Effects of damage on the scaling laws of viscous-plastic sea ice

Antoine Savard & Bruno Tremblay

IICWG-DA-11 — March 22, 2023

Contents

Introduction

- Linear Kinematic Features
- Goals

We should study sea ice because it's sensitive to climate change.

Poles are very sensitive to climate change.
Sea ice variables are a good measure of this change.

Processes affecting sea ice are complex.
And sea ice is a key player in GCMs.

Models are morally good/bad in the same places at high enough resolution. (Bouchat et al., 2022)
We need new ways of studying/parametrizing sea ice.

Different aspects of sea ice can be studied.

Annual/monthly averages

Seasonal with daily averages

Snapshots and scale protractor

- Ice thickness, concentration, velocity
- Deformation statistics
- Angles of fracture

Deformation statistics and Angles of fracture → Linear Kinematic Features

Linear Kinematic Features (LKFs) are lines of high deformation.

- shear
- divergence
- damage
- healing

Credit: Nasa Worldview

LKFs are an important place for atmosphere–ocean interactions.

- salt fluxes
- heat fluxes
- moisture fluxes

Credit: Nasa Worldview

Study of LKFs deformations statistics is the "new" way of characterising models.

The shape of probability density functions
is important for the overall distribution of deformations.

The spatiotemporal scaling laws of LKFs
need to be reproduced for accurate simulations.

The multifractality of sea ice
is critical for the localization/intermittency of LKFs.

Models have various success rate in these metrics, what parametrizations permit that?

- Damage/dilation?
- Memory of past deformations?
- Consideration of elastic deformations?
- Non-normal flow rule?

The goals of this project are to:

Develop a sub-grid scale parametrization for damage.

This is combined with higher resolution models as a first step towards better simulations.

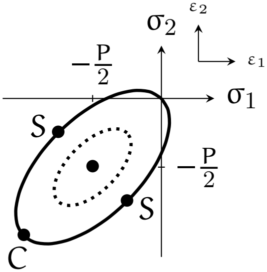
Disentangle the elastic response from the damage response in sea ice models.

Some models have both, and it's unclear which is responsible for their success.

McGill sea ice model

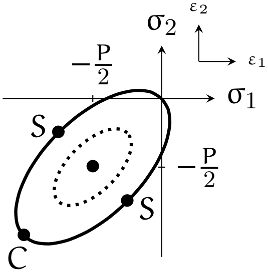
- The original model
- Damage

The McGill sea ice model is a viscous-plastic (VP) model.



$$m\left[\frac{\partial \mathbf{u}}{\partial t} + \overbrace{\cancel{(\mathbf{u}\cdot\nabla)\mathbf{u}}}^{\text{Advection}}\right] = -\underbrace{mf\,\hat{\mathbf{k}}\times\mathbf{u}}_{\text{Coriolis}} + \overbrace{\boldsymbol{\tau}_a + \boldsymbol{\tau}_w}^{\text{Forcing}} - \underbrace{mg\nabla H_d}_{\text{Sea surface height}} + \overbrace{\nabla\cdot\boldsymbol{\sigma}}^{\text{Rheology}}$$

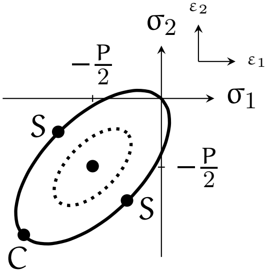
The McGill sea ice model is a viscous-plastic (VP) model.



$$m\left[\frac{\partial \mathbf{u}}{\partial t} + \overbrace{\cancel{(\mathbf{u}\cdot\nabla)\mathbf{u}}}^{\text{Advection}}\right] = -\underbrace{mf\,\hat{\mathbf{k}}\times\mathbf{u}}_{\text{Coriolis}} + \overbrace{\boldsymbol{\tau}_a + \boldsymbol{\tau}_w}^{\text{Forcing}} - \underbrace{mg\nabla H_d}_{\text{Sea surface height}} + \overbrace{\nabla\cdot\boldsymbol{\sigma}}^{\text{Rheology}}$$

$$\sigma_{ij} = 2\eta\dot{\varepsilon}_{ij} + (\zeta-\eta)\dot{\varepsilon}_{kk}\delta_{ij} - \frac{\boxed{P}}{2}\delta_{ij}, \quad \zeta = \frac{\boxed{P}}{2\Delta}, \quad \eta = \frac{\zeta}{e^2}$$

The McGill sea ice model is a viscous-plastic (VP) model.



$$m\left[\frac{\partial \mathbf{u}}{\partial t} + \overbrace{(\mathbf{u}\cdot\nabla)\mathbf{u}}^{\text{Advection}}\right] = -\underbrace{mf\,\hat{\mathbf{k}}\times\mathbf{u}}_{\text{Coriolis}} + \overbrace{\boldsymbol{\tau}_a + \boldsymbol{\tau}_w}^{\text{Forcing}} - \underbrace{mg\nabla H_d}_{\text{Sea surface height}} + \overbrace{\nabla\cdot\boldsymbol{\sigma}}^{\text{Rheology}}$$

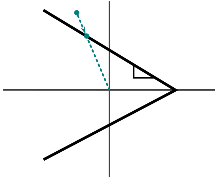
$$\sigma_{ij} = 2\eta\dot{\varepsilon}_{ij} + (\zeta-\eta)\dot{\varepsilon}_{kk}\delta_{ij} - \frac{\boxed{P}}{2}\delta_{ij}, \quad \zeta = \frac{\boxed{P}}{2\Delta}, \quad \eta = \frac{\zeta}{e^2}$$

$$P = P^*\,\boxed{h}\,\exp\left\{-C(1-\boxed{A})\right\}$$

We would like to modify the ice strength P.

$$\boxed{P = P(h, A) \longrightarrow P = P(h, A, d)}$$

Damage comes from rock mechanics.



It has been adapted in Maxwell-Elasto-Brittle (MEB) models (Dansereau et al, 2016; Plante et al., 2020).

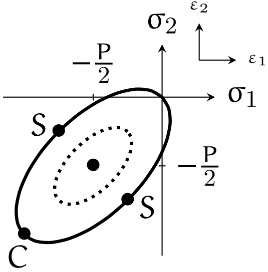
Damage is computed as the stress overshoot.

Damage is coupled to the elastic stiffness and viscosity,

and evolves via a continuity equation.

$$\frac{\partial}{\partial t} d = \frac{(1-\Psi)(1-d)}{t_d} - \frac{1}{t_h}, \quad \boldsymbol{\sigma}^f = \Psi \boldsymbol{\sigma}'$$

We have developed damage for viscous-plastic model.



Maximal damage is reached in plastic regime.
It is reached almost instantly.

Damage is coupled to the ice strength P
in the same way that it is done in MEB.

$$\frac{\partial d}{\partial t} + \nabla \cdot (\mathbf{u} d) = \frac{1 - (\zeta/\zeta_{\mathsf{max}})^{1/n} - d}{t_{\mathsf{d}}} - \frac{d}{t_{\mathsf{h}}}$$

$$P = P^* h \exp\{-C(1 - A)\} \boxed{(1 - d)}$$

But what is damage?
What does it mean to be dammaged?

Here, damage is a scalar field,
similar to A or $\mathfrak{h}$.

Damage encompasses all processes that lead to plastic deformation
like microcracks, microvoids, creep cracks, etc.

Damage is a big box where we put everything,
but future work could include refinements of our definition.

The model is first spun-up, then we do our simulations.

Spin-up is done to find good initial conditions.
We started with a 1 m thickness, and a 100% concentration.

Shuffling the year list prevents biases from low-frequency variability.
Like the Arctic Oscillations or Arctic Ocean Oscillations

Simulations are for January 2002 at a 10 km resolution.

Deformation products from RGPS are used.

We use the three-day gridded data set (Kwok et al., 1997).
So our minimal temporal resolution is 3 days.

The observations are on a 12.5 km $\times$ 12.5 km grid.

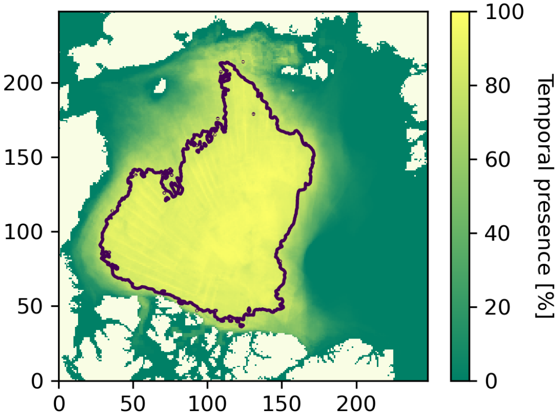
The tracking error is around 100 m (Lindsay and Stern, 2003).
We won't use deformations under $\left|5 \times 10^{-3}\right|$ day^{-1}.

Methods and Results

- Simulated Deformation Fields
- Probability Density Functions (PDFs)
- Spatiotemporal Scaling Laws
- Multifractality

The mask80 is used in every computations.

The mask is obtained via the 80% temporal presence in the observations.



Simulated deformation fields are computed from velocities derivatives.

Divergence

$$\dot{\varepsilon}_{\mathsf{I}} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y},$$

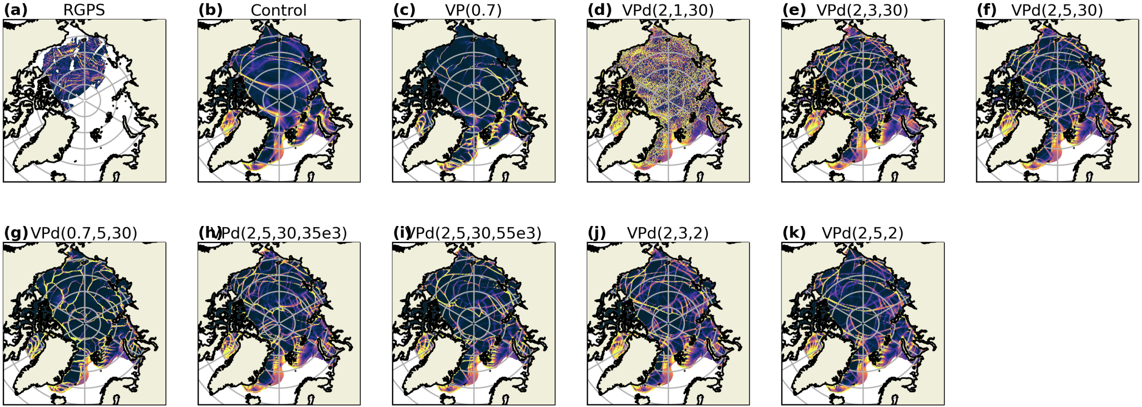
Maximum shear strain rate

$$\dot{\varepsilon}_{\mathsf{II}} = \sqrt{\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2},$$

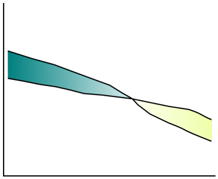
Total deformation

$$\dot{\varepsilon}_{\mathsf{total}} = \sqrt{\dot{\varepsilon}_{\mathsf{I}}^2 + \dot{\varepsilon}_{\mathsf{II}}^2}.$$

Snapshots for January 29–31, 2002 for total deformation.



PDFs used to differentiate simulated and observed distribution of deformation.

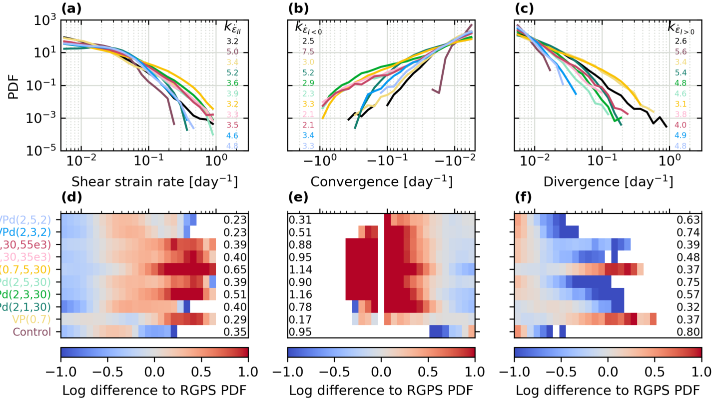


PDFs of shear strain, divergence, and convergence.
Not splitting absolute divergence leads to errors.

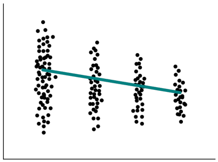
Difference between PDFs of observation and simulations (Bouchat et al., 2022)
will be used as a metric to evaluate models' performances.

Convergence and divergence are linked, but you cannot have both at the same time.

The bottom numbers are the models' performance (lower is better).



Spatiotemporal scaling laws are assumed to follow a power law.



We do a coarse-graining procedure (Marsan et al., 2004)
we take the mean in each cell.

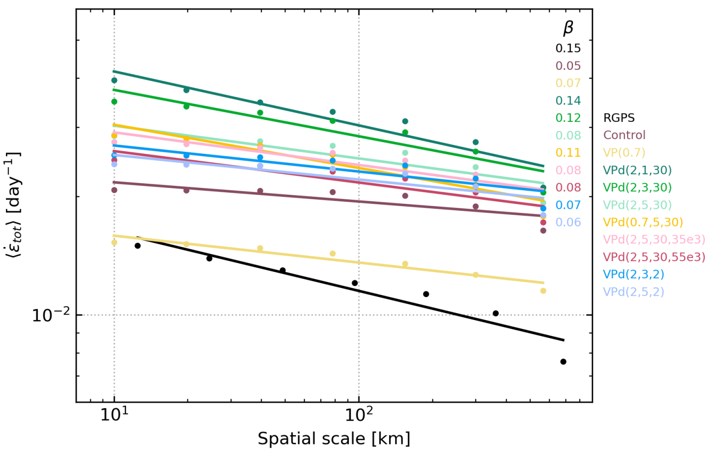
We compare the slopes of simulations and observation.
This tells us the space-time localization of deformations.

$$\langle \dot{\varepsilon}_{tot}(L, T) \rangle \sim L^{-\beta(T)}$$

$$\langle \dot{\varepsilon}_{tot}(L, T) \rangle \sim T^{-\alpha(L)}$$

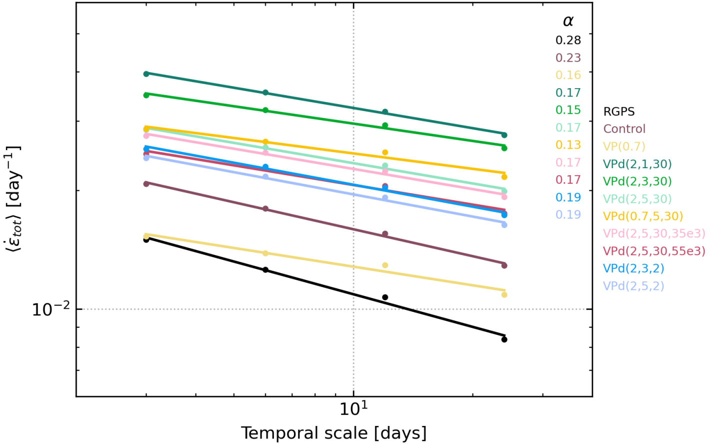
Augmenting P^* brings the curves down with a slope in agreement with obs.

Note how damage increases the mean deformations.

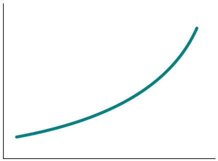


Augmenting P^* brings the curves down with a slope in agreement with obs.

Note how damage increases the mean deformations.



Multifractality to identify how scaling changes.



The scaling exponents are functions of the moment.

We fit the data for the three parameters.

Evaluates sparseness C_1, degree of multifractality ν, and localization H.

Theory says $0 \leqslant \nu \leqslant 2$.

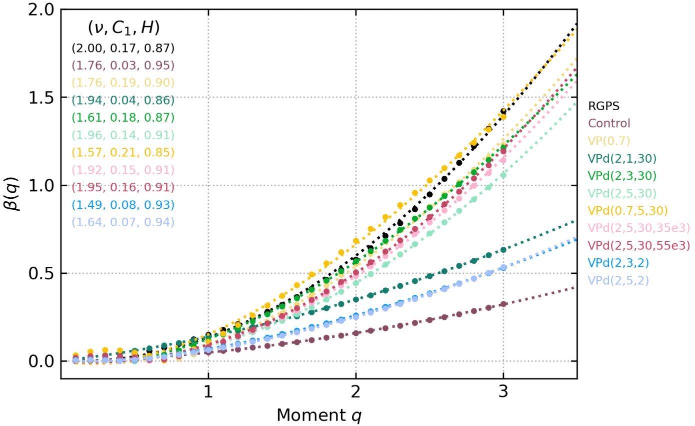
$$\alpha(q), \beta(q) = q(1-H)+K(q) = \frac{C_1}{\nu - 1} q^{\nu} + \left(1 - H - \frac{C_1}{\nu - 1}\right) q$$

$K(q)$ prescribes the distribution of singularities (or points of non-smoothness) across different scales.

It is the moment scaling function exponent.

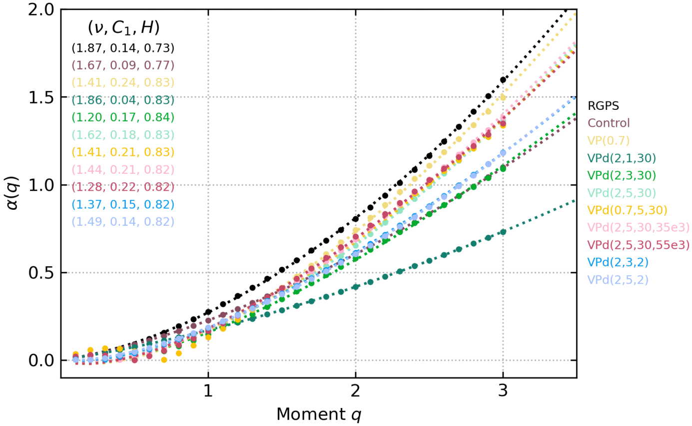
Spatial multifractality is sensitive to n only for high t_h.

Note the new root in the yellow line.



Temporal multifractality is sensitive to n only for high t_h.

Note the new root in the yellow line.



Conclusions

Damage in VP models is an easy-to-implement, powerful tuning knob to adjust deformation statistics.

Optimal values	$n = 5, \qquad t_h = 30$
Effect on statistics	Damage corrects spatial deformation statistics without the need to change the shape of the yield curve.
Multifractal theory	Changing the ellipse ratio or augmenting P^* unveils multifractal behavior that does not fit the theory.
Quantity of memory	Adding this parametrization doesn't make the ice brittle, but the memory of past deformation is still encoded.

Thank you!