Generation of Generalized Spiraling Bessel Beams by The Illumination A Curved Fork-Shaped Hologram with New Hollow Laser Beam

Faroq Saad*
Assistant Professor,
Faculty of Engineering,
Al-Janad University for Science & Technology,
Taiz, Yemen, and
Technical community college,
Taiz, Yemen.

Abdelmajid Belafhal*
Professor,
Laboratory LPNAMME, Laser Physics Group,
Department of Physics,
Faculty of Sciences,
Chouaïb Doukkali University,
P. B 20, 24000 El Jadida, Morocco.

Abstract:- In the present paper, we study the generation of spiraling beams which introduced by the illumination of Flat-topped vortex hollow beam (FtVHB) with curved fork-shaped hologram (CFH). Based on the Fresnel diffraction integral formula, analytical expression for the output amplitudes of the produced beam after diffraction is derived. The variation of the intensity profile of the diffracted beam in longitudinal and radial directions depends on the effect of the incident and the spiraling output fields parameters are given with numerical examples. The present work gives more general characteristic and diffraction by a CFH because the Flat-topped and Gaussian beams.

Keywords:- Spiraling Beams; Curved Fork-Shaped Hologram; Flat-Topped Vortex Hollow Beam.

I. INTRODUCTION

Zero intensity with hollow beam in its center have increasing interest attentions because of their higher applications [1], [2],[3], [4]. The hollow laser beam with different intensity can be generated by applied various methods [5], [6], [7], [8],[9],[10],[11],[12],[13]. Properties of the hollow laser mode have been devoted with several models including: Hollow Gaussian beam [14], High order Bessel beam [15] and Controllable-dark hollow beam [16]. The optical field of Laguerre Gaussian beam [17], Bessel-Gaussian beam [18], Hermite -cosine Gaussian beam [19], Cosh-Gaussian beam [20],[21] and double-half inverse Gaussian hollow beam [22], have been studied. In addition, a novel family of doughnut mode as a superposition of Kummer beam have been investigated [23],[24],[25]. The generalized Humbert beams as a new kind of dark beam is demonstrated by our research group [26],[27]. The vortex beam of Hermite-Cosh-Gaussian and its diffraction have also been introduced [28]. On the other side, Liu et al. [29] have presented a theoretical model to describe dark hollow beam called Flat-topped vortex hollow beam which was obtained from the modulation of Flat-topped mode [30], with spiral phase plate (SPP). Besides, many papers carried out have been studied to this beam in different optical systems [31],[32],[33],[34],[35]. In last few years, the optical field of

vortex beams have found a considerable attention of applications [2],[3],[36],[37],[38]. These vortex beams can be produced by using the diffractive optical elements (DOEs) [39],[40],[41]. Several theoretical and experimental studies are carried out by applied different methods to generate optical vortices with laser beam [42],[43], [44],[45], [46], [47],[48],[49],[50],[51],[52],[53],[54],[55],[56],[57],[58],[59] ,[60],[61],[62],[63]. The nondiffracting vortex beams can be generated by using a SPP with a Gaussian beam [42]. In addition, these authors [43] have investigated an important technique method to produce quasi-Bessel light beams by using the SPP and axicon. In the end of the last century, the conversion of the Gaussian mode by means of fork-shaped holograms has been investigated [44],[45]. On other hand, in 1987, Durin [46] has introduced the first nondiffracting beams which their wave amplitudes consisting of Bessel modes. The higher-order Bessel beams possesses a dark region surrounded with bright and dark rings compared with the zeroth-order Bessel beam that have a central bright intensity. The generation of Bessel beams with zeroth-order through an annular aperture at focal plane of a lens, with various technical methods has been studied [47], [6], [48]. In 1989, Vasara et al have introduced a nondiffracting Bessel beam of higher order [49]. They observed that the higher order Bessel directly generated from Gaussian beams by using the axicon-type-computer generated holograms. Technical method has demonstrated by the illumination of an axicon with Laguerre-Gauss (LG) mode to produce high order Bessel beams [15]. In the end of the last decay, a theoretical analysis about diffraction of Fraunhofer for Gaussian laser amplitude that generated using helical axicon (HA) with SPP [50] and fork-shaped gratings [51] are derived, separately. Furthermore, the diffraction of an incident LG beam by the use of a HA has been presented [52],[53]. In addition, Sun et al. [54],[55] have proposed two different methods to transform the LG beam into a generalized spiraling Bessel beam(GSBB): first one using an aperture axicon and a hologram and the later with a HA. Topuzoski [56] has introduced and observed the optical vortices directly generated by CFH for Gaussian laser beams. Recently, the theoretical analysis of generation GSBB by means of CFH using LG, hypergeometric-Gaussian, Bessel-Gaussian and dark/antidark Gaussian beams have been

investigated [57],[58],[59],[60]. In addition, Saad et al [61] have studied the effect of turbulent atmosphere on GSBB optical field. The (l,n)th mode LG diffracted by a fork - shaped grating has been introduced [62]. More recently, the Fresnel diffraction for generalized Humbert-Gaussian beams by HA has been demonstrated [63]. In the present work, we will explore and investigate how the GSBB can be generated in the Fresnel diffraction process of FtVHB by the CFH. However, to the best of our knowledge, the transformation of the FtVHB by the use of a CFH into GSBB has not been investigated yet. Analytical expressions for the FtVHB diffracted by the CFH are derived in Section 2. Numerical simulations of the diffracted beam are computed and discussed in Section 3. In final Section, we conclude our results.

II. THEORTICAL MODELS

➤ *Distribution incidnet field of the FtVHB*

In the cylindrical coordinate, the field distribution of FtVHB in input plane z=0, is defined by [29]

$$U(r,\varphi,0) = \left(\frac{r}{\omega}\right)^{M} \exp(iM\varphi)\sum_{n=1}^{N} a_n \exp\left(-\frac{nr^2}{\omega^2}\right), \quad (1)$$

Where

$$a_n = \frac{(-1)^{n-1}}{N}\binom{N}{n},$$

N=1, 2,… is the FtVHB order, M is the topological charge and (r, φ represent the cylindrical coordinates. ω is the width of beam at z=0, for Gaussian mode (N=1 and M=0).

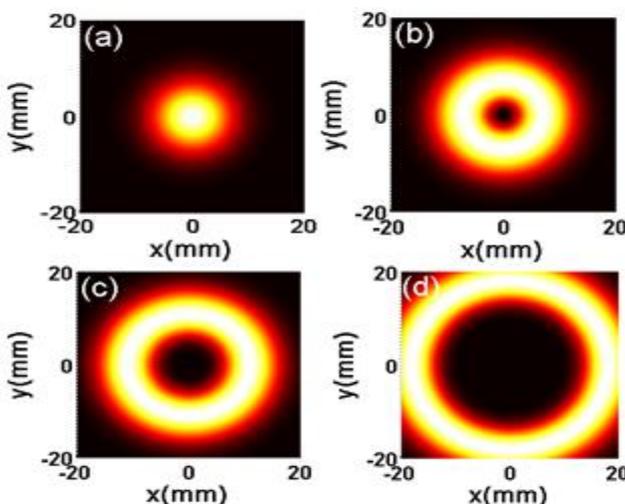Fig 1 Transverse intensity of incident FtVHB at z=0 with ω=10mm for (a) N=1 and M=0 (b) N= M=1 (c) N= M=2 and (d) N= M=6.

From Fig. 1(a), when the order of incident beam are N=1 and M=0, the considered beam intensity changes into a Gaussian shape. For nonzero topological charge (M>0), the central dark spot of zero intensity is shown. This dark center region increases with increasing N and M.

➤ *Conversion of FtVHB into spiraling Bessel beam with CFH*

We present a general theoretical method to produce a novel laser family by converting a FtVHB with a CFH into optical vortex of Bessel beams. Let us first consider based on the Fresnel diffraction process, the interference of a computer-generated hologram by using a wave of optical vortex, whose wave front identical to that of helical axicon phase and a wave of tilted plane with two constant real amplitudes A_1 and A_2. Both interfering waves are given by the following forms [56].

$$U_1(r,\varphi,z) = A_1 \exp[i(kz - \alpha r + p\varphi)], \quad (2)$$

And

$$U_2(r,\varphi,z) = A_2 \exp[i(kz + k_x r \cos(\varphi))], \quad (3)$$

With $\alpha = k(n_r - 1)\gamma$ indicate the axicon parameter, where $k = 2\pi/\lambda$, γ and n_r are being the number of wave with the wavelength λ, the axicon base angle and the refractive index, respectively. p is an integer topological charge of HA, and $k_x = k \sin(\varepsilon)$ represents the wave component with an angle □ along x-axis. The intensity distribution of interfering waves at z=0, is given by

$$I(r,\varphi,z=0) = A_1^2 + A_2^2 + 2A_1A_2\cos(\alpha r - p\varphi + k_x r \cos\varphi). \quad (4)$$

Fig. 2 describe the effect of γ on the transverse intensity of the holograms at z = 0, calculated from Eq. (4), with fixed parameters are $A_1 = A_2 = 1$, $\varepsilon = 5°$, p = 2, $n_r = 1.48$ and λ= 810 nm.
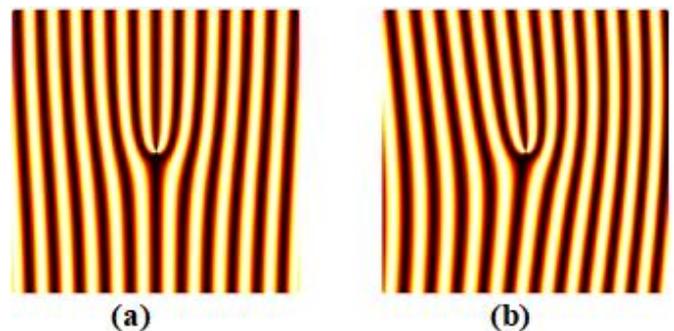

Fig 2 (a) Fork-shaped hologram with γ =0°, and **(b)** curved fork-shaped hologram with γ =1.35°

Fig. 2a shows the description of the hologram with a fixed value of p = 2 and zero angle γ. We observe from this figure that the fork shaped is clear while in Fig. 2(b) show that the curve of the fork-shaped is formed and becomes more observable with increasing γ. In the approximated of thin transparency, the general function of transmission for the CFH in polar coordinate system for amplitude and phase holograms are defined by [56]

$$T(r,\varphi) = \sum_{m=-\infty}^{m=+\infty} t_m \exp[-im(\alpha r - p\varphi + \beta r \cos\varphi)], \quad (5)$$

With t_m represents the transmission coefficients defined in Ref. [51], $\exp[-i(\alpha r - p\varphi)]$, denotes the HA transmittance and $\beta = 2\pi/d$, being the spatial frequency for x direction, and d is a constant determining the hologram period. We are considering in Fig.3, the schematic view to describe the optical setup by CFH of an arbitrary integer charge p, which leads the conversion of FtVHB of topological charge order M, into spiraling Bessel-like beams with orders have opposite signs $(M - mp)$ and $(M + mp)$.
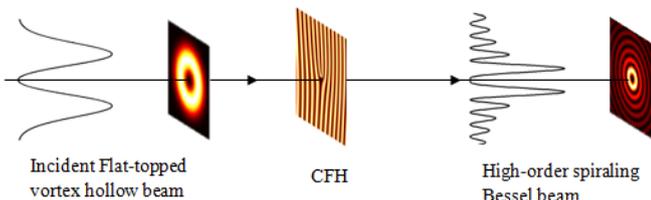

Fig 3 An optical setup description for generating spiraling beams.

In order to study the creation of spiraling beam transforming of FtVHB by CFH, an optical field distribution placed a distance z from the plane of CFH is described by the use of Fresnel-Kirchhoff diffraction integral [64]

$$U(\rho,\theta,z) = \frac{ik}{2\pi z}\exp\left[-ik\left(z+\frac{\rho^2}{2z}\right)\right]\iint_{\Delta}T(r,\varphi)U(r,\varphi,0)$$

$$\times\exp\left[-i\frac{k}{2}\left(\frac{r^2}{z}-\frac{2r\rho\cos(\varphi-\theta)}{z}\right)\right]rdrd\varphi, \quad (6)$$

Where $U(r,\varphi,0)$ represents the incident field amplitude at z=0 and Δ being the area of the element optical diffraction of hologram. This equation is expressed by the sum of the zeroth order U_0 and both positive U_{+m} and negative U_{-m} higher orders diffraction, respectively.

$$U(\rho,\theta,z)=U_0(\rho,\theta,z)+\sum_{m=1}^{+\infty}U_{+m}(\rho,\theta,z)+\sum_{m=1}^{+\infty}U_{-m}(\rho,\theta,z), \quad (7)$$

Where $U_0(\rho,\theta,z)$ is zeroth–diffraction order and $U_{\pm m}(\rho,\theta,z)$ is higher–diffraction order for the output field amplitudes. By inserting Eqs. (1) and (5) in Eq. (6), we obtain

$$U_0(\rho,\theta,z) = \frac{ik}{2\pi z}\left(\frac{1}{\omega}\right)^M t_0 \exp\left[-ik\left(z+\frac{\rho^2}{2z}\right)\right]$$

$$\times\sum_{n=1}^{N}a_n\int_{0}^{\infty}\int_{0}^{2\pi}r^M\exp\left(-\frac{nr^2}{\omega^2}\right)\exp\left(-i\frac{kr^2}{2z}\right) \quad (8)$$

$$\times\exp(iM\varphi)\exp\left[ik\left(\frac{r\rho\cos(\varphi-\theta)}{z}\right)\right]rdrd\varphi,$$

And

$$U_{\pm m}(\rho,\theta,z) = \frac{ik}{2\pi z}\left(\frac{1}{\omega}\right)^M t_{\pm m}\exp\left[-ik\left(z+\frac{\rho^2}{2z}\right)\right]$$

$$\times\sum_{n=1}^{N}a_n\int_{0}^{\infty}\int_{0}^{2\pi}r^M\exp\left(-\frac{nr^2}{\omega^2}\right)\exp\left(-i\frac{kr^2}{2z}\right) \quad (9)$$

$$\times\exp(iM\varphi)\exp\left[ik\left(\frac{r\rho\cos(\varphi-\theta)}{z}\right)\right]$$

$$\times\exp\left[\pm im(p\varphi-\alpha r)\right]\exp(\mp im\beta r\cos\varphi)rdrd\varphi.$$

These last equations can be calculated analytically. To calculate the first part of Eq. (8), we will use the following well-known integrals [65], [66]

$$\int_{0}^{2\pi}\exp\left[in\varphi+i\frac{kr\rho}{z}\cos(\varphi-\theta)\right]d\varphi = 2\pi(i)^n\exp(in\theta)J_n\left(\frac{kr\rho}{z}\right), \quad (10)$$

And

$$\int_{0}^{\infty}x^{\nu+1}\exp(-\alpha x^2)J_\nu(\beta x)dx = \frac{\beta^\nu}{(2\alpha)^{\nu+1}}\exp\left(\frac{-\beta^2}{4\alpha}\right), \quad (11)$$

With $\mathrm{Re}\,\alpha>0$, $\mathrm{Re}\,\nu>-1$ and after some developments, we obtain the output amplitude field for zeroth-order diffraction order as

$$U_0(\rho,\theta,z) = t_0\left(\frac{\rho}{\omega}\right)^M\exp(iM\theta)$$

$$\times\sum_{n=1}^{N}a_n\left(\frac{q(0)}{q_n(z)}\right)^{M+1}\exp\left(-\frac{n\rho^2}{\omega_n^2(z)}\right) \quad (12)$$

$$\times\exp\left[-ik\left(z+\frac{\rho^2}{2R_n(z)}\right)\right],$$

Where $\omega_n(z)=\omega(1+(nz/z_0)^2)^{1/2}$ denotes the radius of the transverse beam amplitude, $z_0=k\omega^2/2$ being the Rayleigh distance and $q_n(z)=nz+iz_0$ is the complex beam parameter at distance z, which is related with its curvature wavefront radius $R_n(z)=z(1+(z_0/nz)^2)$. Note that, from Eq. (12), we can deduce two particular cases. The first case, for $N=1$ and $M=0$ one obtains a formula which is in agreement with by Eq. (4) of Ref. [56] which represents the zeroth-diffraction order for Gaussian mode. The second case, when $N>1$ and $M=0$, Eq. (12) gives the output field amplitude of the zeroth-diffraction order in the form of a chargeless Flat-topped beam as follows

$$U_0(\rho,\theta,z) = t_0\sum_{n=1}^{N}a_n\frac{q(0)}{q_n(z)}$$

$$\times\exp\left(-\frac{n\rho^2}{\omega_n^2(z)}\right)\exp\left[-ik\left(z+\frac{\rho^2}{2R_n(z)}\right)\right]. \quad (13)$$

In the second part and in order to calculate the wave amplitudes of higher-diffraction-order presented into Eq. (9), we will write the transformation variables concerning in the

observation plane as $\rho\cos\theta \mp mz\lambda/D = \rho_{\pm m}\cos\theta_{\pm m}$, and $\rho\sin\theta = \rho_{\pm m}\sin\theta_{\pm m}$ [51]. Then, by the use the well-known integral of Eq. (10) again, and after some algebraic calculations, Eq. (9) can be expressed as

$$U_{\pm m}\left(\rho_{\pm m},\theta_{\pm m},z\right) = \frac{ik}{z}t_{\pm m}\frac{(\pm 1)^{M+|mp|}}{\omega^M}\exp\left[-ik\left(z+\frac{\rho_{\pm m}^2}{2z}\right)\right]$$
$$\times \exp\left[i\left(M+|mp|\right)\left(\theta_{\pm m}\pm\frac{\pi}{2}\right)\right]\sum_{n=1}^{N}a_n Y_{\pm m}^n, \quad (14)$$

Where

$$Y_{\pm m}^n = \int_0^\infty r^M \exp\left(-\frac{nr^2}{\omega^2}\right)J_{M+|mp|}\left(\frac{k\rho_{\pm m}}{z}r\right)\exp\left[-ik\left(\frac{r^2}{2z}\pm\frac{m\alpha}{k}r\right)\right]r\,dr. \quad (15)$$

The integral of Eq. (15) can be expanded analytically from the function of axicon to Taylor series [66] as:

$$\exp(\mp im\alpha r) = \sum_{s=0}^{\infty}\frac{(\mp im\alpha r)^s}{s!}, \quad (16a)$$

And then recalling the following radial integral in Eq. (6.631-1) of Ref. [66] as

$$\int_0^\infty x^\mu e^{-\alpha x^2}J_\nu(\beta x)dx \quad (16b)$$
$$= \frac{\beta^\nu\Gamma\left(\frac{1}{2}(\mu+\nu+1)\right)}{2^{\nu+1}\alpha^{\frac{1}{2}(\mu+\nu+1)}\Gamma(\nu+1)}\,_1F_1\left(\frac{1}{2}(\mu+\nu+1);\nu+1;-\frac{\beta^2}{4\alpha}\right)$$

With $\left[\mathrm{Re}\,\alpha > 0,\ \mathrm{Re}(\mu+\nu) > -1\right]$, we obtain after some manipulations, the output field of the produced beam as

$$U_{\pm m}\left(\rho_{\pm m},\theta_{\pm m},z\right) = it_{\pm m}(-1)^{M+|mp|}\frac{(k/2z)^{M+|mp|+1}}{\omega^M}$$
$$\times\exp\left[i\left(M+|mp|\right)\left(\theta_{\pm m}\pm\frac{\pi}{2}\right)\right]\exp\left[-ik\left(z+\frac{\rho_{\pm m}^2}{2z}\right)\right] \quad (17)$$
$$\times\sum_{s=0}^{\infty}\sum_{n=1}^{N}a_n\frac{(\mp im\alpha)^s\left(z\omega^2/q_n(z)\right)^{M+\frac{s+|mp|}{2}+1}}{\Gamma(s+1)\Gamma(M+|mp|+1)}\Gamma\left(M+\frac{s+|mp|}{2}+1\right)$$
$$\times\rho_{\pm m}^{M+|mp|}\,_1F_1\left(M+\frac{s+|mp|}{2}+1;M+|mp|+1;-\frac{k^2\omega^2\rho_{\pm m}^2}{4zq_n(z)}\right),$$

Where $_1F_1(.)$ and $\Gamma(.)$ are the Kummer function and the Gamma function, respectively. It is clearly from Eq. (17) that, the output beams have a phase singularity carrying topological charge $M+|mp|$. This analytical result is obtained as a function of Kummer with complex argument and the diffracted field expressed in infinite sum of Kummer function. Thus, it is not suitable for analytical expression in the case which can generate the vortex radius. Vasara et al

[49] have proposed a simple approximation for solving the Fresnel diffraction problem for plane wave by axicon-type diffractive elements using the radial integral of stationary phase method given by $\int_0^d rJ_n(kr\rho/z)\exp\left[ik\left(r^2/2z - 2\pi r/r_0 k\right)\right]dr$ with d is the radius of thin circular hologram and r_0 is a constant.

The waves, produced by such optical elements, are described by Bessel function. In 1996, Paterson and Smith [8] have investigated the theoretical study to improve the validity of the method of stationary phase starting to consider the variation in the integrand due to the Bessel function. Then, by comparing the result of ideal Bessel wave with that one obtained by the method of stationary phase in Ref. [49], they have observed that a very good approximation. The authors in Ref. [8] show that the validity of obtained results when the variation in the small factor $rJ_n(kr\rho/z)$ over the region of stationary phase, i.e., the Bessel function region required a much larger compared the width of the stationary phase. This requirement leads to the proved condition $\rho^2 \ll \lambda z/4$. Then, based on the stationary phase [15,49,53,64], we will evaluate the radial integral of Eq. (15), thus obtaining the output field expression for describing the diffraction interval and vortex radius as [49]

$$\int_0^\infty f(r)\exp\left[-ik\,\mu(r)\right]dr = \frac{f\left(r_{c,\pm}\right)\exp\left(-ik\,\mu\left(r_{c,\pm}\right)\right)}{\sqrt{k\,\mu''\left(r_{c,\pm}\right)}} \quad (18)$$

Where

$$f(r) = r^{M+1}\exp\left(-\frac{nr^2}{\omega^2}\right)J_{M+|mp|}\left(\frac{k\rho_{\pm m}}{z}r\right), \quad (19a)$$

And

$$\mu(r) = \left(\frac{r^2}{2z}\pm\frac{m\alpha}{k}r\right) \quad (19b)$$

Where the stationary points for the integrals $Y_{\pm m}^n(r)$, obtained as the root of the equations $\mu'(r) = 0$, are written as: $r_{c,\pm} = \mp m\alpha z/k$. $\mu''(r_{c,\pm}) = 1/z$ denotes the second derivative value at the critical point. Then, after inserting the integral representation of Eq. (18) into Eq. (14), the final expression for the amplitudes of higher-diffraction-order of spiraling Bessel is

$$U_{\pm m}\left(\rho_{\pm m},\theta_{\pm m},z\right) = (\mp i)\frac{(-1)^{|mp|}}{(\sqrt{2})^M}t_{\pm m}\left(\frac{\omega ka_m}{\sqrt{2}}\right)^{1/2}\exp\left[i\left(M+|mp|\right)\left(\theta_{\pm m}+\frac{\pi}{2}\right)\right]$$
$$\times\exp\left[-ik\left(z+\frac{\rho^2}{2z}-\frac{za_m^2}{2}\right)\right]\left(\frac{\sqrt{2}z}{(\omega/a_m)}\right)^{M+1/2} \quad (20)$$
$$\times\sum_{n=1}^{N}a_n\exp\left(-\frac{nz^2}{(\omega/a_m)^2}\right)J_{M+|mp|}\left(ka_m\rho_{\pm m}\right),$$

Where $a_m = m\alpha/k$. Eq. (20) describes the created spiraling Bessel beam with phase singularity carrying topological charge $M+|mp|$. Eq. (20) shows both $\pm m$-diffraction-order for optical vortex beams. $M+mp$. and $M-mp$, indicate the opposite helicity directions of their wave fronts. Then, the intensity distribution for higher-diffraction-order beams defined by

$$I(\rho_{\pm m}, \theta_{\pm m}, z) \propto |U(\rho_{\pm m}, \theta_{\pm m}, z)|^2$$

$$I_{\pm m}(\rho_{\pm m}, \theta_{\pm m}, z) = \frac{1}{2^M}|t_{\pm m}|^2 \left(\frac{\omega k a_m}{\sqrt{2}}\right)\left(\frac{2z^2}{(\omega/a_m)^2}\right)^{M+1/2}$$

$$\times \left[\sum_{n=1}^{N} a_n \exp\left(-\frac{nz^2}{(\omega/a_m)^2}\right) J_{M+|mp|}(ka_m\rho_{\pm m})\right]^2. \quad (21)$$

Eq. (21) is the main theoretical result, which describes the intensity profile of the higher-diffraction-order FtVHB with CFH. A central bright surrounds the vortex rings for higher-diffraction order. Moreover, the non-diverging vortex radius in the first bright ring from the center is unchanged and it expressed as $\rho_{max} = \mu_{M+p,1}/\alpha$ with $\mu_{M+p,1}$ being the Bessel function arguments values, for which the first derivative defines their first maxima. It denotes that the Bessel function argument value is found as a root of the following expression: $|J_{M+mp+1}(ka_m\rho_{\pm m})| = |J_{M+mp-1}(ka_m\rho_{\pm m})|$.

The case $M=0$, corresponds the Flat-topped beam diffracted by the CFH, the Eq. (20) and Eq. (21), reduce to

$$U_{\pm m}(\rho_{\pm m}, \theta_{\pm m}, z) = (\mp i)(-1)^{|mp|} t_{\pm m}\left(\frac{\omega k a_m}{\sqrt{2}}\right)^{1/2}$$

$$\times \exp\left[\pm imp\left(\theta_{\pm m} + \frac{\pi}{2}\right)\right]\exp\left[-ik\left(z + \frac{\rho^2}{2z} - \frac{za_m^2}{2}\right)\right] \quad (22)$$

$$\times \left(\frac{\sqrt{2}z}{(\omega/a_m)}\right)^{1/2} \sum_{n=1}^{N} a_n \exp\left(-\frac{nz^2}{(\omega/a_m)^2}\right) J_{|mp|}(ka_m\rho_{\pm m}).$$

And

$$I_{\pm m}(\rho_{\pm m}, \theta_{\pm m}, z) = |t_{\pm m}|^2 \left(\frac{\omega k a_m}{\sqrt{2}}\right)\left(\frac{2z^2}{(\omega/a_m)^2}\right)^{1/2}$$

$$\times \left[\sum_{n=1}^{N} a_n \exp\left(-\frac{nz^2}{(\omega/a_m)^2}\right) J_{|mp|}(ka_m\rho_{\pm m})\right]^2. \quad (23)$$

The case $N=1$ and $M=0$ corresponds the transformation of Gaussian beam into high-order spiraling Bessel beam by means of CFH, then, Eq. (20) and Eq. (21) reduce to

$$U_{\pm m}(\rho_{\pm m}, \theta_{\pm m}, z) = (\mp i)(-1)^{|mp|} t_{\pm m}\left(\frac{\omega k a_m}{\sqrt{2}}\right)^{1/2} \exp\left[\pm imp\left(\theta_{\pm m} + \frac{\pi}{2}\right)\right]$$

$$\times \exp\left[-ik\left(z + \frac{\rho^2}{2z} - \frac{za_m^2}{2}\right)\right]\exp\left(-\frac{z^2}{(\omega/a_m)^2}\right) \quad (24)$$

$$\times \left(\frac{\sqrt{2}z}{(\omega/a_m)}\right)^{1/2} J_{|mp|}(ka_m\rho_{\pm m}).$$

And

$$I_{\pm m}(\rho_{\pm m}, \theta_{\pm m}, z) = |t_{\pm m}|^2 \left(\frac{\omega k a_m}{\sqrt{2}}\right)\left(\frac{2z^2}{(\omega/a_m)^2}\right)^{1/2}$$

$$\times \exp\left(-\frac{2z^2}{(\omega/a_m)^2}\right) J_{|mp|}^2(ka_m\rho_{\pm m}). \quad (25)$$

Note that, this obtained result is in agreement with that given in Eqs. (7) and (8) of Ref. [56], for Gaussian beam diffracted by CFH.

III.   NUMERICAL CALCULATIONS AND DISCUSSIONS

In order to investigate the output spiraling Bessel beam created by converting the FtVHB by the CFH, numerical simulations are calculated in this Section using the above result elaborated in Eq. (21). The intensity distribution of the diffracted beam is numerically calculated at plane placed at distance z and using the following parameters are $m=1$, $\lambda=810$nm and $\omega=10$mm. The axicon parameters: $n_r=1.48$ and $\gamma=1.35°=0.0235$ rad. Fig. 4 presents the longitudinal intensity of the considered beam, which diffracted by CFH, for $p=1$ and $m=1$ at three values of M (=0,2 and 6) and each value for three different beam orders N (=1,2 and 6). In the simulation, the radial position is $\rho_{max}=0.02$mm at the first bright ring. From the curves of this figure, it can be shown that, for $N=1$ and $M=0$ the obtained curve is in agreement with that in the case of the Fresnel diffraction by CFH for a Gaussian beam elaborated by Topuzoski [56]. When the beam order N increasing and a fixed $M=0$, the intensity changes and its maximum decreases gradually.

Similarly, we investigate in Fig. 4(b and c), the intensity distribution versus the propagation distance z with nonzero M. The curves of this figure see that the intensity profile is zero within the first several hundred meters and then increases with increasing distance z up to maximum intensity value z_{max}. This distance z_{max} increases with topological charge increasing. In addition, it is clearly from the plots of these figures that, the intensity has a maximum that decreases with increasing N and M. We also observe that the intensity profiles have a similar shape for all beam order N. These numerical results can be applied for several various applications, such as the optical trapping and tweezers.

Figs. 5 and 6 illustrate the transverse and the radial intensity distributions of the diffracted FtVHB by CFH for two values of p (= 0 and 1) and three values of M (=0, 2 and 6), for each value of topological charge $M+|mp|$ and three values of N (=1, 2 and 6). The other parameter considered in the numerical simulations is z=450m Fig. 5 investigates the variation of the intensity distribution of the produced output beam, for p =0, and three values of M. For each value of M, three curves for N are plotted
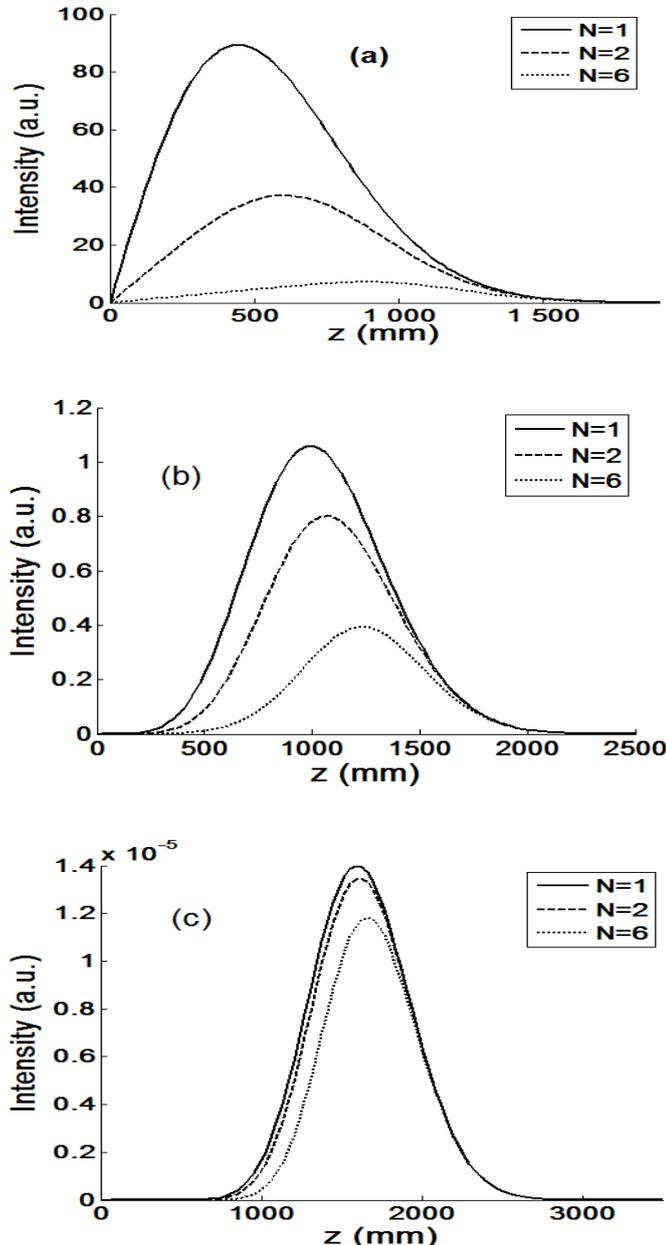
Other views of the illustrations of this figure are shown in Fig. 5 (B and C), for nonzero topological charge with a phase singularity $(M+|mp|\neq 0)$. Note that, for this case, the topological charge of the diffracted beam is an identical to that of the incident beam. From the plots of this figure, it can be observed that the intensity profile gets a central dark hollow beam of zero intensity with several different peaks of vortex bright rings.
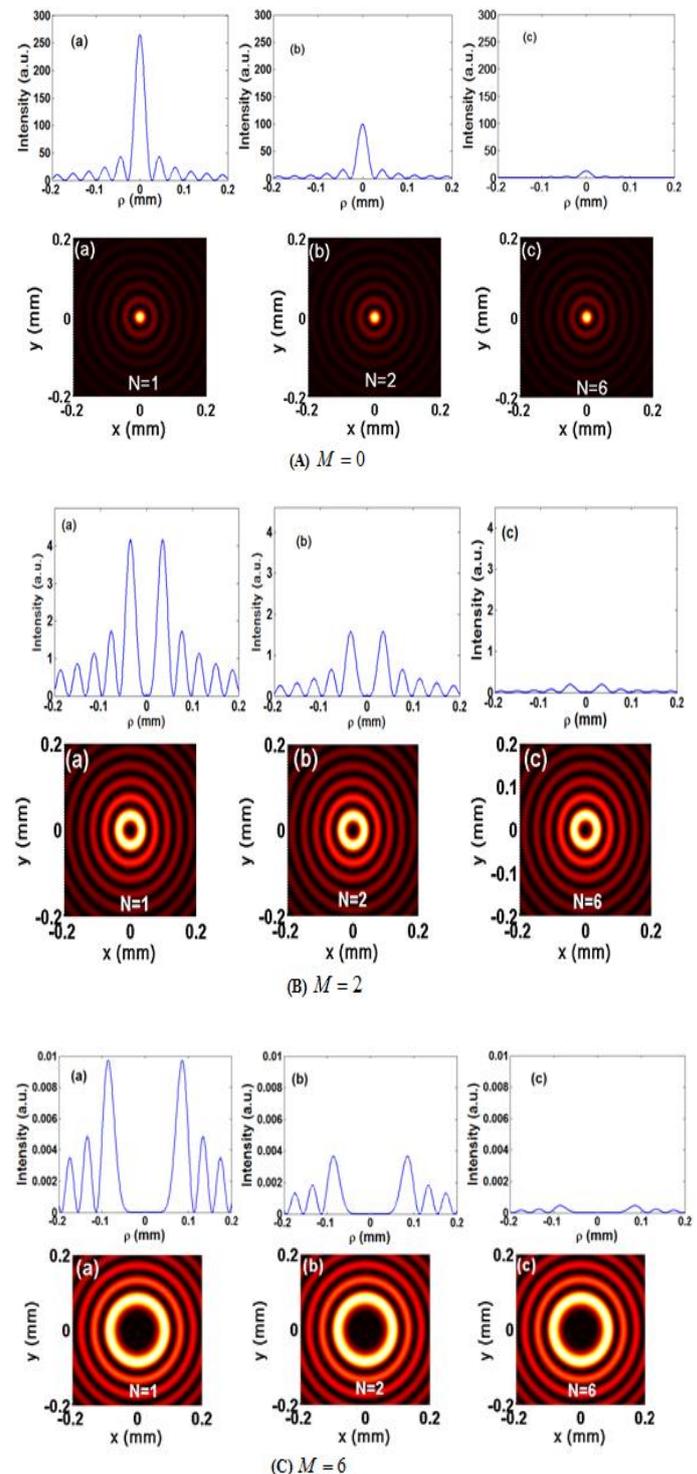

Fig 4 Longitudinal intensity profile of FtVHB diffracted by CFH for p=1, at (a) M=0, (b) M=2, and (c) M=6.

From the plots of this figure, we shows that when the produced output beams have a zero topological charge (the hologram without a phase singularity $M+|mp|$.=0, the wave field amplitude for higher diffraction orders can be described by zeroth-order Bessel functions. The intensity profile has a central bright spot with maximum intensity surrounded by five bright rings with different intensities. The maximum intensity decreases gradually with increasing N (see Fig. 5A).

Fig 5 Transverse and radial intensity distribution of FtVHB diffracted by CFH for p=0 and (A) M=0, (B) M=2 and (C) M=6.

We conclude from these plots that as long as the *N* increases, when *M* is fixed as much as the intensity maximum of vortex radius, which are formed decreases and the region radius of central dark of the created beam slightly increases. Moreover, we also observe from that, the influence of *M* on the intensity profile is clearly shown. The dark hollow region increases but the vortex radius number and their intensity decreases with the increase of value of *M* with the same value of *N*. In Fig. 6, the helical axicon effect is observed (*p*=1). The general variation rule of intensity distribution in this figure is similar to that in Fig. 6 (B and C). However, from Fig. 6A, we can clearly see that with the increase of *p* up to one and *M* is equal to zero at three values of *N*, the higher diffraction orders of the output field will be described by first-order Bessel functions.

So, the output beam profile gives a dark hollow at the center with four values of the bright vortex radius of different intensities. This dark spot slightly increases and a maximum peak of intensity of the vortex radius which are formed, decreases (see Fig. 6A (a-b)).
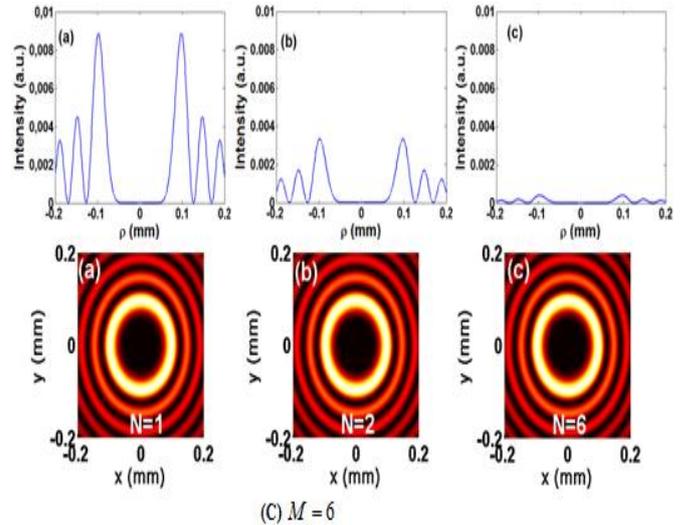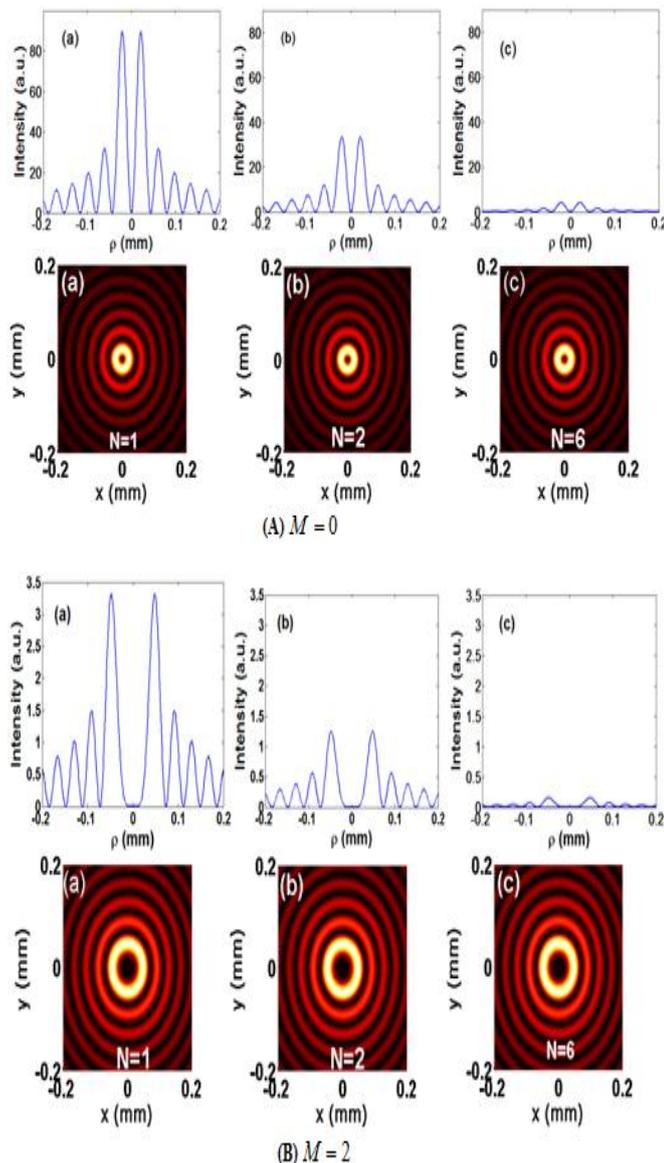

(A) *M* = 0

(B) *M* = 2

(C) *M* = 6

Fig 6 Transverse and radial intensity distribution of FtVHB diffracted by CFH for *p*=1 and (A)*M*=0, (B) *M*=2 and (C)*M*=6.

The similar remarks of Fig. 6A are seen in Fig. 6(B and C), but in the later shapes the influence of the parameter *M* is shown. From the illustration of these plots, we can see that, with the increase of the topological charge for the same values of *N* are chosen in Fig. 6A, the central dark radius of the produced output beam increases but the vortex radius number and their maximal intensities decrease.

On the other hand, we can see in Fig. 7, the transverse intensity profile for the generated spiraling Bessel beam in the case when the topological charges of helical axicon *p* and of an incident beam *M* have same values, but opposite signs (*p*= -*M*). Therefore, in this case, the phase singularity annihilates and the zero order Bessel beams whose distribution of intensity has a bright central spot will be generated. From the drawings of this figure, it can be seen that the profile of the outgoing beam gets a central maximum intensity of bright spot surrounded with five weak bright rings of various peaks intensity i.e., the obtained spiraling Bessel beams are chargeless. In addition, the effect of order *N* on the distribution of intensity of the produced beam is clearly shown. The maximum of intensity peaks, for vortex radius that is generated, decreases.
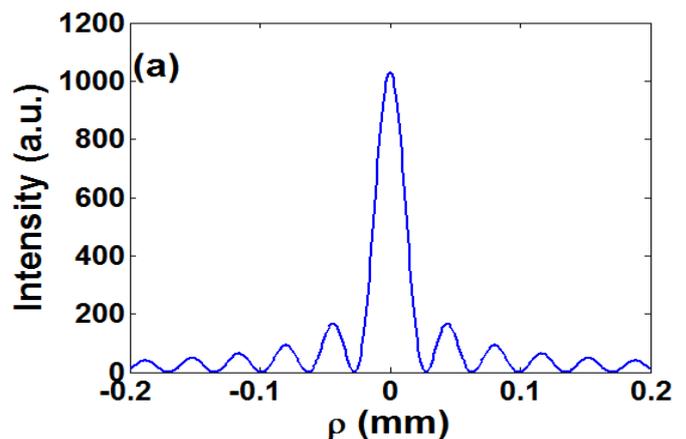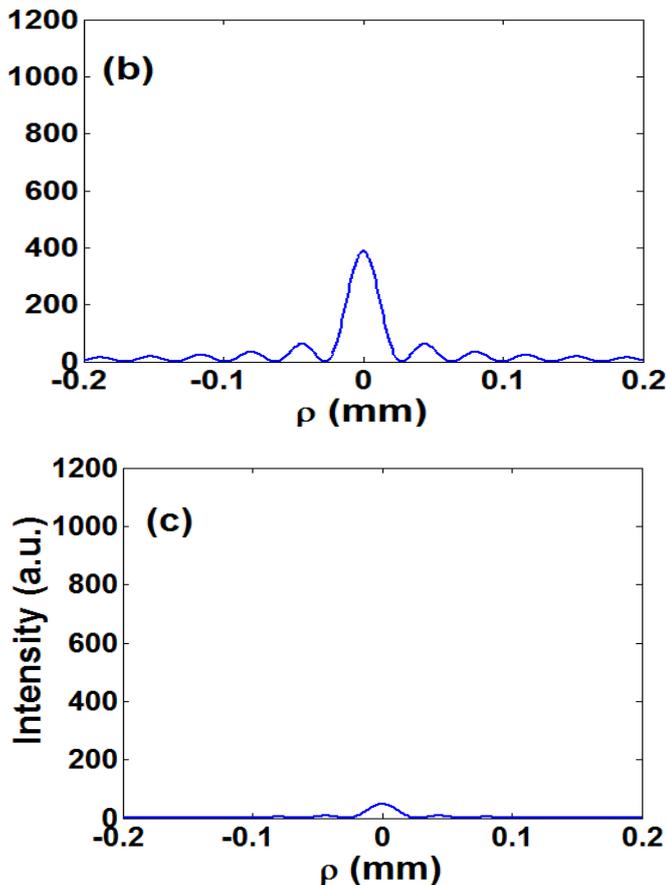
Fig 7 Radial intensity distribution of FtVHB diffracted by CFH for *P*=1 and *M*= -1 with (a) *N*=1, (b) *N*=2 and (c) *N*=6

IV. CONCLUSION

In the presented paper, we have investigated a new Bessel-like beam family called spiraling Bessel beam generated by converting the diffracted FtVHB with the CFH. Based on the Fresnel diffraction integral formula, the analytical expressions of the output field amplitudes of the produced beam are derived. In numerical calculations, the intensity distributions in longitudinal and radial directions are analyzed and discussed, by studying the effect of the incident beam order N and the topological charge of the produced beam. Our obtained results in this paper, were generalized the diffraction cases of flat-topped and Gaussian modes by the CFH, respectively. Results of this research can find interest applications in optical field of communication.

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