^2√x/d^2t}**

Out[11]=

Input interpretation: +

simplify $d^2 \times \frac{x}{d^2} t + d^2 \times \frac{\pi^x}{d^2} t - d^2 \times \frac{\sqrt[3]{x}}{d^2} t - d^2 \times \frac{\sqrt{x}}{d^2} t$

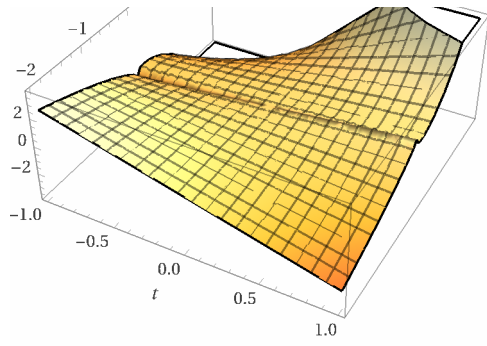
Result: +

$t(x - \sqrt{x} - \sqrt[3]{x} + \pi^x)$

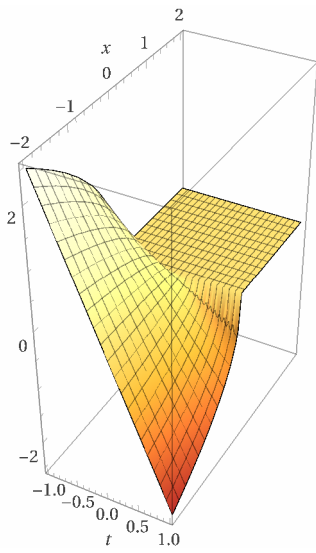
3D plots: +

Real part:



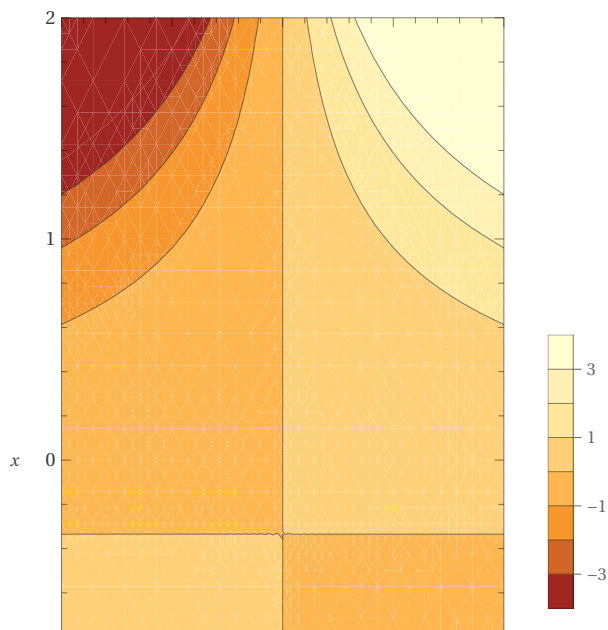


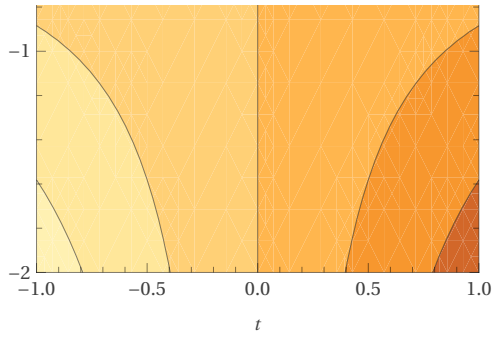
Imaginary part:



Contour plots:

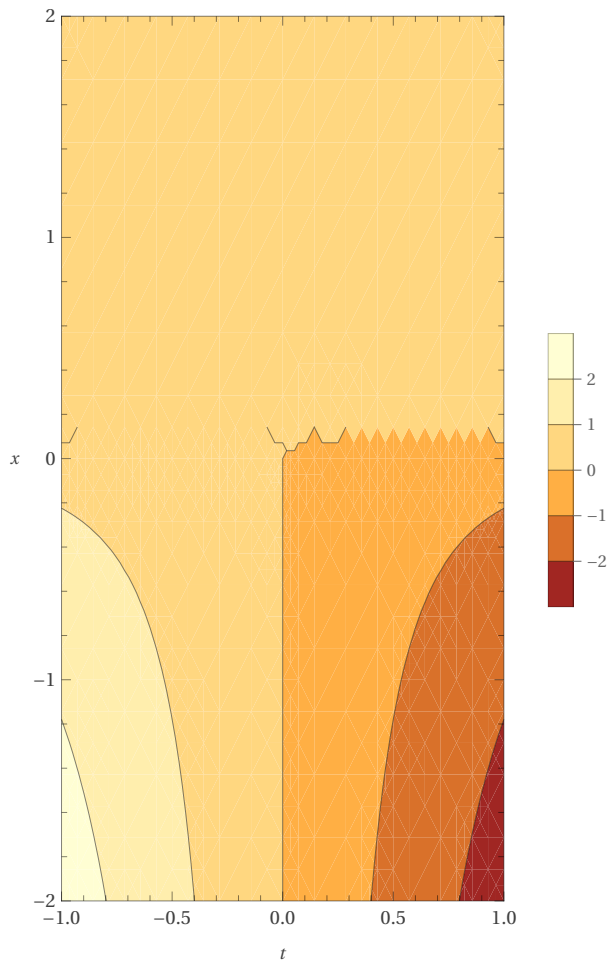
Real part:





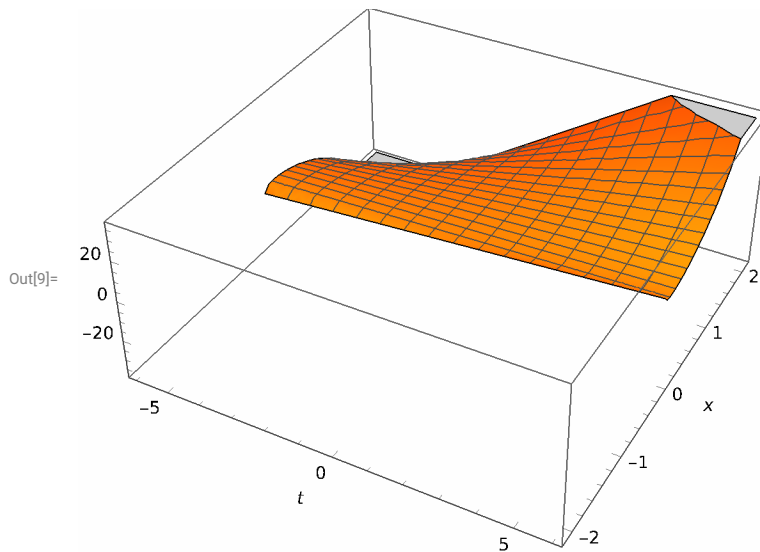
t_{\min} t_{\max}
 x_{\min} x_{\max}

Imaginary part:



t_{\min} t_{\max}
 x_{\min} x_{\max}

In[9]:= `Plot3D[t (πx - x1/3 - √x + x), {t, -6., 6.}, {x, -2.10603, 2.10603}]`



`⊞ simplify{d2x/d2t}+{d2ex/d2t}-{d2x1/3/d2t}-{d2√x/d2t}` Eq 10

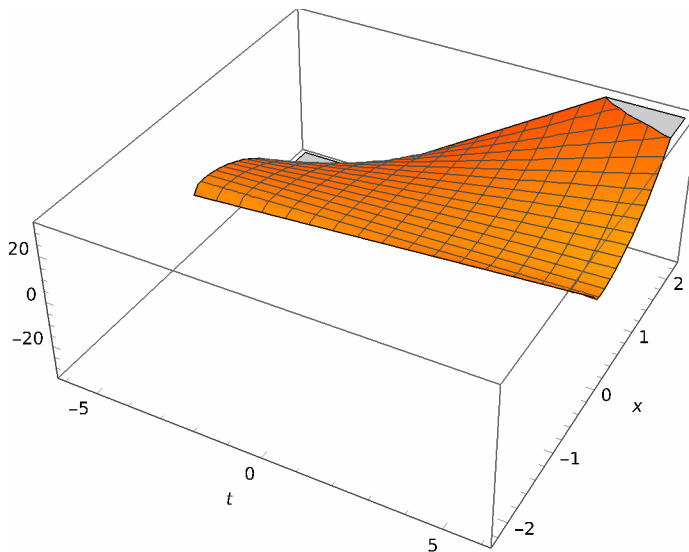
In[10]:= `Simplify[d2*(x/d2)*t + d2*(Ex/d2)*t - d2*(x(1/3)/d2)*t - d2*(Sqrt[x]/d2)*t]`

Out[10]=

$$t(x - \sqrt{x} - \sqrt[3]{x} + e^x)$$

In[12]:= `Plot3D[t (ex - x1/3 - √x + x), {t, -6., 6.}, {x, -2.24762, 2.24762}]`

Out[12]=



$$\text{Eq 12} \quad \frac{d^2 x/d^2 t + \{d^2 e^x/d^2 t\}}{[d^2 x^{1/3}/d^2 t] + \{d^2 \sqrt{x}/d^2 t\}}$$

In[1]:= $d^2*(x/d^2)*t + (d^2*(E^x/d^2)*t)/(d^2*(x^(1/3)/d^2)*t + d^2*(Sqrt[x]/d^2)*t)$

Out[1]= $tx + \frac{te^x}{t\sqrt{x} + t\sqrt[3]{x}}$

In[4]:= **✶** $\frac{[d^2 x/d^2 t + \{d^2 e^x/d^2 t\}]}{[\{d^2 x^{1/3}/d^2 t\} + \{d^2 \sqrt{x}/d^2 t\}]}$

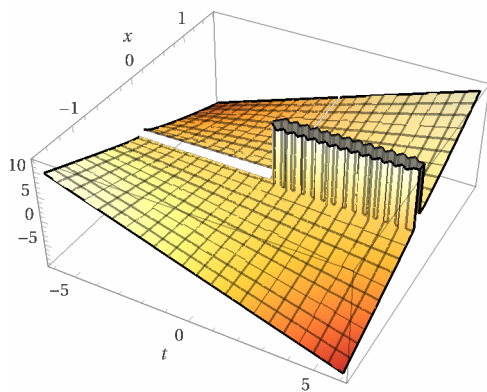
An attempt was made to fix mismatched parentheses, brackets, or braces.

Input: $d^2 \times \frac{x}{d^2} t + \frac{d^2 \times \frac{e^x}{d^2} t}{d^2 \times \frac{\sqrt[3]{x}}{d^2} t + d^2 \times \frac{\sqrt{x}}{d^2} t}$

Result: $tx + \frac{te^x}{t\sqrt{x} + t\sqrt[3]{x}}$

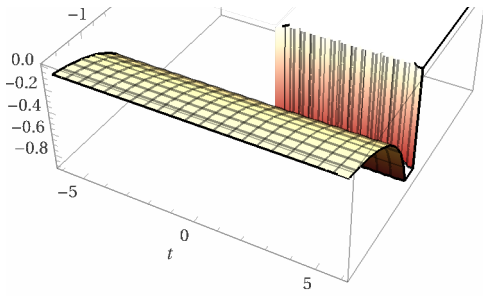
3D plots: **+**

Real part:



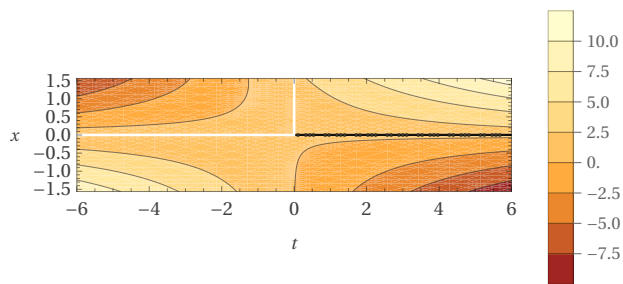
Imaginary part:





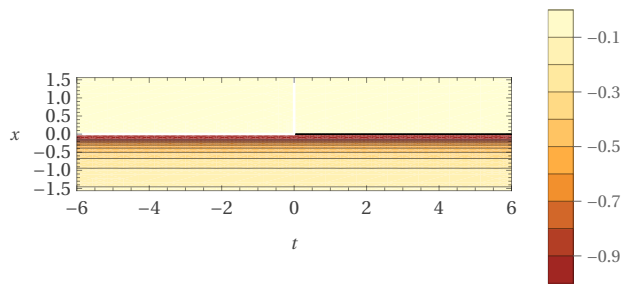
Contour plots:

Real part:



Out[4]=

Imaginary part:



Alternate forms:

More

$$t x + \frac{e^x}{\sqrt{x} + \sqrt[3]{x}}$$

$$e^{-x}$$

$$t x + \frac{e^x}{(\sqrt[6]{x} + 1) \sqrt[3]{x}}$$

$$t \left(\frac{e^x}{\sqrt[3]{x} (t \sqrt[6]{x} + t)} + x \right)$$

Series expansion at x = 0:

$$O\left(\frac{1}{x^7}\right)$$

(Taylor series)

[Big-O notation »](#)

Series expansion at x = ∞:

$$\left(t x + O\left(\left(\frac{1}{x}\right)^6\right) \right) +$$

$$e^x \left(\sqrt{\frac{1}{x}} - \left(\frac{1}{x}\right)^{2/3} + \left(\frac{1}{x}\right)^{5/6} - \frac{1}{x} + \left(\frac{1}{x}\right)^{7/6} - \left(\frac{1}{x}\right)^{4/3} + \left(\frac{1}{x}\right)^{3/2} - \left(\frac{1}{x}\right)^{5/3} + \left(\frac{1}{x}\right)^{11/6} - \left(\frac{1}{x}\right)^2 + \right. \\ \left. \left(\frac{1}{x}\right)^{13/6} - \left(\frac{1}{x}\right)^{7/3} + \left(\frac{1}{x}\right)^{5/2} - \left(\frac{1}{x}\right)^{8/3} + \left(\frac{1}{x}\right)^{17/6} - \left(\frac{1}{x}\right)^3 + \left(\frac{1}{x}\right)^{19/6} - \right. \\ \left. \left(\frac{1}{x}\right)^{10/3} + \left(\frac{1}{x}\right)^{7/2} - \left(\frac{1}{x}\right)^{11/3} + \left(\frac{1}{x}\right)^{23/6} - \left(\frac{1}{x}\right)^4 + \left(\frac{1}{x}\right)^{25/6} - \left(\frac{1}{x}\right)^{13/3} + \right. \\ \left. \left(\frac{1}{x}\right)^{9/2} - \left(\frac{1}{x}\right)^{14/3} + \left(\frac{1}{x}\right)^{29/6} - \left(\frac{1}{x}\right)^5 + \left(\frac{1}{x}\right)^{31/6} - \left(\frac{1}{x}\right)^{16/3} + O\left(\left(\frac{1}{x}\right)^{11/2}\right) \right)$$

[Big-O notation »](#)

Derivative:

[Step-by-step solution](#)

$$\frac{\partial}{\partial x} \left(\frac{d^2 x t}{d^2} + \frac{d^2 e^x t}{d^2 \left(\frac{d^2 \sqrt[3]{x} t}{d^2} + \frac{d^2 \sqrt{x} t}{d^2} \right)} \right) = t + \frac{e^x (6 x^{7/6} + 6 x - 3 \sqrt{x} - 2)}{6 (\sqrt[6]{x} + 1)^2 x^{4/3}}$$

WolframAlpha

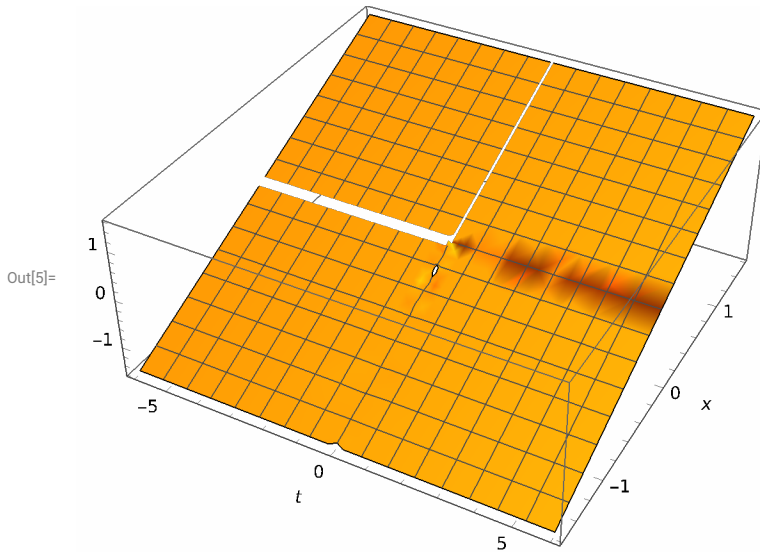
In[2]:= Simplify[$\frac{e^x t}{t x^{1/3} + t \sqrt{x}} + t x$]

Out[2]= $t \left(\frac{e^x}{\sqrt[3]{x} (t \sqrt[6]{x} + t)} + x \right)$

$$\text{In[3]:= } \partial_t \left(t \left(\frac{e^x}{(t + t x^{1/6}) x^{1/3}} + x \right) \right)$$

$$\text{Out[3]:= } -\frac{t e^D (\sqrt[6]{x} + 1)}{\sqrt[3]{x} (t \sqrt[6]{x} + t)^2} + \frac{e^D}{\sqrt[3]{x} (t \sqrt[6]{x} + t)} + x$$

$$\text{In[5]:= } \text{Plot3D}\left[-\frac{e^U t (1 + x /)}{(t + t x /) x /} + \frac{e^U}{(t + t x /) x /} + x, \{t, -6., 6.\}, \{x, -1.56716, 1.56716\}\right]$$



$$\text{Eq 12} \quad \frac{[d^2 x/d^2 t] + [d^2 e^x/d^2 t]}{[d^2 x^{1/3}/d^2 t] + [d^2 \sqrt{x}/d^2 t]}$$

$$\text{In[6]:= } \frac{(d^2 * (x/d^2) * t + d^2 * (E^x/d^2) * t)}{(d^2 * (x^{1/3}/d^2) * t + d^2 * (Sqrt[x]/d^2) * t)}$$

$$\text{Out[6]:= } \frac{t e^x + t x}{t \sqrt{x} + t \sqrt[3]{x}}$$



Input:

$$\frac{d^2 \times \frac{x}{d^2} t + d^2 \times \frac{e^x}{d^2} t}{d^2 \times \frac{\sqrt[3]{x}}{d^2} t + d^2 \times \frac{\sqrt{x}}{d^2} t}$$

Result:

$$\frac{t e^x + t x}{t \sqrt{x} + t \sqrt[3]{x}}$$

Alternate forms: +

$$\frac{x + e^x}{\sqrt{x} + \sqrt[3]{x}}$$

$$\frac{x + e^x}{(\sqrt[6]{x} + 1)\sqrt[3]{x}}$$

$$\frac{x^{2/3}}{\sqrt[6]{x} + 1} + \frac{e^x}{(\sqrt[6]{x} + 1)\sqrt[3]{x}}$$

Expanded form: Step-by-step solution +

$$\frac{tx}{t\sqrt{x} + t\sqrt[3]{x}} + \frac{te^x}{t\sqrt{x} + t\sqrt[3]{x}}$$

Series expansion at $x = 0$: +

$$O\left(\frac{1}{x^7}\right)$$

(Taylor series)

[Big-O notation »](#)Series expansion at $x = \infty$: +

$$e^x \left(\sqrt{\frac{1}{x}} - \left(\frac{1}{x}\right)^{2/3} + \left(\frac{1}{x}\right)^{5/6} - \frac{1}{x} + \left(\frac{1}{x}\right)^{7/6} - \left(\frac{1}{x}\right)^{4/3} + \left(\frac{1}{x}\right)^{3/2} - \left(\frac{1}{x}\right)^{5/3} + \left(\frac{1}{x}\right)^{11/6} - \left(\frac{1}{x}\right)^2 + \left(\frac{1}{x}\right)^{13/6} - \right. \\ \left. \left(\frac{1}{x}\right)^{7/3} + \left(\frac{1}{x}\right)^{5/2} - \left(\frac{1}{x}\right)^{8/3} + \left(\frac{1}{x}\right)^{17/6} - \left(\frac{1}{x}\right)^3 + \left(\frac{1}{x}\right)^{19/6} - \left(\frac{1}{x}\right)^{10/3} + \left(\frac{1}{x}\right)^{7/2} - \left(\frac{1}{x}\right)^{11/3} + \right. \\ \left. \left(\frac{1}{x}\right)^{23/6} - \left(\frac{1}{x}\right)^4 + \left(\frac{1}{x}\right)^{25/6} - \left(\frac{1}{x}\right)^{13/3} + \left(\frac{1}{x}\right)^{9/2} - \left(\frac{1}{x}\right)^{14/3} + \left(\frac{1}{x}\right)^{29/6} - \left(\frac{1}{x}\right)^5 + O\left(\left(\frac{1}{x}\right)^{31/6}\right) \right) + \\ \left(\sqrt{x} - \sqrt[3]{x} + \sqrt[6]{x} - 1 + \sqrt[6]{\frac{1}{x}} - \sqrt[3]{\frac{1}{x}} + \sqrt{\frac{1}{x}} - \left(\frac{1}{x}\right)^{2/3} + \left(\frac{1}{x}\right)^{5/6} - \frac{1}{x} + \left(\frac{1}{x}\right)^{7/6} - \right. \\ \left. \left(\frac{1}{x}\right)^{4/3} + \left(\frac{1}{x}\right)^{3/2} - \left(\frac{1}{x}\right)^{5/3} + \left(\frac{1}{x}\right)^{11/6} - \left(\frac{1}{x}\right)^2 + \left(\frac{1}{x}\right)^{13/6} - \left(\frac{1}{x}\right)^{7/3} + \left(\frac{1}{x}\right)^{5/2} - \right. \\ \left. \left(\frac{1}{x}\right)^{8/3} + \left(\frac{1}{x}\right)^{17/6} - \left(\frac{1}{x}\right)^3 + \left(\frac{1}{x}\right)^{19/6} - \left(\frac{1}{x}\right)^{10/3} + \left(\frac{1}{x}\right)^{7/2} - \left(\frac{1}{x}\right)^{11/3} + \left(\frac{1}{x}\right)^{23/6} - \right. \\ \left. \left(\frac{1}{x}\right)^4 + \left(\frac{1}{x}\right)^{25/6} - \left(\frac{1}{x}\right)^{13/3} + \left(\frac{1}{x}\right)^{9/2} - \left(\frac{1}{x}\right)^{14/3} + \left(\frac{1}{x}\right)^{29/6} - \left(\frac{1}{x}\right)^5 + O\left(\left(\frac{1}{x}\right)^{31/6}\right) \right)$$

[Big-O notation »](#)Derivative: Step-by-step solution +[Step-by-step solution +](#)

Out[7]=

$$\frac{\partial}{\partial x} \left(\frac{\frac{d^2 x t}{d^2} + \frac{d^2 e^x t}{d^2}}{\frac{d^2 \sqrt[3]{x} t}{d^2} + \frac{d^2 \sqrt{x} t}{d^2}} \right) = \frac{e^x (6 x^{7/6} + 6 x - 3 \sqrt[6]{x} - 2) + (3 \sqrt[6]{x} + 4) x}{6 (\sqrt[6]{x} + 1)^2 x^{4/3}}$$

Limit: +

$$\lim_{t \rightarrow -\infty} \frac{e^x t + t x}{t \sqrt[3]{x} + t \sqrt{x}} = \frac{-x - e^x}{-\sqrt{x} - \sqrt[3]{x}} \approx \frac{-x - 2.71828^x}{-\sqrt{x} - \sqrt[3]{x}}$$

$$\lim_{t \rightarrow \infty} \frac{e^x t + t x}{t \sqrt[3]{x} + t \sqrt{x}} = \frac{x + e^x}{\sqrt{x} + \sqrt[3]{x}} \approx \frac{x + 2.71828^x}{\sqrt{x} + \sqrt[3]{x}}$$

WolframAlpha +

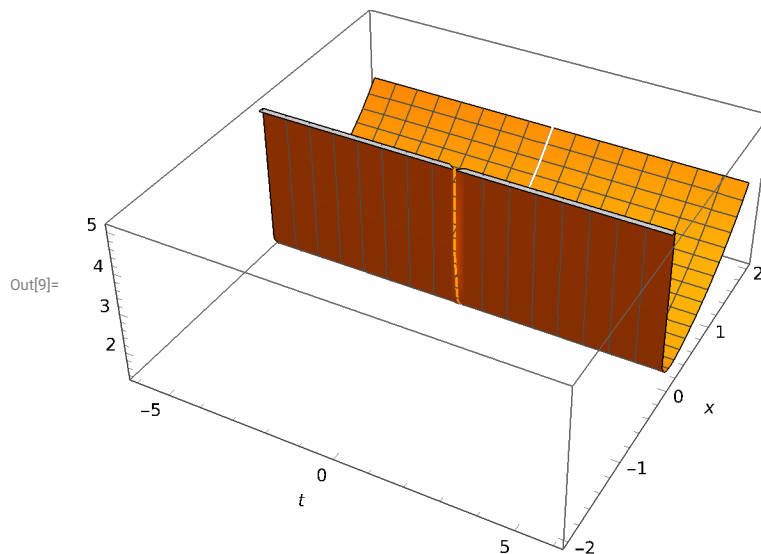
$$\boxed{\frac{\{d^2 x/d^2 t\} + \{d^2 e^x/d^2 t\}}{\{d^2 x^{1/3}/d^2 t\} + \{d^2 \sqrt{x}/d^2 t\}}}$$

Eq 12

In[8]:= $(d^2 * (x/d^2) * t + d^2 * (E^x/d^2) * t) / (d^2 * (x^{1/3}/d^2) * t + d^2 * (Sqrt[x]/d^2) * t)$

Out[8]= $\frac{t e^x + t x}{t \sqrt{x} + t \sqrt[3]{x}}$

In[9]:= $\text{Plot3D}\left[\frac{e^U t + t x}{t x / + t \sqrt{x}}, \{t, -6., 6.\}, \{x, -2.00691, 2.00691\}\right]$



$$\frac{[d^2 x/d^2 t] \sin x + [d^2 e^x/d^2 t] \tan x}{[d^2 x^{1/3}/d^2 t] + [d^2 \sqrt{x}/d^2 t] \sin x}$$

$$\text{In[10]:= } \frac{(d^2 * (x/d^2) * t) * \text{Sin}[x] + (d^2 * (E^x/d^2) * t) * \text{Tan}[x]}{(d^2 * (x^{1/3}/d^2) * t + (d^2 * (\text{Sqrt}[x]/d^2) * t) * \text{Sin}[x])}$$

$$\text{Out[10]= } \frac{t x \sin(x) + t e^x \tan(x)}{t \sqrt[3]{x} + t \sqrt{x} \sin(x)}$$

$$\text{In[12]:= } \frac{[d^2 x/d^2 t] \sin x + [d^2 e^x/d^2 t] \tan x}{[d^2 x^{1/3}/d^2 t] + [d^2 \sqrt{x}/d^2 t] \sin x}$$

Out[12]=

Input:

$$\frac{(d^2 \times \frac{x}{d^2} t) \sin(x) + (d^2 \times \frac{e^x}{d^2} t) \tan(x)}{d^2 \times \frac{\sqrt[3]{x}}{d^2} t + (d^2 \times \frac{\sqrt{x}}{d^2} t) \sin(x)}$$

Result:

$$\frac{t x \sin(x) + t e^x \tan(x)}{t \sqrt[3]{x} + t \sqrt{x} \sin(x)}$$

Alternate forms:

More 

$$\frac{(e^x + x \cos(x)) \tan(x)}{\sqrt[3]{x} + \sqrt{x} \sin(x)}$$

$$\frac{(e^x + x \cos(x)) \tan(x)}{\sqrt[3]{x} (\sqrt[3]{x} \sin(x) + 1)}$$

$$\frac{x \sin(x) + e^x \tan(x)}{\sqrt[3]{x} (\sqrt[6]{x} \sin(x) + 1)}$$

Expanded form:

$$\frac{t x \sin(x)}{t \sqrt[3]{x} + t \sqrt{x} \sin(x)} + \frac{t e^x \tan(x)}{t \sqrt[3]{x} + t \sqrt{x} \sin(x)}$$

Series expansion at x = 0:

$$x^{2/3} + 2 x^{5/3} - x^{11/6} + \frac{5 x^{8/3}}{6} - 2 x^{17/6} + x^3 + \frac{x^{11/3}}{3} - \frac{2 x^{23/6}}{3} + 2 x^4 - x^{25/6} + \frac{41 x^{14/3}}{120} + \frac{x^5}{2} - 2 x^{31/6} + x^{16/3} + O(x^{17/3})$$

(Puiseux series)

[Big-O notation »](#)

Derivative:

[Step-by-step solution](#) +

$$\frac{\partial}{\partial x} \left(\frac{\frac{(d^2 x t) \sin(x)}{d^2} + \frac{(d^2 e^x t) \tan(x)}{d^2}}{\frac{d^2 \sqrt[6]{x} t}{d^2} + \frac{(d^2 \sqrt{x} t) \sin(x)}{d^2}} \right) = \frac{1}{6 x^{4/3} (\sqrt[6]{x} \sin(x) + 1)^2}$$

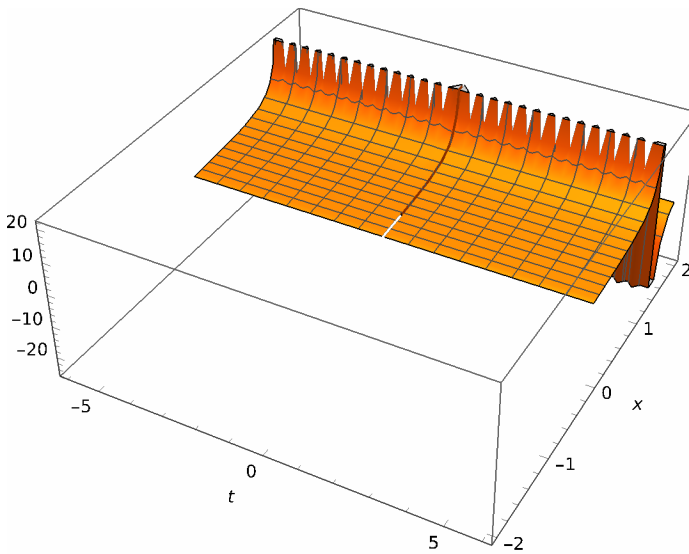
$$(6 x (\sqrt[6]{x} \sin(x) + 1) (\sin(x) + x \cos(x) + e^x \tan(x) + e^x \sec^2(x)) - (x \sin(x) + e^x \tan(x)) (6 x^{7/6} \cos(x) + 3 \sqrt[6]{x} \sin(x) + 2))$$

[sec\(x\) is the secant function »](#)

WolframAlpha +

```
In[11]:= Plot3D[ $\frac{t x \sin[x] + e^{U t} t \tan[x]}{t x / + t \sqrt{x} \sin[x]}$ , {t, -6., 6.}, {x, -2.00691, 2.00691}]
```

Out[11]=



Eq 13
$$\frac{\{d^2 x/d^2 t\} \sin x + \{d^2 e^x/d^2 t\} \tan x}{\{d^2 x^{1/3}/d^2 t\} * \{d^2 \sqrt{x}/d^2 t\} \sin x}$$

```
In[13]:= ((d^2*(x/d^2)*t)*Sin[x] + (d^2*(E^x/d^2)*t)*Tan[x])/((d^2*(x^(1/3)/d^2)*t)*(d^2*(Sqrt[x]/d^2)*t)*Sin[x])
```

Out[13]=

$$\frac{\csc(x) (t x \sin(x) + t e^x \tan(x))}{t^2 x^{5/6}}$$

```
In[15]:=  
$$\frac{\{d^2 x/d^2 t\} \sin x + \{d^2 e^x/d^2 t\} \tan x}{\{d^2 x^{1/3}/d^2 t\} * \{d^2 \sqrt{x}/d^2 t\} \sin x}$$

```

Out[15]=

Input:

$$\frac{\left(d^2 \times \frac{x}{d^2} t\right) \sin(x) + \left(d^2 \times \frac{e^x}{d^2} t\right) \tan(x)}{\left(d^2 \times \frac{\sqrt[3]{x}}{d^2} t\right) \left(d^2 \times \frac{\sqrt{x}}{d^2} t\right) \sin(x)}$$

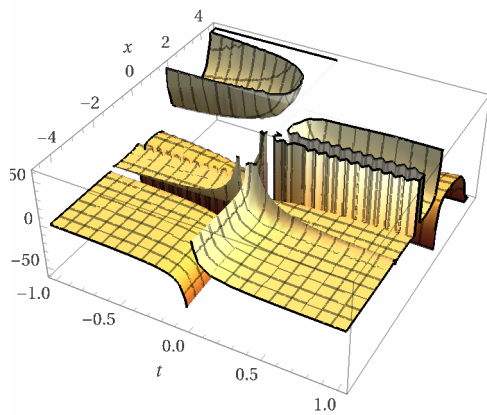
Result:

$$\frac{\csc(x) (t x \sin(x) + t e^x \tan(x))}{t^2 x^{5/6}}$$

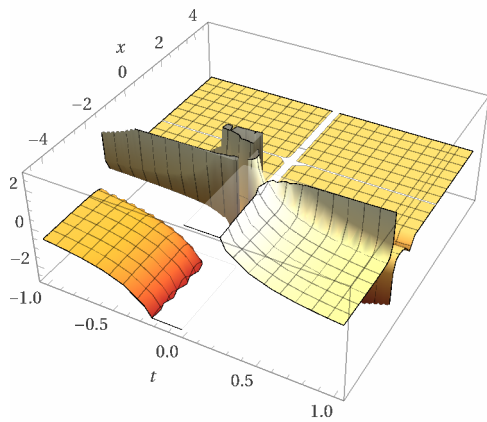
[csc\(x\) is the cosecant function »](#)

3D plots:

Real part:

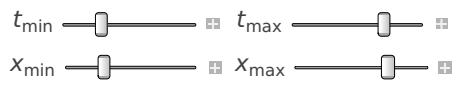
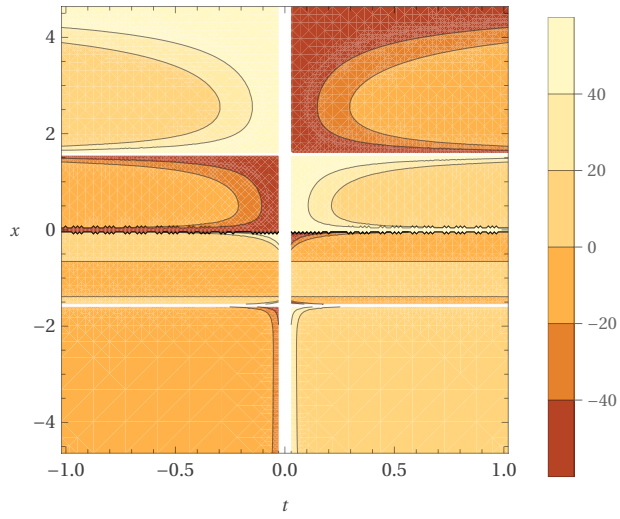


Imaginary part:

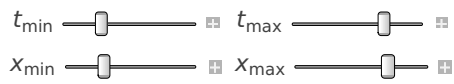
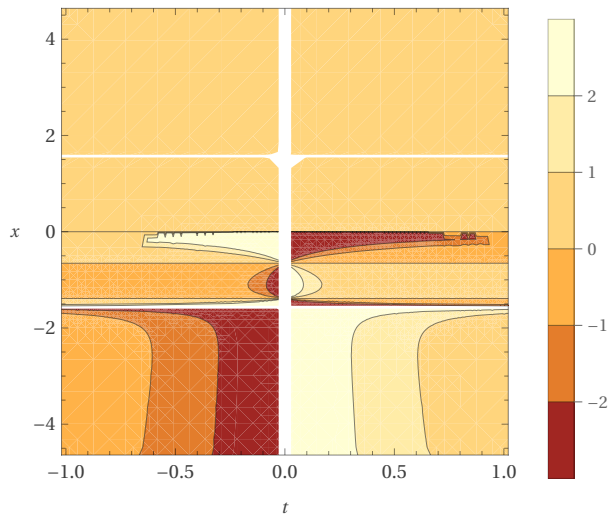


Contour plots:

Real part:



Imaginary part:



Alternate forms:

More

$$\frac{x + e^x \sec(x)}{t x^{5/6}}$$

$$\frac{(e^x + x \cos(x)) \sec(x)}{t x^{5/6}}$$

$$\frac{\frac{e^x \sec(x)}{x^{5/6}} + \sqrt[6]{x}}{t}$$

[sec\(x\) is the secant function »](#)

Expanded form: +

$$\frac{e^x \sec(x)}{t x^{5/6}} + \frac{\sqrt[6]{x}}{t}$$

Series expansion at x = 0: +

$$O\left(\frac{1}{x^{25}}\right)$$

(Taylor series)

[Big-O notation »](#)

Derivative:

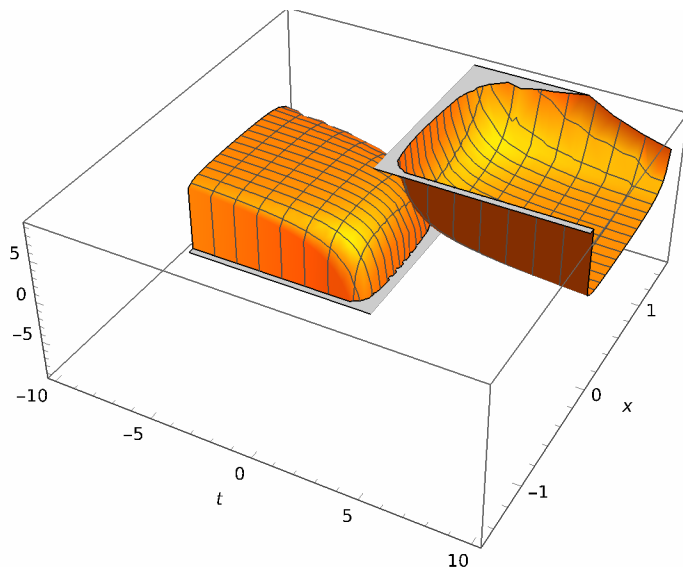
[Step-by-step solution](#) +

$$\frac{\partial}{\partial x} \left(\frac{\frac{(d^2 x t) \sin(x)}{d^2} + \frac{(d^2 e^x t) \tan(x)}{d^2}}{\frac{(d^2 \sqrt[3]{x} t) (d^2 \sqrt{x} t) \sin(x)}{d^2 d^2}} \right) = \frac{x + e^x (6 x - 5) \sec(x) + 6 e^x x \csc(x) (\sec^2(x) - 1)}{6 t x^{11/6}}$$

WolframAlpha +

```
In[14]:= Plot3D[Csc[x] (t x Sin[x] + e^x t Tan[x]) / (t^2 x^(5/6)), {t, -10., 10.}, {x, -1.50958, 1.50958}]
```

Out[14]=



$$\boxed{= \left[\frac{d^2 x}{d^2 t} \sin x + \frac{d^2 e^x}{d^2 t} \tan x \right]^2 / \left[\frac{d^2 x^{1/3}}{d^2 t} * \frac{d^2 \sqrt{x}}{d^2 t} \sin x \right]} \quad \text{Eq 13}$$

$$\text{In[1]:= } \frac{\left(\frac{d^2 (x/d^2) * t}{d^2} * \text{Sin}[x] + \frac{d^2 (E^x/d^2) * t}{d^2} * \text{Tan}[x] \right)^2}{\left(\frac{d^2 (x^{1/3}/d^2) * t}{d^2} * \frac{d^2 (\text{Sqrt}[x]/d^2) * t}{d^2} * \text{Sin}[x] \right)}$$

$$\text{Out[1]= } \frac{\text{csc}(x) \left(t x \sin(x) + t e^x \tan(x) \right)^2}{t^2 x^{5/6}}$$

$$\text{In[3]:= } \text{☠ } \left[\frac{d^2 x}{d^2 t} \sin x + \frac{d^2 e^x}{d^2 t} \tan x \right]^2 / \left[\frac{d^2 x^{1/3}}{d^2 t} * \frac{d^2 \sqrt{x}}{d^2 t} \sin x \right]$$

Input: +

$$\frac{\left((d^2 \times \frac{x}{d^2} t) \sin(x) + (d^2 \times \frac{e^x}{d^2} t) \tan(x) \right)^2}{\left(d^2 \times \frac{\sqrt[6]{x}}{d^2} t \right) \left(d^2 \times \frac{\sqrt{x}}{d^2} t \right) \sin(x)}$$

Result: +

$$\frac{\csc(x) (t x \sin(x) + t e^x \tan(x))^2}{t^2 x^{5/6}}$$

[csc\(x\) is the cosecant function »](#)

Alternate forms: More +

$$\frac{\sin(x) (x + e^x \sec(x))^2}{x^{5/6}}$$

$$\frac{(x^2 \cos^2(x) + e^{2x} + 2 e^x x \cos(x)) \tan(x) \sec(x)}{x^{5/6}}$$

$$\frac{\left(t x \sin(x) + \frac{e^x t \sin(x)}{\cos(x)} \right)^2}{t^2 x^{5/6} \sin(x)}$$

[sec\(x\) is the secant function »](#)

Out[3]=

Expanded form: +

$$x^{7/6} \sin(x) + \frac{e^{2x} \tan(x) \sec(x)}{x^{5/6}} + 2 e^x \sqrt[6]{x} \tan(x)$$

Series expansion at x = 0: +

$$\sqrt[6]{x} + 4 x^{7/6} + \frac{35 x^{13/6}}{6} + O(x^{19/6})$$

(Puiseux series)

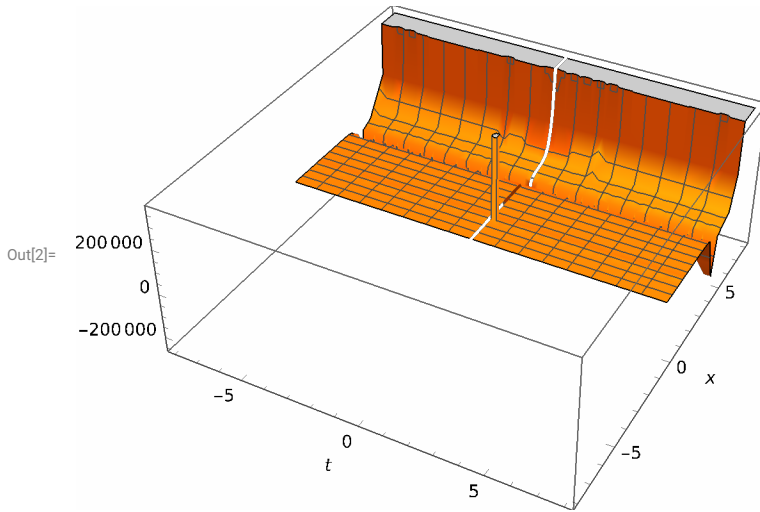
[Big-O notation »](#)

Derivative: Step-by-step solution +

$$\frac{\partial}{\partial x} \left(\frac{\left(\frac{(d^2 x t) \sin(x)}{d^2} + \frac{(d^2 e^x t) \tan(x)}{d^2} \right)^2}{\frac{(d^2 \sqrt[6]{x} t) (d^2 \sqrt{x} t) \sin(x)}{d^2 d^2}} \right) =$$

$$\frac{\cos(x) (x + e^x \sec(x)) (x (6 x + 7 \tan(x)) + 12 e^x x \sec^3(x) + e^x ((12 x - 5) \tan(x) - 6 x) \sec(x))}{6 x^{11/6}}$$

```
In[2]:= Plot3D[ $\frac{\text{Csc}[x] (t x \text{Sin}[x] + e^x t \text{Tan}[x])^2}{t^2 x^{5/6}}$ , {t, -8, 8}, {x, -8, 8}]
```



Eq 14
$$\frac{\{d^2 x/d^2 t\} \sin x + \{d^2 e^x/d^2 t\} \tan x}{\{d^2 x^{1/3}/d^2 t\} * \{d^2 \sqrt{x}/d^2 t\} \sin x}^2$$

```
In[4]:= ((d^2*(x/d^2)*t)*Sin[x] + (d^2*(E^x/d^2)*t)*Tan[x])/
((d^2*(x^(1/3)/d^2)*t)*(d^2*(Sqrt[x]/d^2)*t)*Sin[x])^2
```

```
Out[4]=  $\frac{\text{csc}^2(x) (t x \sin(x) + t e^x \tan(x))}{t^4 x^{5/3}}$ 
```

```
In[6]:=  [{"d^2 x/d^2 t} sin x + {d^2 e^x/d^2 t} tan x] / [{"d^2 x^1/3/d^2 t} * {d^2 sqrt(x)/d^2 t} sin x]^2
```

Input: +

$$\frac{(d^2 \times \frac{x}{d^2} t) \sin(x) + (d^2 \times \frac{e^x}{d^2} t) \tan(x)}{\left((d^2 \times \frac{\sqrt[3]{x}}{d^2} t) (d^2 \times \frac{\sqrt{x}}{d^2} t) \sin(x) \right)^2}$$

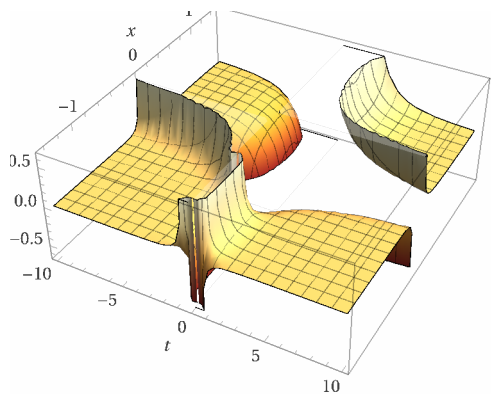
Result: +

$$\frac{\text{csc}^2(x) (t x \sin(x) + t e^x \tan(x))}{t^4 x^{5/3}}$$

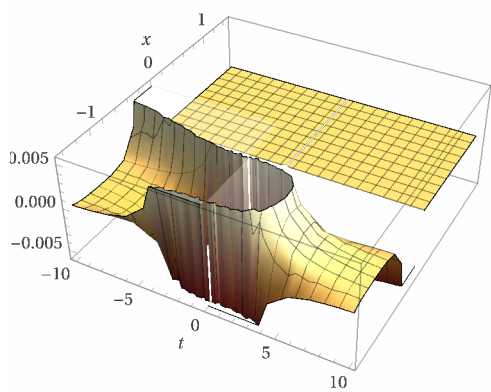
[csc\(x\) is the cosecant function »](#)

3D plots: +

Real part:



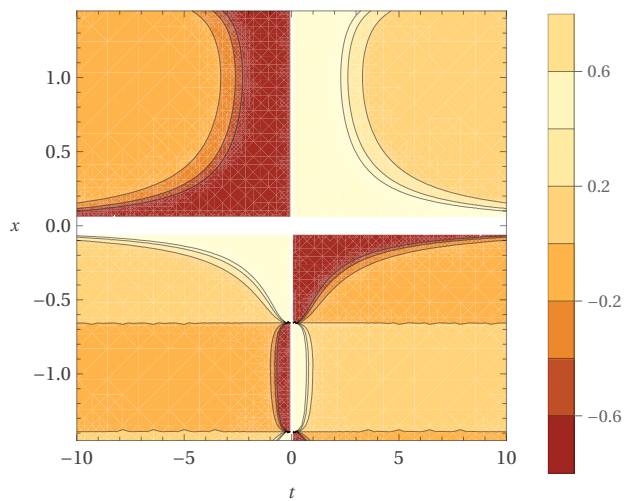
Imaginary part:



Contour plots:



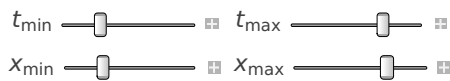
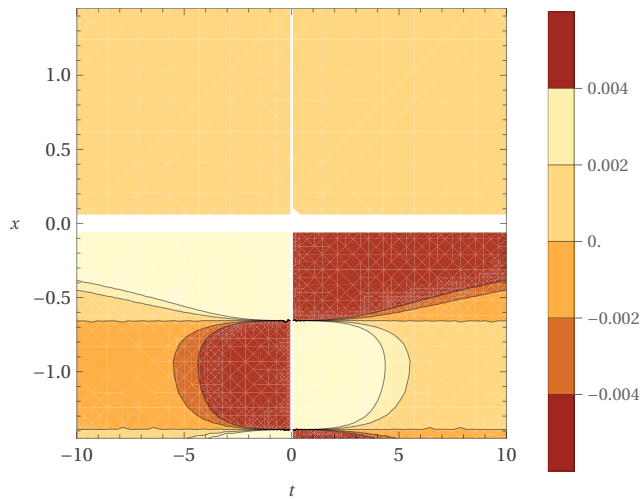
Real part:



Out[6]=

t_{\min} t_{\max}
 x_{\min} x_{\max}

Imaginary part:



Alternate forms:

More

$$\frac{\csc(x) (x + e^x \sec(x))}{t^3 x^{5/3}}$$

$$\frac{(e^x + x \cos(x)) \csc(x) \sec(x)}{t^3 x^{5/3}}$$

$$\frac{\csc(x) \left(\frac{e^x \sec(x)}{x^{5/3}} + \frac{1}{x^{2/3}} \right)}{t^3}$$

sec(x) is the secant function [»](#)

Expanded form:

$$\frac{e^x \csc(x) \sec(x)}{t^3 x^{5/3}} + \frac{\csc(x)}{t^3 x^{2/3}}$$

Series expansion at x = 0:

$$O\left(\frac{1}{x^{19}}\right)$$

(Taylor series)

Big-O notation [»](#)

Derivative:

Step-by-step solution

$$\frac{\partial}{\partial x} \left(\frac{(d^2 x t) \sin(x)}{d^2} + \frac{(d^2 e^x t) \tan(x)}{d^2} \right) = \frac{\cot(x) (\sec(x) (2 x - e^x (3 x - 5) \sec(x)) + 3 x \csc(x) (x - e^x \sec^3(x) + 2 e^x \sec(x)))}{3 t^3 x^{8/3}}$$

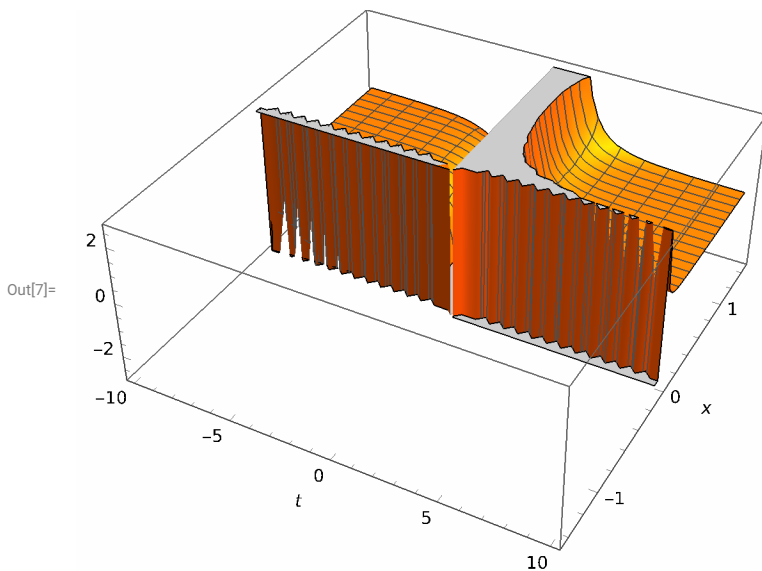
cot(x) is the cotangent function »

WolframAlpha

```
In[5]:= FullSimplify[ $\frac{\text{Csc}[x]^2 (t x \text{Sin}[x] + e^x t \text{Tan}[x])}{t^4 x^{5/3}}$ ]
```

Out[5]= $\frac{\csc(x) (x + e^x \sec(x))}{t^3 x^{5/3}}$

```
In[7]:= Plot3D[ $\frac{\text{Csc}[x] (x + e^x \text{Sec}[x])}{t^3 x^{5/3}}$ , {t, -10., 10.}, {x, -1.4497, 1.4497}]
```



$\frac{\{d^2 x/d^2 t\} \sin x + \{d^2 \pi^x/d^2 t\} \tan x}{\{d^2 x^{1/3}/d^2 t\} \{d^2 \sqrt{x}/d^2 t\} \sin x^2}$ Eq 15

```
In[8]:= ((d^2*(x/d^2)*t)*Sin[x] + (d^2*(Pi^x/d^2)*t)*Tan[x])/
((d^2*(x^(1/3)/d^2)*t)*(d^2*(Sqrt[x]/d^2)*t)*Sin[x]^2)
```

Out[8]= $\frac{\csc^2(x) (t x \sin(x) + t \pi^x \tan(x))}{t^4 x^{5/3}}$

In[11]:= 
$$\frac{\{d^2 x/d^2 t\} \sin x + \{d^2 \pi^x/d^2 t\} \tan x}{\{d^2 x^{1/3}/d^2 t\} * \{d^2 \sqrt{x}/d^2 t\} \sin x}^2$$

Out[11]=

Input: +

$$\frac{\left(d^2 \times \frac{x}{d^2} t\right) \sin(x) + \left(d^2 \times \frac{\pi^x}{d^2} t\right) \tan(x)}{\left(\left(d^2 \times \frac{\sqrt[3]{x}}{d^2} t\right) \left(d^2 \times \frac{\sqrt{x}}{d^2} t\right) \sin(x)\right)^2}$$

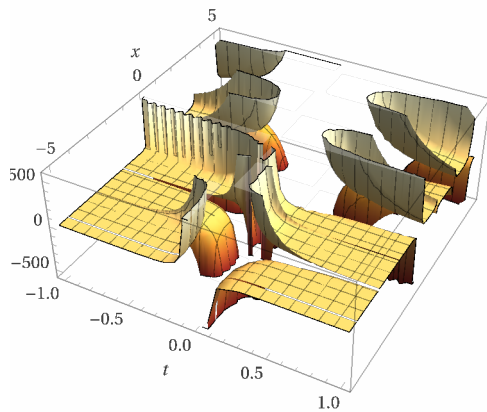
Result: +

$$\frac{\csc^2(x) (t x \sin(x) + t \pi^x \tan(x))}{t^4 x^{5/3}}$$

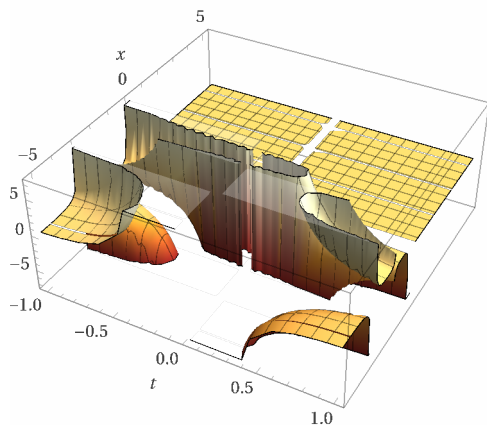
[csc\(x\) is the cosecant function »](#)

3D plots: +

Real part:



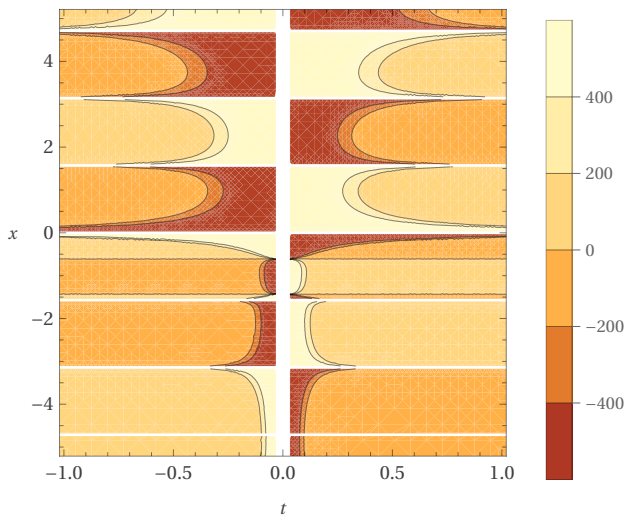
Imaginary part:



Contour plots:

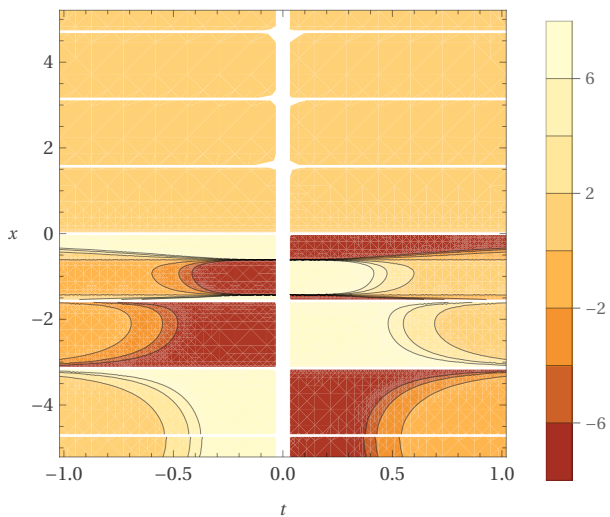


Real part:



t_{\min} t_{\max}
 x_{\min} x_{\max}

Imaginary part:



t_{\min} t_{\max}
 x_{\min} x_{\max}

Alternate forms:

More

$$\frac{\csc(x) (x + \pi^x \sec(x))}{t^3 x^{5/3}}$$

$$\frac{(\pi^x + x \cos(x)) \csc(x) \sec(x)}{t^3 x^{5/3}}$$

$$\frac{\csc(x) \left(\frac{\pi^x \sec(x)}{x^{5/3}} + \frac{1}{x^{2/3}} \right)}{t^3}$$

[sec\(x\) is the secant function »](#)

Partial fraction expansion:

[Step-by-step solution](#) 

$$\frac{\pi^x \csc(x) \sec(x)}{t^3 x^{5/3}} + \frac{\csc(x)}{t^3 x^{2/3}}$$

Series expansion at $x = 0$:



$$\begin{aligned} & \frac{1}{t^3 x^{8/3}} + \frac{1 + \log(\pi)}{t^3 x^{5/3}} + \frac{4 + 3 \log^2(\pi)}{6 t^3 x^{2/3}} + \frac{\sqrt[3]{x} (1 + \log^3(\pi) + 4 \log(\pi))}{6 t^3} + \\ & \frac{x^{4/3} (112 + 15 \log^4(\pi) + 120 \log^2(\pi))}{360 t^3} + \frac{x^{7/3} (7 + 3 \log^5(\pi) + 40 \log^3(\pi) + 112 \log(\pi))}{360 t^3} + \\ & \frac{x^{10/3} (1984 + 21 \log^6(\pi) + 420 \log^4(\pi) + 2352 \log^2(\pi))}{15\,120 t^3} + \\ & \frac{x^{13/3} (31 + 3 \log^7(\pi) + 84 \log^5(\pi) + 784 \log^3(\pi) + 1984 \log(\pi))}{15\,120 t^3} + O(x^{16/3}) \end{aligned}$$

(Puiseux series)

[log\(x\) is the natural logarithm »](#)

[Big-O notation »](#)

Derivative:

[Approximate form](#)

[Step-by-step solution](#) 

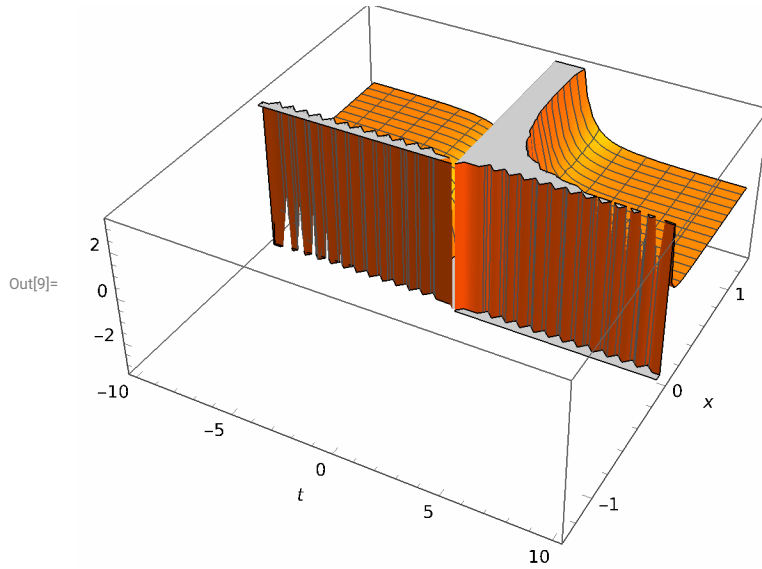
$$\frac{\partial}{\partial x} \left(\frac{\frac{(d^2 x t) \sin(x)}{d^2} + \frac{(d^2 \pi^x t) \tan(x)}{d^2}}{\left(\frac{(d^2 \sqrt[3]{x} t) (d^2 \sqrt{x} t) \sin(x)}{d^2 d^2} \right)^2} \right) =$$

$$-\frac{\cot(x) (\sec(x) (2x - \pi^x (3x \log(\pi) - 5) \sec(x)) + 3x \csc(x) (x - \pi^x \sec^3(x) + 2\pi^x \sec(x)))}{3 t^3 x^{8/3}}$$

[cot\(x\) is the cotangent function »](#)

WolframAlpha 

$$\text{In[9]:= Plot3D}\left[\frac{\text{Csc}[x]^2 (t x \text{Sin}[x] + \pi^x t \text{Tan}[x])}{t^4 x^{5/3}}, \{t, -10., 10.\}, \{x, -1.30812, 1.30812\}\right]$$



Eq 16
$$\frac{[d^2x/d^2t] \sin x + [d^2\pi^x/d^2t] \tan x]^2}{[d^2x^{1/3}/d^2t] * [d^2\sqrt{x}/d^2t] \sin x}$$

In[12]:=
$$\frac{((d^2*(x/d^2)*t)*\text{Sin}[x] + (d^2*(\text{Pi}^x/d^2)*t)*\text{Tan}[x])^2}{((d^2*(x^{1/3}/d^2)*t)*(d^2*(\text{Sqrt}[x]/d^2)*t)*\text{Sin}[x])}$$

Out[12]=
$$\frac{\text{csc}(x) (t x \sin(x) + t \pi^x \tan(x))^2}{t^2 x^{5/6}}$$

In[14]:= **⚠**
$$[d^2x/d^2t] \sin x + [d^2\pi^x/d^2t] \tan x]^2 / [d^2x^{1/3}/d^2t] * [d^2\sqrt{x}/d^2t] \sin x$$

Out[14]=

Input: +

$$\frac{\left(d^2 \times \frac{x}{d^2} t\right) \sin(x) + \left(d^2 \times \frac{\pi^x}{d^2} t\right) \tan(x)^2}{\left(d^2 \times \frac{\sqrt[6]{x}}{d^2} t\right) \left(d^2 \times \frac{\sqrt{x}}{d^2} t\right) \sin(x)}$$

Result: +

$$\frac{\csc(x) (t x \sin(x) + t \pi^x \tan(x))^2}{t^2 x^{5/6}}$$

[csc\(x\) is the cosecant function »](#)Alternate forms: More +

$$\frac{\sin(x) (x + \pi^x \sec(x))^2}{x^{5/6}}$$

$$\frac{(x^2 \cos^2(x) + \pi^{2x} + 2 \pi^x x \cos(x)) \tan(x) \sec(x)}{x^{5/6}}$$

$$\frac{\left(t x \sin(x) + \frac{\pi^x t \sin(x)}{\cos(x)}\right)^2}{t^2 x^{5/6} \sin(x)}$$

[sec\(x\) is the secant function »](#)Expanded form: +

$$x^{7/6} \sin(x) + \frac{\pi^{2x} \tan(x) \sec(x)}{x^{5/6}} + 2 \pi^x \sqrt[6]{x} \tan(x)$$

Series expansion at x = 0: +

$$\sqrt[6]{x} + x^{7/6} (2 + 2 \log(\pi)) + x^{13/6} \left(\frac{11}{6} + 2 \log^2(\pi) + 2 \log(\pi) \right) + O(x^{19/6})$$

(Puiseux series)

[log\(x\) is the natural logarithm »](#)[Big-O notation »](#)Derivative: Approximate form Step-by-step solution +

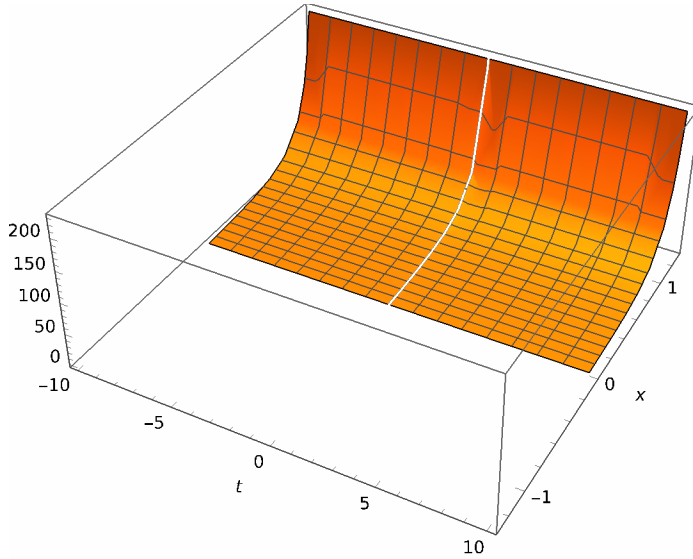
$$\frac{\partial}{\partial x} \left(\frac{\left(\frac{(d^2 x t) \sin(x)}{d^2} + \frac{(d^2 \pi^x t) \tan(x)}{d^2} \right)^2}{\frac{(d^2 \sqrt[6]{x} t) (d^2 \sqrt{x} t) \sin(x)}{d^2 d^2}} \right) = \frac{1}{6 x^{11/6}} \cos(x) (x + \pi^x \sec(x))$$

$$(x (6 x + 7 \tan(x)) + 12 \pi^x x \sec^3(x) + \pi^x \sec(x) ((12 x \log(\pi) - 5) \tan(x) - 6 x))$$

WolframAlpha +

In[13]:= Plot3D[$\frac{\text{Csc}[x] (t x \text{Sin}[x] + \pi^x t \text{Tan}[x])^2}{t^2 x^{5/6}}$, {t, -10., 10.}, {x, -1.29231, 1.29231}]

Out[13]=



⊞ $\frac{\{d^2x/d^2t\} \sin x + \{d^2\pi^x/d^2t\} \tan x}{\{d^2x^{1/3}/d^2t\} \{d^2\sqrt{x}/d^2t\} \sin x}$ Eq 17

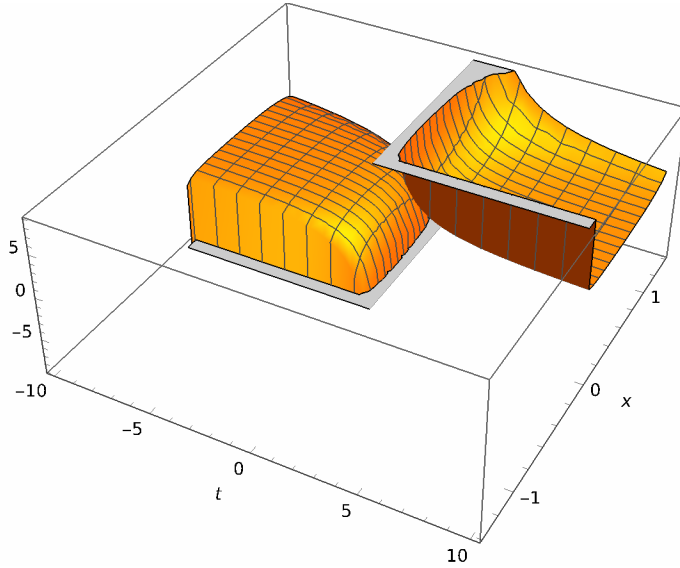
In[15]:= $\frac{(d^2(x/d^2)t) * \text{Sin}[x] + (d^2(\text{Pi}^x/d^2)t) * \text{Tan}[x]}{(d^2(x^{1/3}/d^2)t) * (d^2(\text{Sqrt}[x]/d^2)t) * \text{Sin}[x]}$

Out[15]=

$$\frac{\text{csc}(x) (t x \sin(x) + t \pi^x \tan(x))}{t^2 x^{5/6}}$$

In[16]:= Plot3D[$\frac{\text{Csc}[x] (t x \text{Sin}[x] + \pi^x t \text{Tan}[x])}{t^2 x^{5/6}}$, {t, -10., 10.}, {x, -1.36799, 1.36799}]

Out[16]=



☰ $\frac{\{d^{2x/d^2t}\} \sin x + \{d^{2\pi^x/d^2t}\} \tan x}{\{d^{2x^{1/3}/d^2t}\} \{d^{2\sqrt{x}/d^2t}\} \sin x^3}$ Eq 18

In[17]:= $\frac{(d^{2*(x/d^2)*t}) * \text{Sin}[x] + (d^{2*(\text{Pi}^x/d^2)*t}) * \text{Tan}[x]}{(d^{2*(x^{1/3}/d^2)*t}) * (d^{2*(\text{Sqrt}[x]/d^2)*t}) * \text{Sin}[x]^3}$

Out[17]=

$$\frac{\text{csc}^3(x) (t x \sin(x) + t \pi^x \tan(x))}{t^6 x^{5/2}}$$

In[19]:= 🌟 $\frac{\{d^{2x/d^2t}\} \sin x + \{d^{2\pi^x/d^2t}\} \tan x}{\{d^{2x^{1/3}/d^2t}\} \{d^{2\sqrt{x}/d^2t}\} \sin x^3}$

Out[19]=

Input:

$$\frac{(d^2 \times \frac{x}{d^2} t) \sin(x) + (d^2 \times \frac{\pi^x}{d^2} t) \tan(x)}{\left((d^2 \times \frac{\sqrt[3]{x}}{d^2} t) (d^2 \times \frac{\sqrt{x}}{d^2} t) \sin(x) \right)^3}$$

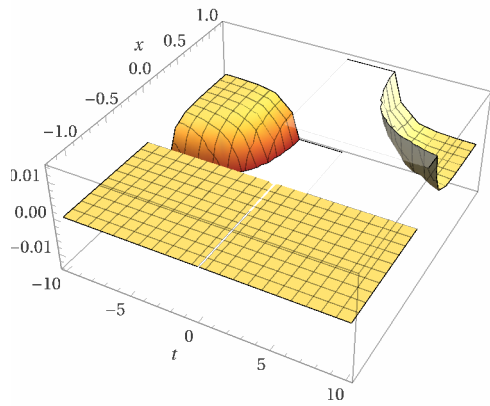
Result:

$$\frac{\text{csc}^3(x) (t x \sin(x) + t \pi^x \tan(x))}{t^6 x^{5/2}}$$

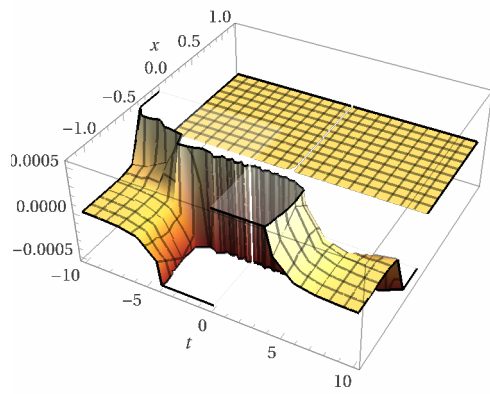
csc(x) is the cosecant function >>

3D plots:

Real part:



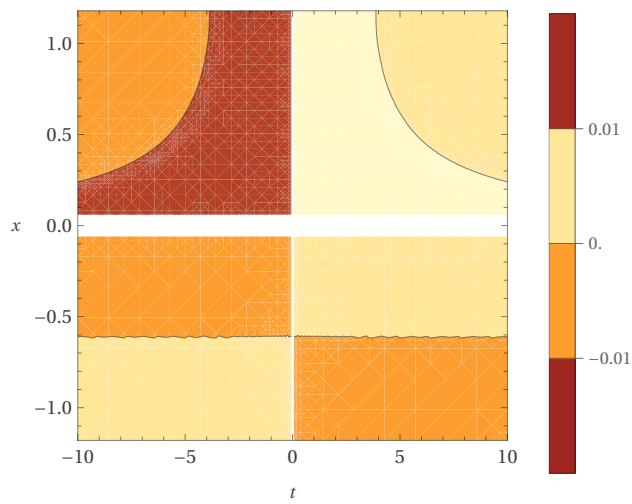
Imaginary part:



Contour plots:

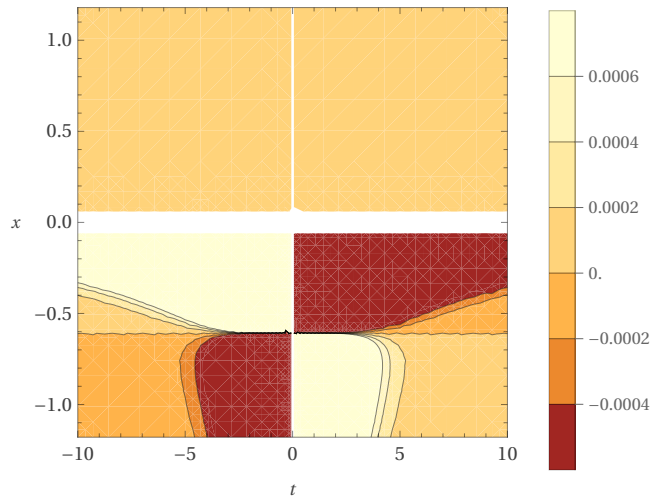


Real part:



t_{\min} t_{\max}
 x_{\min} x_{\max}

Imaginary part:



t_{\min} t_{\max}
 x_{\min} x_{\max}

Alternate forms:

More

$$\frac{\csc^2(x) (x + \pi^x \sec(x))}{t^5 x^{5/2}}$$

$$\frac{(\pi^x + x \cos(x)) \csc^2(x) \sec(x)}{t^5 x^{5/2}}$$

$$\frac{\csc^2(x) \left(\frac{\pi^x \sec(x)}{x^{5/2}} + \frac{1}{x^{3/2}} \right)}{t^5}$$

sec(x) is the secant function »

Partial fraction expansion:

Step-by-step solution

$$\frac{\pi^x \csc^2(x) \sec(x)}{t^5 x^{5/2}} + \frac{\csc^2(x)}{t^5 x^{3/2}}$$

Derivative:


Approximate form

Step-by-step solution

$$\frac{\partial}{\partial x} \left(\frac{\frac{(d^2 x t) \sin(x)}{d^2} + \frac{(d^2 \pi^x t) \tan(x)}{d^2}}{\left(\frac{(d^2 \sqrt[3]{x} t) (d^2 \sqrt{x} t) \sin(x)}{\rho \rho} \right)^3} \right) = -\frac{1}{2 t^5 x^{7/2}} \cot(x) \csc(x)$$

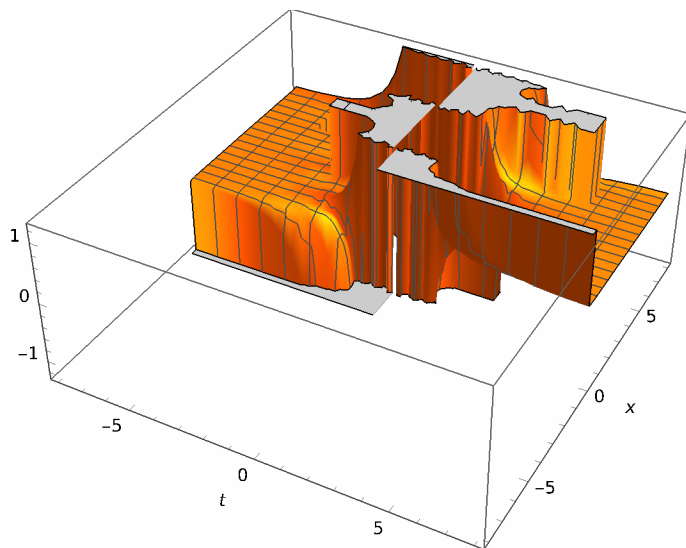
$$(\sec(x) (3 x - \pi^u (2 x \log(\pi) - 5) \sec(x)) + 2 x \csc(x) (2 x - \pi^u \sec^3(x) + 3 \pi^u \sec(x)))$$


cot(x) is the cotangent function >>
 log(x) is the natural logarithm >>

WolframAlpha 

In[18]:= `Plot3D[Csc[x]^3 (t x Sin[x] + Pi^x t Tan[x]) / (t^6 x^(5/2)), {t, -8, 8}, {x, -8, 8}]`

Out[18]=




$$\frac{\{d^2 x / d^2 t\} \sin x + \{d^2 \pi^x / d^2 t\} \tan x}{\{d^2 x^{1/3} / d^2 t\} + \{d^2 \sqrt{x} / d^2 t\} \sin x} \quad \text{Eq 19}$$

In[20]:= `((d^2*(x/d^2)*t)*Sin[x] + (d^2*(Pi^x/d^2)*t)*Tan[x]) / (d^2*(x^(1/3)/d^2)*t + (d^2*(Sqrt[x]/d^2)*t)*Sin[x])`

Out[20]=

$$\frac{t x \sin(x) + t \pi^x \tan(x)}{t \sqrt[3]{x} + t \sqrt{x} \sin(x)}$$

In[22]:=  `[(d^2 x / d^2 t) sin x + (d^2 pi^x / d^2 t) tan x] / [(d^2 x^(1/3) / d^2 t) + (d^2 sqrt(x) / d^2 t) sin x]`

Out[22]=

Input: 

$$\frac{(d^2 \times \frac{x}{d^2} t) \sin(x) + (d^2 \times \frac{\pi^x}{d^2} t) \tan(x)}{d^2 \times \frac{\sqrt[3]{x}}{d^2} t + (d^2 \times \frac{\sqrt{x}}{d^2} t) \sin(x)}$$

Result:

$$\frac{t x \sin(x) + t \pi^x \tan(x)}{t \sqrt[3]{x} + t \sqrt{x} \sin(x)}$$

Alternate forms:

More 

$$\frac{(\pi^x + x \cos(x)) \tan(x)}{\sqrt[3]{x} + \sqrt{x} \sin(x)}$$

$$\frac{(\pi^x + x \cos(x)) \tan(x)}{\sqrt[3]{x} (\sqrt[6]{x} \sin(x) + 1)}$$

$$\frac{x \sin(x) + \pi^x \tan(x)}{\sqrt[3]{x} (\sqrt[6]{x} \sin(x) + 1)}$$

Expanded form:



$$\frac{t x \sin(x)}{t \sqrt[3]{x} + t \sqrt{x} \sin(x)} + \frac{t \pi^x \tan(x)}{t \sqrt[3]{x} + t \sqrt{x} \sin(x)}$$

Series expansion at $x = 0$:

$$\begin{aligned} & x^{2/3} + x^{5/3} (1 + \log(\pi)) - x^{11/6} + \frac{1}{6} x^{8/3} (2 + 3 \log^2(\pi)) + x^{17/6} (-1 - \log(\pi)) + \\ & x^3 + \frac{1}{6} x^{11/3} (-1 + \log^3(\pi) + 2 \log(\pi)) + \frac{1}{6} x^{23/6} (-1 - 3 \log^2(\pi)) + \\ & x^4 (1 + \log(\pi)) - x^{25/6} + \frac{1}{120} x^{14/3} (16 + 5 \log^4(\pi) + 20 \log^2(\pi)) + \\ & \frac{1}{6} x^{29/6} (2 - \log^3(\pi) - \log(\pi)) + \frac{1}{2} x^5 \log^2(\pi) + x^{31/6} (-1 - \log(\pi)) + x^{16/3} + O(x^{17/3}) \end{aligned}$$

(Puiseux series)

[log\(x\) is the natural logarithm »](#)[Big-O notation »](#)

Derivative:

Approximate form

Step-by-step solution 

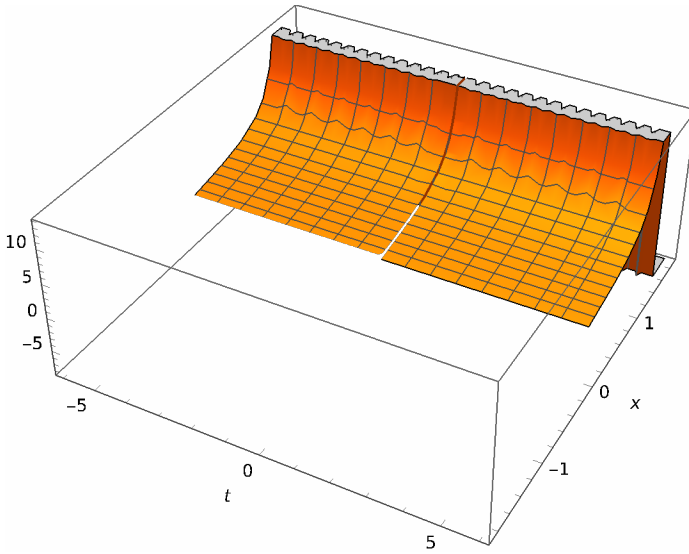
$$\frac{\partial}{\partial x} \left(\frac{\frac{(d^2 x t) \sin(x)}{d^2} + \frac{(d^2 \pi^x t) \tan(x)}{d^2}}{\frac{d^2 \sqrt[3]{x} t}{d^2} + \frac{(d^2 \sqrt{x} t) \sin(x)}{d^2}} \right) = \frac{1}{6 x^{4/3} (\sqrt[6]{x} \sin(x) + 1)^2}$$

$$\begin{aligned} & (6 x (\sqrt[6]{x} \sin(x) + 1) (\sin(x) + x \cos(x) + \pi^x \sec^2(x) + \pi^x \log(\pi) \tan(x)) - \\ & (x \sin(x) + \pi^x \tan(x)) (6 x^{7/6} \cos(x) + 3 \sqrt[6]{x} \sin(x) + 2)) \end{aligned}$$

[sec\(x\) is the secant function »](#)WolframAlpha 

In[21]:= Plot3D[$\frac{t x \text{Sin}[x] + \pi^x t \text{Tan}[x]}{t x^{1/3} + t \sqrt{x} \text{Sin}[x]}$, {t, -6., 6.}, {x, -1.87476, 1.87476}]

Out[21]=



Eq 20

$$\frac{\{d^2 x/d^2 t\} \sin x + \{d^2 \pi^x/d^2 t\} \tan x}{\{d^2 x^{1/3}/d^2 t\} + \{d^2 \sqrt{x}/d^2 t\} \sin x}^2$$

In[23]:= $\frac{(d^2 * (x/d^2) * t) * \text{Sin}[x] + (d^2 * (\text{Pi}^x/d^2) * t) * \text{Tan}[x]}{(d^2 * (x^{1/3}/d^2) * t + (d^2 * (\text{Sqrt}[x]/d^2) * t) * \text{Sin}[x])^2}$

Out[23]=

$$\frac{t x \sin(x) + t \pi^x \tan(x)}{(t \sqrt[3]{x} + t \sqrt{x} \sin(x))^2}$$

In[25]:= $\frac{\{d^2 x/d^2 t\} \sin x + \{d^2 \pi^x/d^2 t\} \tan x}{\{d^2 x^{1/3}/d^2 t\} + \{d^2 \sqrt{x}/d^2 t\} \sin x}^2$

Out[25]=

Input: +

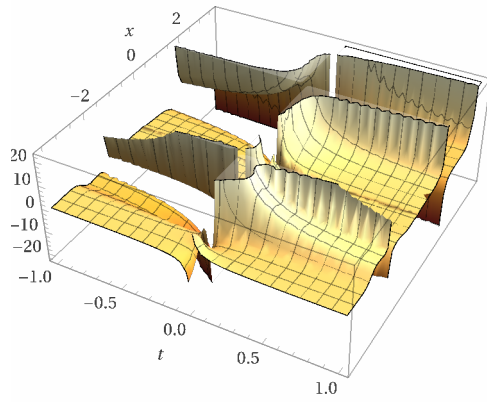
$$\frac{(d^2 \times \frac{x}{d^2} t) \sin(x) + (d^2 \times \frac{\pi^x}{d^2} t) \tan(x)}{(d^2 \times \frac{\sqrt[3]{x}}{d^2} t + (d^2 \times \frac{\sqrt{x}}{d^2} t) \sin(x))^2}$$

Result: +

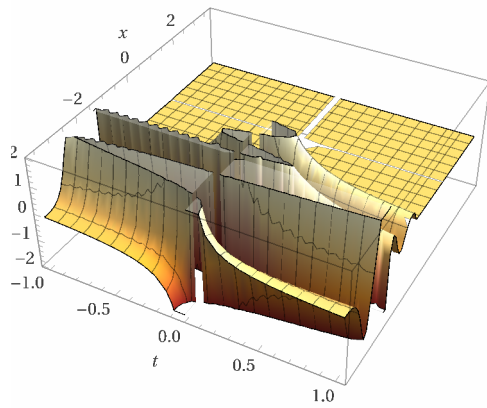
$$\frac{t x \sin(x) + t \pi^x \tan(x)}{(t \sqrt[3]{x} + t \sqrt{x} \sin(x))^2}$$

3D plots: +

Real part:

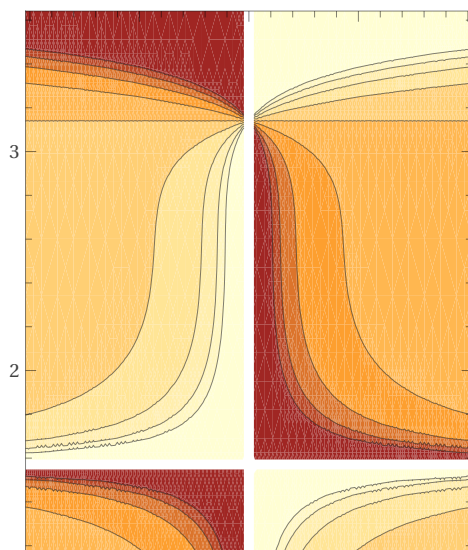


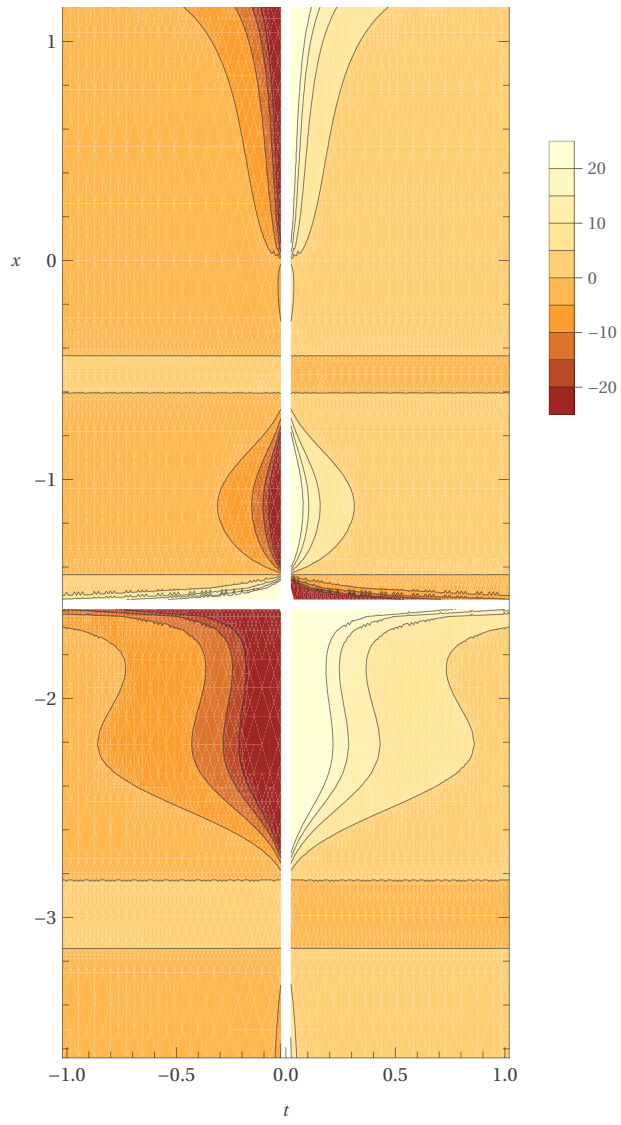
Imaginary part:



Contour plots:

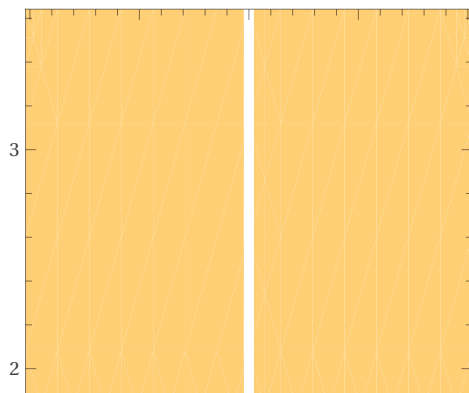
Real part:

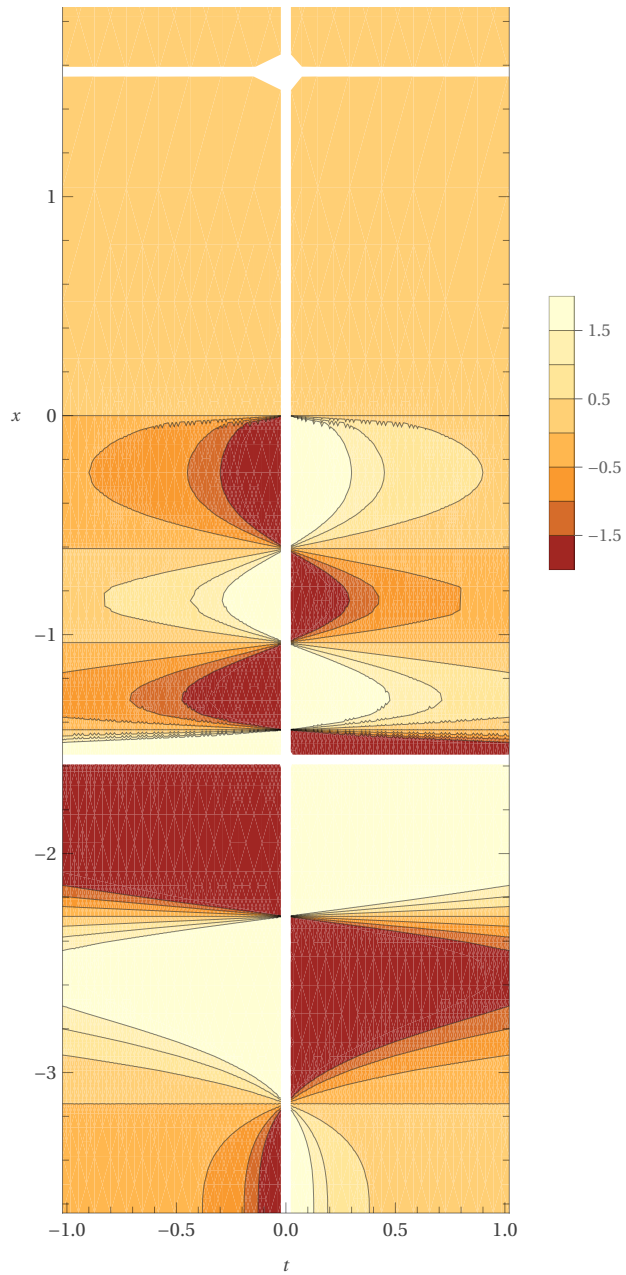




t_{\min} t_{\max}
 x_{\min} x_{\max}

Imaginary part:





t_{\min} t_{\max}
 x_{\min} x_{\max}

Alternate forms:

More

$$\frac{(\pi^x + x \cos(x)) \tan(x)}{t x^{2/3} (\sqrt[6]{x} \sin(x) + 1)^2}$$

$$\frac{x \sin(x) + \pi^x \tan(x)}{t x^{2/3} (\sqrt[6]{x} \sin(x) + 1)^2}$$

$$\frac{t x \sin(x) + \frac{\pi^x t \sin(x)}{\cos(x)}}{(t \sqrt[3]{x} + t \sqrt{x} \sin(x))^2}$$

Partial fraction expansion:

[Step-by-step solution](#) 

$$\frac{\pi^x \tan(x)}{t x^{2/3} (\sqrt[6]{x} \sin(x) + 1)^2} + \frac{\sqrt[6]{x} \sin(x)}{t (\sqrt[6]{x} \sin(x) + 1)^2}$$

Expanded forms:



$$\frac{t x \sin(x)}{(t \sqrt[3]{x} + t \sqrt{x} \sin(x))^2} + \frac{t \pi^x \tan(x)}{(t \sqrt[3]{x} + t \sqrt{x} \sin(x))^2}$$

$$\frac{t x \sin(x)}{2 t^2 x^{5/6} \sin(x) + t^2 x^{2/3} + t^2 x \sin^2(x)} + \frac{t \pi^x \tan(x)}{2 t^2 x^{5/6} \sin(x) + t^2 x^{2/3} + t^2 x \sin^2(x)}$$

Derivative:

[Approximate form](#)

[Step-by-step solution](#) 

$$\frac{\partial}{\partial x} \left(\frac{\frac{(d^2 x t) \sin(x)}{d^2} + \frac{(d^2 \pi^x t) \tan(x)}{d^2}}{\left(\frac{d^2 \sqrt[3]{x} t}{d^2} + \frac{(d^2 \sqrt{x} t) \sin(x)}{d^2} \right)^2} \right) =$$

$$\frac{(3 x (\sqrt[6]{x} \sin(x) + 1) (\sin(x) + x \cos(x) + \pi^x \sec^2(x) + \pi^x \log(\pi) \tan(x)) - (x \sin(x) + \pi^x \tan(x)) (6 x^{7/6} \cos(x) + 3 \sqrt[6]{x} \sin(x) + 2))}{(3 t x^{5/3} (\sqrt[6]{x} \sin(x) + 1)^3)}$$

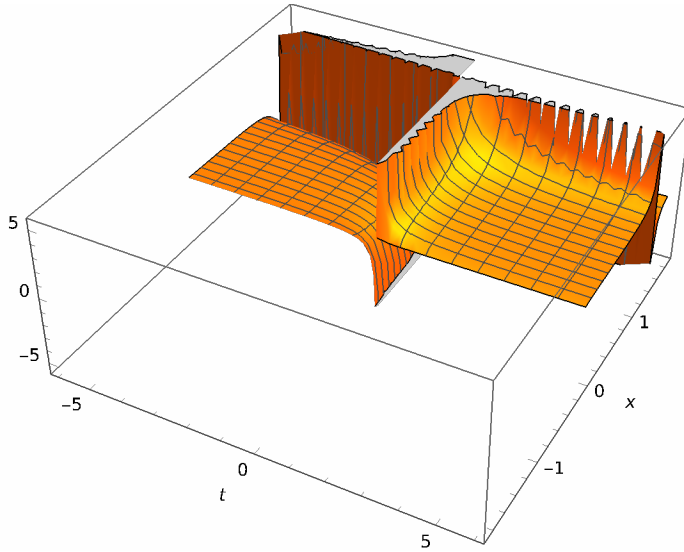
sec(x) is the secant function »

log(x) is the natural logarithm »

WolframAlpha 

```
In[24]:= Plot3D[ $\frac{t x \text{Sin}[x] + \pi^x t \text{Tan}[x]}{(t x^{1/3} + t \sqrt{x} \text{Sin}[x])^2}$ , {t, -6., 6.}, {x, -1.87476, 1.87476}]
```

Out[24]=



$$\text{Eq 21: } \frac{[(d^2 x/d^2 t) \sin x + (d^2 \pi^x/d^2 t) \tan x]^2}{[d^2 x^{1/3}/d^2 t + (d^2 \sqrt{x}/d^2 t) \sin x]}$$

In[26]=

$$\frac{(d^2 (x/d^2) * t) * \text{Sin}[x] + (d^2 * (\text{Pi}^x/d^2) * t) * \text{Tan}[x]^2}{(d^2 * (x^{1/3}/d^2) * t + (d^2 * (\text{Sqrt}[x]/d^2) * t) * \text{Sin}[x])}$$

Out[26]=

$$\frac{(t x \sin(x) + t \pi^x \tan(x))^2}{t \sqrt[3]{x} + t \sqrt{x} \sin(x)}$$

$$\text{Eq 21: } \frac{[(d^2 x/d^2 t) \sin x + (d^2 \pi^x/d^2 t) \tan x]^2}{[d^2 x^{1/3}/d^2 t + (d^2 \sqrt{x}/d^2 t) \sin x]}$$

Out[29]=

Input: +

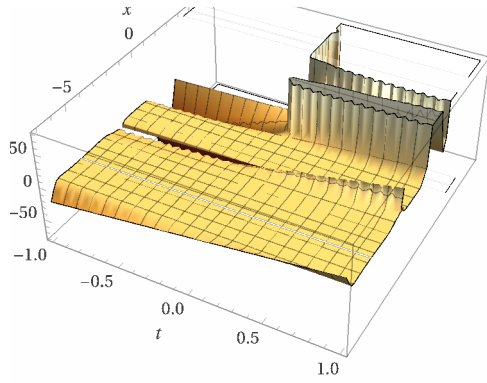
$$\frac{((d^2 \times \frac{x}{d^2} t) \sin(x) + (d^2 \times \frac{\pi^x}{d^2} t) \tan(x))^2}{d^2 \times \frac{\sqrt[3]{x}}{d^2} t + (d^2 \times \frac{\sqrt{x}}{d^2} t) \sin(x)}$$

Result: +

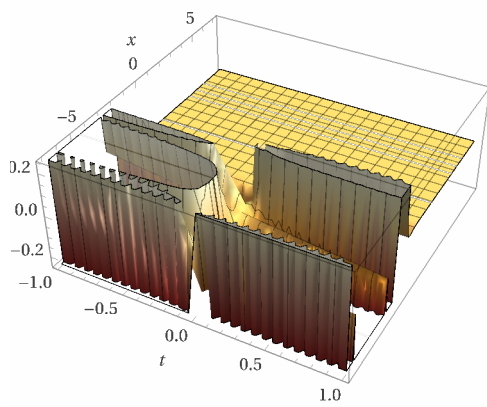
$$\frac{(t x \sin(x) + t \pi^x \tan(x))^2}{t \sqrt[3]{x} + t \sqrt{x} \sin(x)}$$

3D plots: +

Real part:



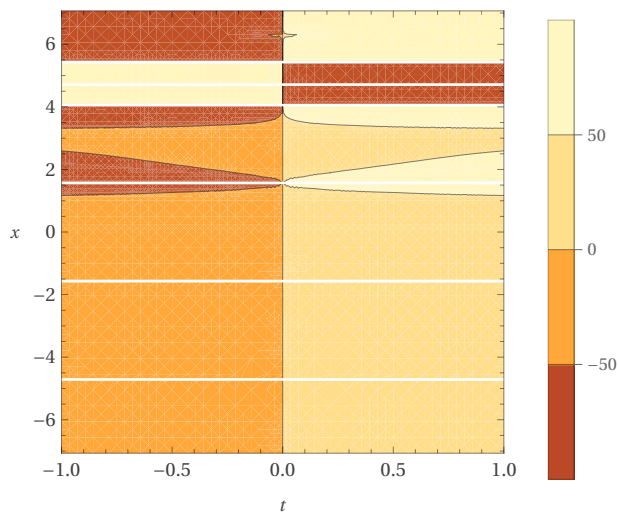
Imaginary part:



Contour plots:

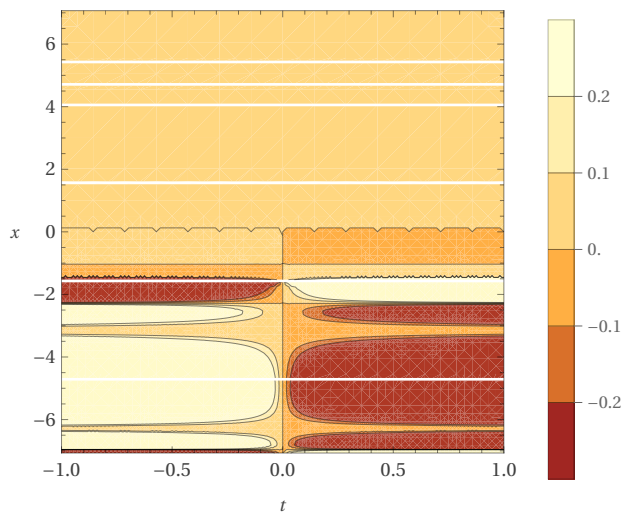


Real part:



t_{\min} t_{\max}
 x_{\min} x_{\max}

Imaginary part:



t_{\min} t_{\max}
 x_{\min} x_{\max}

Alternate forms:

[More](#)

$$\frac{t (x \sin(x) + \pi^x \tan(x))^2}{\sqrt[3]{x} + \sqrt{x} \sin(x)}$$

$$\frac{t (x \sin(x) + \pi^x \tan(x))^2}{\sqrt[3]{x} (\sqrt[6]{x} \sin(x) + 1)}$$

$$\frac{t (\pi^x + x \cos(x))^2 \tan^2(x)}{\sqrt[3]{x} (\sqrt[6]{x} \sin(x) + 1)}$$

Expanded form:

$$\frac{t^2 x^2 \sin^2(x)}{t \sqrt[3]{x} + t \sqrt{x} \sin(x)} + \frac{t^2 \pi^{2x} \tan^2(x)}{t \sqrt[3]{x} + t \sqrt{x} \sin(x)} + \frac{2 t^2 \pi^x x \sin(x) \tan(x)}{t \sqrt[3]{x} + t \sqrt{x} \sin(x)}$$

Derivative:

[Approximate form](#)[Step-by-step solution](#)

$$\frac{\partial}{\partial x} \left(\frac{\left(\frac{(d^2 x t) \sin(x)}{d^2} + \frac{(d^2 \pi^x t) \tan(x)}{d^2} \right)^2}{\frac{d^2 \sqrt[3]{x} t}{d^2} + \frac{(d^2 \sqrt{x} t) \sin(x)}{d^2}} \right) =$$

$$\frac{2 (t x \sin(x) + t \pi^x \tan(x)) (t \sin(x) + t x \cos(x) + t \pi^x \sec^2(x) + t \pi^x \log(\pi) \tan(x))}{t \sqrt[3]{x} + t \sqrt{x} \sin(x)}$$

$$\frac{(t x \sin(x) + t \pi^x \tan(x))^2 \left(\frac{t}{3x^{2/3}} + \frac{t \sin(x)}{2\sqrt{x}} + t \sqrt{x} \cos(x) \right)}{(t \sqrt[3]{x} + t \sqrt{x} \sin(x))^2}$$

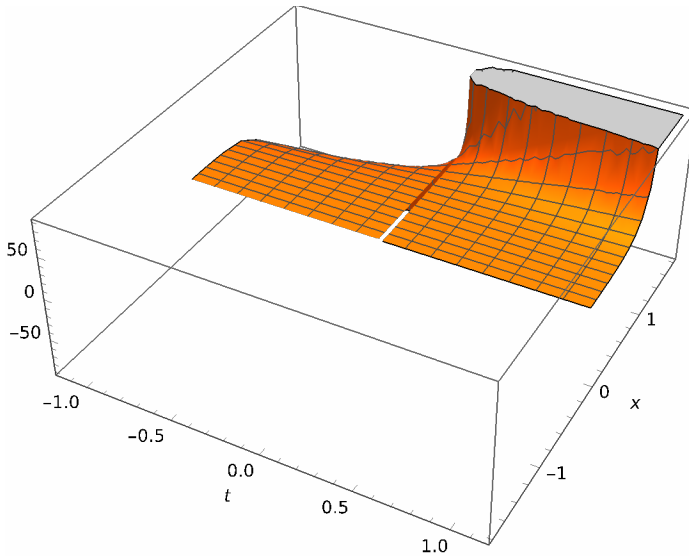
sec(x) is the secant function »
log(x) is the natural logarithm »

WolframAlpha

```
In[28]:= Plot3D[ $\frac{(t x \text{Sin}[x] + \pi^x t \text{Tan}[x])^2}{t x^{1/3} + t \sqrt{x} \text{Sin}[x]}$ , {t, -1.14412, 1.14412}, {x, -1.78005, 1.78005}]
```

```
In[27]:= Plot3D[ $\frac{(t x \text{Sin}[x] + \pi^x t \text{Tan}[x])^2}{t x^{1/3} + t \sqrt{x} \text{Sin}[x]}$ , {t, -1.14412, 1.14412}, {x, -1.78005, 1.78005}]
```

Out[27]=



$\{ \{d^{2x}/d^{2t}\} \sin x + \{d^{2\pi^x}/d^{2t}\} \tan x \} / [\{d^{2x^{1/3}}/d^{2t}\} + \{d^{2\sqrt{x}}/d^{2t}\} \sin x]^3$ Eq 22

```
In[30]:=  $\frac{(d^{2*(x/d^2)*t})*\text{Sin}[x] + (d^{2*(\text{Pi}^x/d^2)*t})*\text{Tan}[x]}{(d^{2*(x^{1/3}/d^2)*t} + (d^{2*(\text{Sqrt}[x]/d^2)*t})*\text{Sin}[x])^3}$ 
```

Out[30]=

$$\frac{t x \sin(x) + t \pi^x \tan(x)}{(t \sqrt[3]{x} + t \sqrt{x} \sin(x))^3}$$

```
In[33]:=   $\{ \{d^{2x}/d^{2t}\} \sin x + \{d^{2\pi^x}/d^{2t}\} \tan x \} / [ \{d^{2x^{1/3}}/d^{2t}\} + \{d^{2\sqrt{x}}/d^{2t}\} \sin x ]^3$ 
```

Out[33]=

Input:

input:

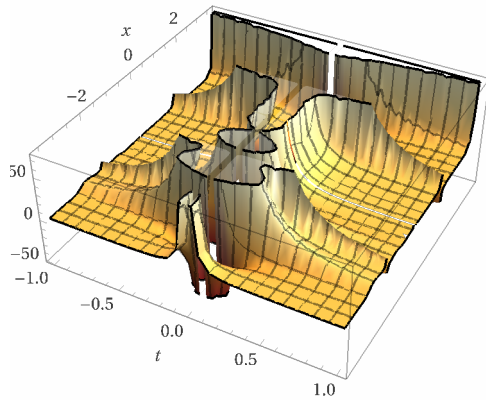
$$\frac{(d^2 \times \frac{x}{d^2} t) \sin(x) + (d^2 \times \frac{\pi^x}{d^2} t) \tan(x)}{(d^2 \times \frac{\sqrt{x}}{d^2} t + (d^2 \times \frac{\sqrt{x}}{d^2} t) \sin(x))^3}$$

Result:

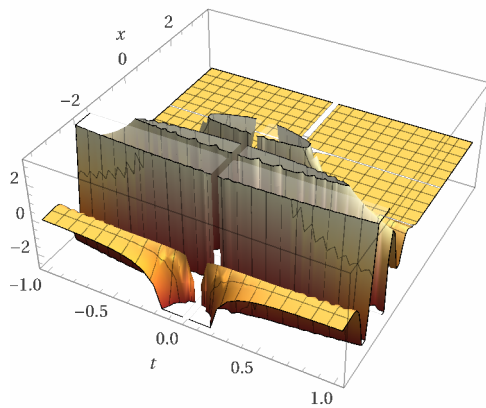
$$\frac{t x \sin(x) + t \pi^x \tan(x)}{(t \sqrt[3]{x} + t \sqrt{x} \sin(x))^3}$$

3D plots:

Real part:



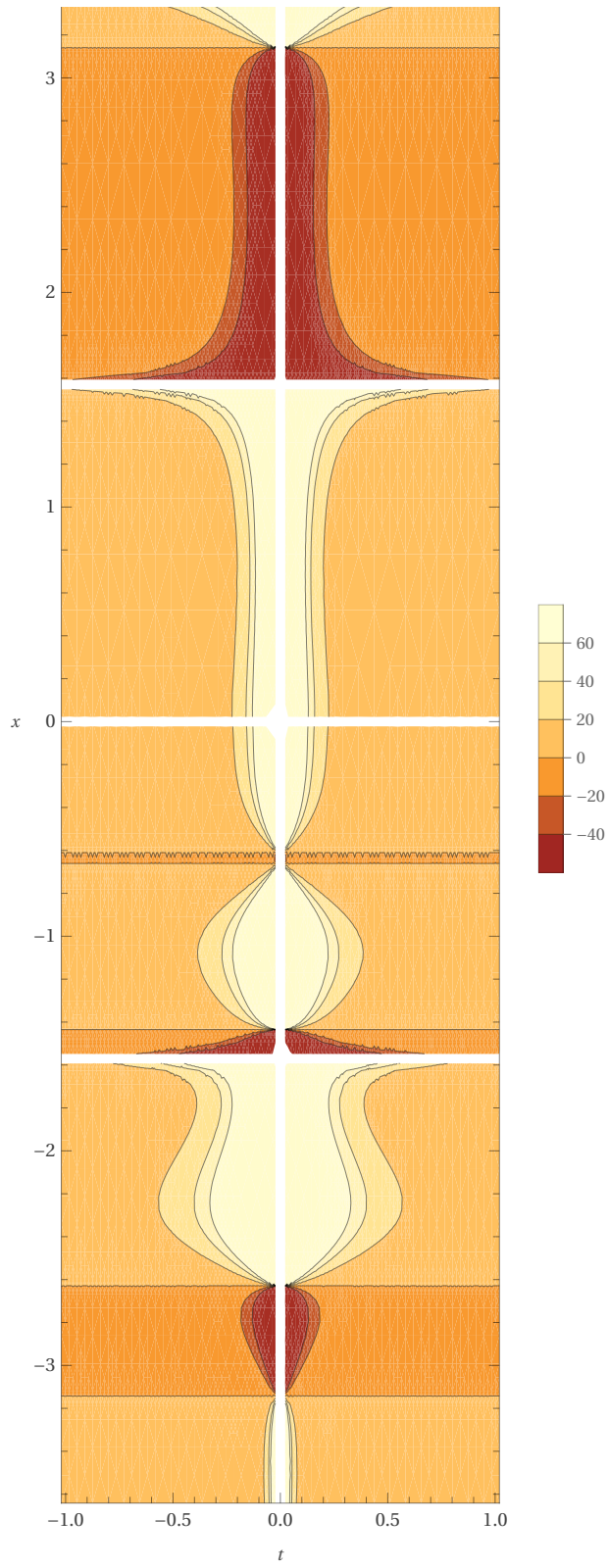
Imaginary part:



Contour plots:

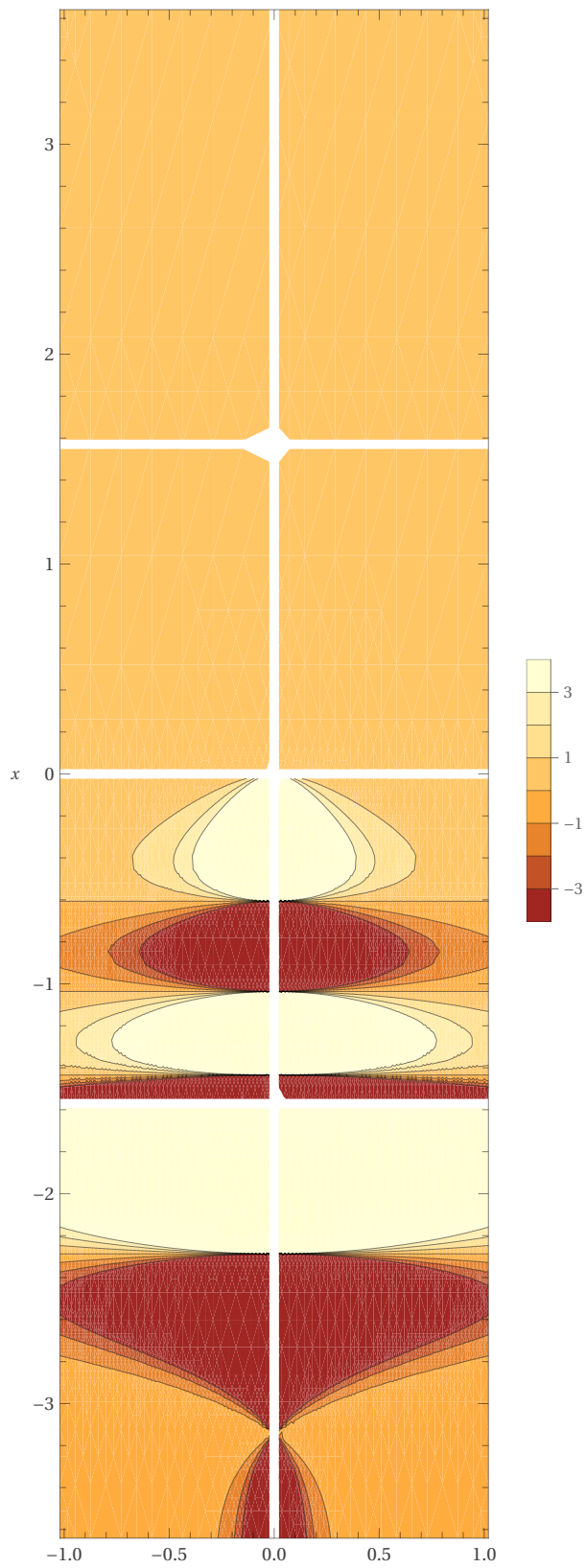
Real part:





t_{\min} t_{\max}
 x_{\min} x_{\max}

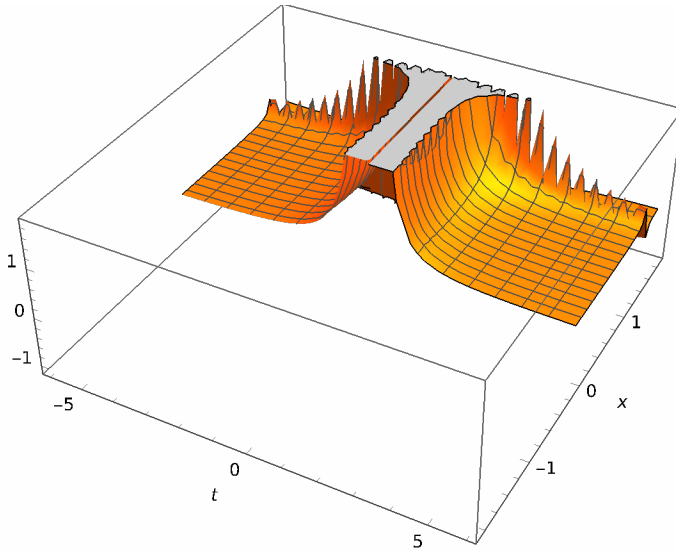
Imaginary part:




```
In[32]:= Plot3D[ $\frac{t x \sin[x] + \pi^U t \tan[x]}{(t x / + t \sqrt{x} \sin[x])}$ , {t, -6., 6.}, {x, -1.87476, 1.87476}]
```

```
In[31]:= Plot3D[ $\frac{t x \sin[x] + \pi^U t \tan[x]}{(t x / + t \sqrt{x} \sin[x])}$ , {t, -6., 6.}, {x, -1.87476, 1.87476}]
```

Out[31]=



Eq 23
$$\frac{(d^2 x / d^2 t) \sin x + (d^2 \pi^x / d^2 t) \tan x}{(d^2 x^{1/3} / d^2 t) + (d^2 \sqrt{x} / d^2 t) \sin x}$$

```
In[34]:= ((d^2*(x/d^2)*t)*Sin[x] + (d^2*(Pi^x/d^2)*t)*Tan[x])^3 / (d^2*(x^(1/3)/d^2)*t + (d^2*(Sqrt[x]/d^2)*t)*Sin[x])
```

Out[34]=

$$\frac{(t x \sin(x) + t \pi^x \tan(x))^3}{t \sqrt[3]{x} + t \sqrt{x} \sin(x)}$$

```
In[36]:=  [(d^2 x / d^2 t) sin x + (d^2 pi^x / d^2 t) tan x]^3 / [(d^2 x^1/3 / d^2 t) + (d^2 sqrt(x) / d^2 t) sin x]
```

Out[36]=

Input: +

$$\frac{\left(d^2 \times \frac{x}{d^2} t \sin(x) + d^2 \times \frac{\pi^x}{d^2} t \tan(x) \right)^3}{d^2 \times \frac{\sqrt[3]{x}}{d^2} t + \left(d^2 \times \frac{\sqrt{x}}{d^2} t \right) \sin(x)}$$

Result: +

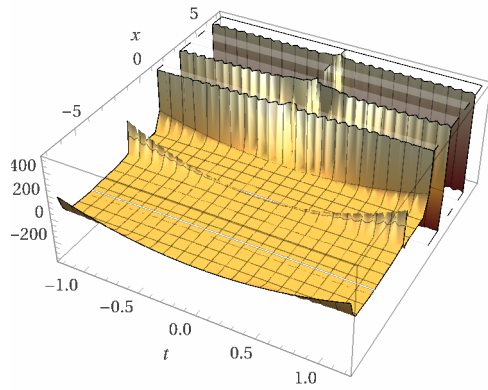
$$\frac{(t x \sin(x) + t \pi^x \tan(x))^3}{t \sqrt[3]{x} + t \sqrt{x} \sin(x)}$$

$$i \sqrt{x} + i \sqrt{x} \sin(x)$$

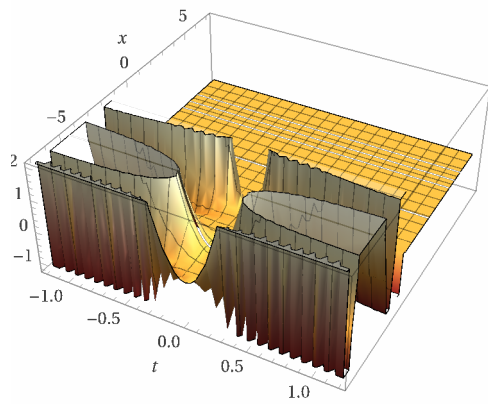
3D plots:



Real part:



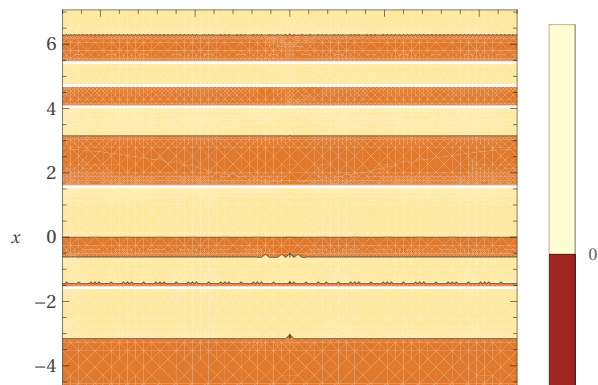
Imaginary part:

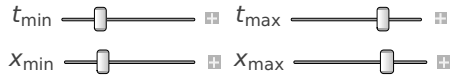
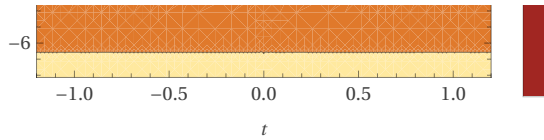


Contour plots:

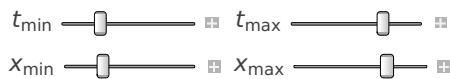
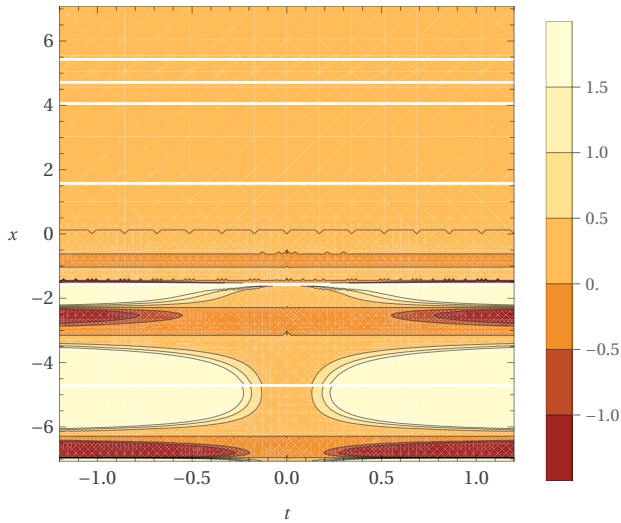


Real part:





Imaginary part:



Alternate forms:

More

$$\frac{t^2 (x \sin(x) + \pi^x \tan(x))^3}{\sqrt[3]{x} + \sqrt{x} \sin(x)}$$

$$\frac{t^2 (x \sin(x) + \pi^x \tan(x))^3}{\sqrt[3]{x} (\sqrt[3]{x} \sin(x) + 1)}$$

$$\frac{t^2 (\pi^x + x \cos(x))^3 \tan^3(x)}{\sqrt[3]{x} (\sqrt[6]{x} \sin(x) + 1)}$$

Expanded form:

$$\frac{t^3 x^3 \sin^3(x)}{t \sqrt[3]{x} + t \sqrt{x} \sin(x)} + \frac{3 t^3 \pi^x x^2 \sin^2(x) \tan(x)}{t \sqrt[3]{x} + t \sqrt{x} \sin(x)} + \frac{t^3 \pi^{3x} \tan^3(x)}{t \sqrt[3]{x} + t \sqrt{x} \sin(x)} + \frac{3 t^3 \pi^{2x} x \sin(x) \tan^2(x)}{t \sqrt[3]{x} + t \sqrt{x} \sin(x)}$$

Derivative:

Approximate form

Step-by-step solution

$$\frac{\partial}{\partial x} \left(\frac{\left(\frac{(d^2 x t) \sin(x)}{d^2} + \frac{(d^2 \pi^x t) \tan(x)}{d^2} \right)^3}{\frac{d^2 \sqrt[3]{x} t}{d^2} + \frac{(d^2 \sqrt{x} t) \sin(x)}{d^2}} \right) =$$

$$\frac{3 (t x \sin(x) + t \pi^x \tan(x))^2 (t \sin(x) + t x \cos(x) + t \pi^x \sec^2(x) + t \pi^x \log(\pi) \tan(x))}{t \sqrt[3]{x} + t \sqrt{x} \sin(x)}$$

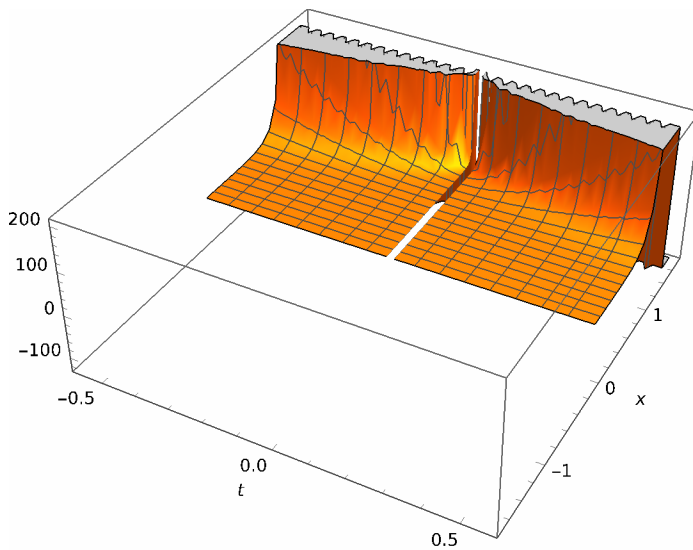
$$\frac{(t x \sin(x) + t \pi^x \tan(x))^3 \left(\frac{t}{3x^{2/3}} + \frac{t \sin(x)}{2 \sqrt{x}} + t \sqrt{x} \cos(x) \right)}{(t \sqrt[3]{x} + t \sqrt{x} \sin(x))^2}$$

sec(x) is the secant function >>
log(x) is the natural logarithm >>

WolframAlpha +

```
In[35]:= Plot3D[(t x Sin[x] + Pi^x t Tan[x])^3 / (t x^(1/3) + t Sqrt[x] Sin[x]), {t, -0.569059, 0.569059}, {x, -1.72149, 1.72149}]
```

Out[35]=



Eq 23

$$\frac{(d^2 x/d^2 t) \sin x + (d^2 \pi^x/d^2 t) \tan x^3}{(d^2 x^{1/3}/d^2 t) + (d^2 \sqrt{x}/d^2 t) \sin x}$$

```
In[37]:= ((d^2*(x/d^2)*t)*Sin[x] + (d^2*(Pi^x/d^2)*t)*Tan[x])^3 / ((d^2*(x^(1/3)/d^2)*t + (d^2*(Sqrt[x]/d^2)*t)*Sin[x])
```

Out[37]=

$$\frac{(t x \sin(x) + t \pi^x \tan(x))^3}{t \sqrt[3]{x} + t \sqrt{x} \sin(x)}$$

In[38]:= Simplify $\left[\frac{(t x \sin[x] + \pi^x t \tan[x])^3}{t x^{1/3} + t \sqrt{x} \sin[x]}\right]$

Out[38]=

$$\frac{t^2 (x \sin(x) + \pi^x \tan(x))^3}{\sqrt[3]{x} + \sqrt{x} \sin(x)}$$

E $\left\{ \left[\frac{d^2 x}{d^2 t} \sin x + \left(\frac{d^2 \pi^x}{d^2 t} \tan x \right)^3 \right] / \left[\left(\frac{d^2 x^{1/3}}{d^2 t} + \left(\frac{d^2 \sqrt{x}}{d^2 t} \right) \sin x \right)^{1/3} \right] \right\}$ Eq 24

In[1]:= $\left(\frac{d^2 (x/d^2) * \sin[x] + (d^2 (\pi^x/d^2) * t) * \tan[x]^3}{d^2 (x^{1/3}/d^2) * t + (d^2 (\sqrt{x}/d^2) * t) * \sin[x]} \right)^{1/3}$

Out[1]= $\frac{(t x \sin(x) + t \pi^x \tan(x))^3}{\sqrt[3]{t \sqrt[3]{x} + t \sqrt{x} \sin(x)}}$

In[2]:= **⚠** $\left[\frac{d^2 x}{d^2 t} \sin x + \left(\frac{d^2 \pi^x}{d^2 t} \tan x \right)^3 \right] / \left[\left(\frac{d^2 x^{1/3}}{d^2 t} + \left(\frac{d^2 \sqrt{x}}{d^2 t} \right) \sin x \right)^{1/3} \right]$

Input:

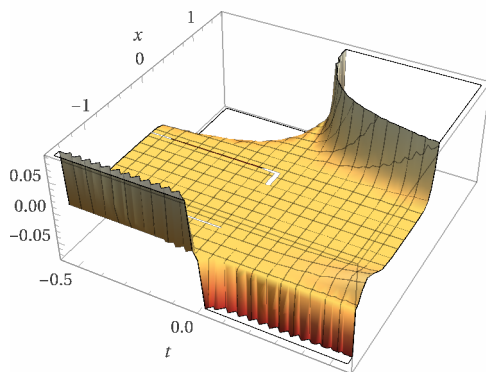
$$\frac{\left(\left(d^2 \times \frac{x}{d^2} t \right) \sin(x) + \left(d^2 \times \frac{\pi^x}{d^2} t \right) \tan(x) \right)^3}{\sqrt[3]{d^2 \times \frac{\sqrt[3]{x}}{d^2} t + \left(d^2 \times \frac{\sqrt{x}}{d^2} t \right) \sin(x)}}$$

Result:

$$\frac{(t x \sin(x) + t \pi^x \tan(x))^3}{\sqrt[3]{t \sqrt[3]{x} + t \sqrt{x} \sin(x)}}$$

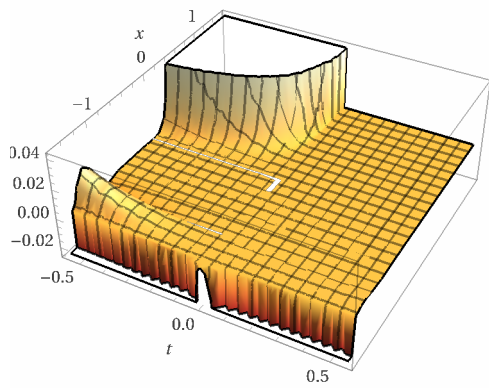
3D plots:

Real part:



0.5

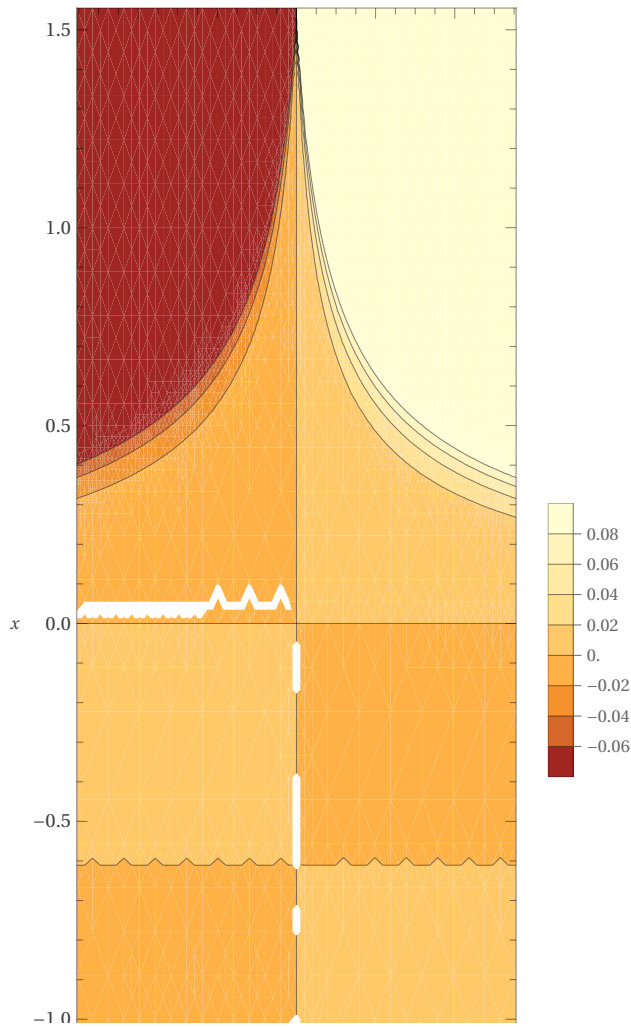
Imaginary part:



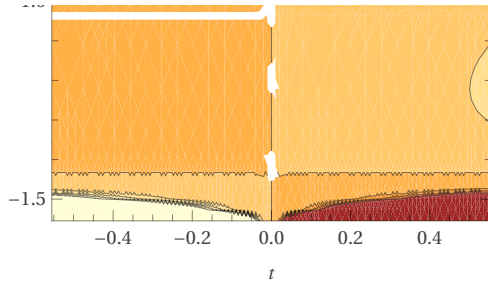
Contour plots:



Real part:

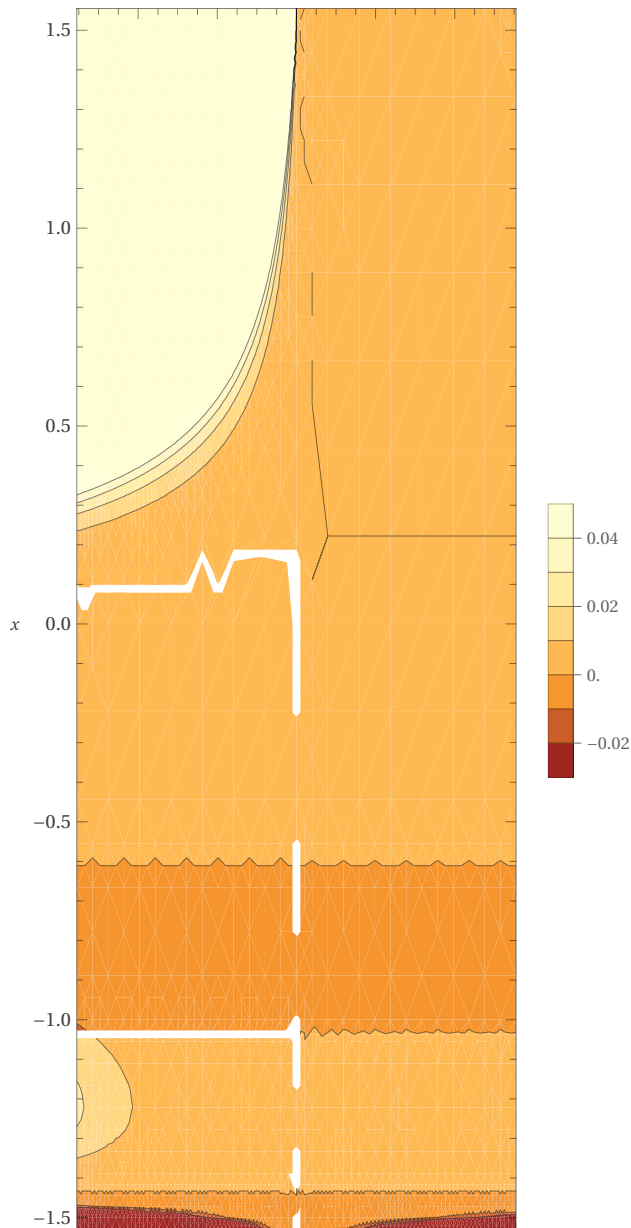


Out[2]=



t_{\min} t_{\max}
 x_{\min} x_{\max}

Imaginary part:





Alternate forms:

More +

$$\frac{t^3 (\pi^x + x \cos(x))^3 \tan^3(x)}{\sqrt[3]{t (\sqrt[3]{x} + \sqrt{x} \sin(x))}}$$

$$\frac{t^3 (x \sin(x) + \pi^x \tan(x))^3}{\sqrt[3]{t \sqrt[3]{x} (\sqrt[6]{x} \sin(x) + 1)}}$$

$$\frac{t^3 (x \sin(x) + \pi^x \tan(x))^3}{\sqrt[3]{t \sqrt[3]{x} (\sqrt[6]{x} \sin(x) + 1)}}$$

$$\frac{\left(t x \sin(x) + \frac{\pi^x t \sin(x)}{\cos(x)}\right)^3}{\sqrt[3]{t \sqrt[3]{x} + t \sqrt{x} \sin(x)}}$$

$$\frac{\left(t x \sin(x) + \frac{\pi^x t \sin(x)}{\cos(x)}\right)^3}{\sqrt[3]{t \sqrt[3]{x} + t \sqrt{x} \sin(x)}}$$

$$\frac{\left(t x \sin(x) + \frac{\pi^x t \sin(x)}{\cos(x)}\right)^3}{\sqrt[3]{t \sqrt[3]{x} + t \sqrt{x} \sin(x)}}$$

Expanded form:

+

$$\frac{t^3 x^3 \sin^3(x)}{\sqrt[3]{t \sqrt[3]{x} + t \sqrt{x} \sin(x)}} + \frac{3 t^3 \pi^x x^2 \sin^2(x) \tan(x)}{\sqrt[3]{t \sqrt[3]{x} + t \sqrt{x} \sin(x)}} +$$

$$\frac{t^3 \pi^{3x} \tan^3(x)}{\sqrt[3]{t \sqrt[3]{x} + t \sqrt{x} \sin(x)}} + \frac{3 t^3 \pi^{2x} x \sin(x) \tan^2(x)}{\sqrt[3]{t \sqrt[3]{x} + t \sqrt{x} \sin(x)}}$$

WolframAlpha +

Eq 25 $\left\{ \frac{d^2 x \sin(x) \tan(x)}{d^2 \pi^x} \right\} / \left\{ \frac{d^2 x^{1/3}}{d^2} \sqrt{x} \sin(x) \right\}$

In[3]:= $\frac{(d^2 * (x / d^2) * t) * \sin[x] * (d^2 * (\pi^x / d^2) * t) * \tan[x]}{(d^2 * (x^{1/3} / d^2) * t) * (d^2 * (\sqrt{x} / d^2) * t) * \sin[x]}$

Out[3]= $\pi^x \sqrt[6]{x} \tan(x)$

Eq 25 $\left\{ \frac{d^2 x \sin(x) \tan(x)}{d^2 \pi^x} \right\} / \left\{ \frac{d^2 x^{1/3}}{d^2} \sqrt{x} \sin(x) \right\}$

In[5]:= **Eq 25** $\left\{ \frac{d^2 x \sin(x) \tan(x)}{d^2 \pi^x} \right\} / \left\{ \frac{d^2 x^{1/3}}{d^2} \sqrt{x} \sin(x) \right\}$

Input:

+

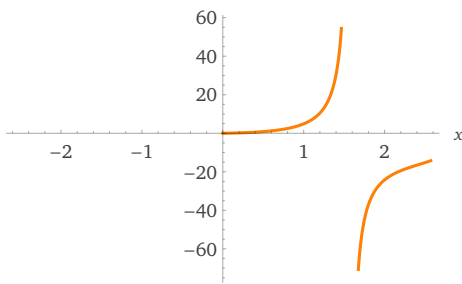
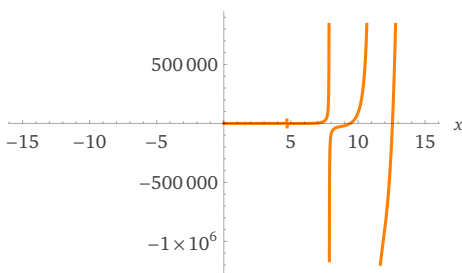
$$\left(d^2 \times \frac{x}{d^2} t\right) \sin(x) \left(d^2 \times \frac{\pi^x}{d^2} t\right) \tan(x)$$

$$\left(d^2 \times \frac{\sqrt[3]{x}}{d^2} t\right) \left(d^2 \times \frac{\sqrt{x}}{d^2} t\right) \sin(x)$$

Result:

$$\pi^x \sqrt[6]{x} \tan(x)$$

Plots:

Real-valued plots +min max min max

Alternate forms:

$$\frac{\pi^x \sqrt[6]{x} \sin(x)}{\cos(x)}$$

$$\frac{i(e^{-ix} - e^{ix}) \pi^x \sqrt[6]{x}}{e^{-ix} + e^{ix}}$$

Roots:

Step-by-step solution +

(no roots exist)

Series expansion at x = 0:

$$x^{7/6} + x^{13/6} \log(\pi) + \frac{1}{6} x^{19/6} (2 + 3 \log^2(\pi)) + \frac{1}{6} x^{25/6} \log(\pi) (2 + \log^2(\pi)) + O(x^{31/6})$$

Out[5]=

(Puiseux series)

log(x) is the natural logarithm »

Big-O notation »

Derivative:

Approximate form

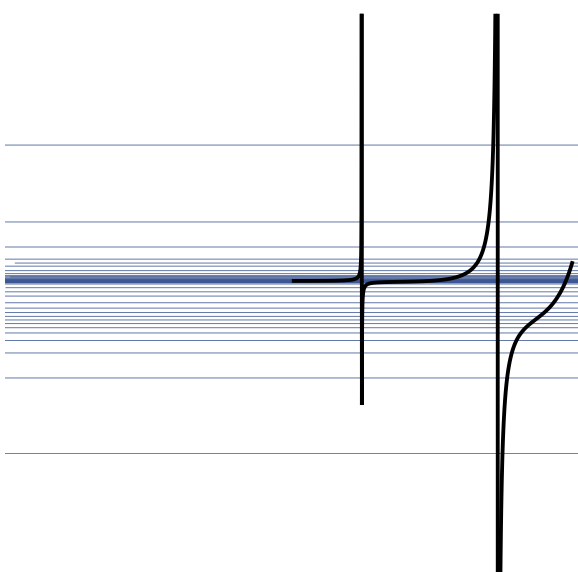
Step-by-step solution +

$$\frac{d}{dx} \left(\frac{(d^2 x t) \sin(x) (d^2 \pi^x t) \tan(x)}{(d^2 d^2) ((d^2 \sqrt[3]{x} t) (d^2 \sqrt{x} t) \sin(x))} \right) = \frac{\pi^x (\tan(x) + 6 x \sec^2(x) + 6 x \log(\pi) \tan(x))}{6 x^{5/6}}$$

sec(x) is the secant function »

Differential geometric curves:

+



— $\pi^x \sqrt[6]{x} \tan(x)$ — normals

Horizontal plot range:

x_{\min} x_{\max} symmetric

+ More controls

WolframAlpha +

```
In[4]:= Plot[ $\pi^x x^{1/6} \tan[x]$ , {x, -15.4248, 15.4248}]
```

