Hawking temperature and the quantum pressure of the Schwarzschild black hole

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Abstract

There is no term for pressure ($P\nabla V$) in the first law of black hole thermodynamics. To address this question, we study the first law of black hole thermodynamics and derive an expression for it. We report that this pressure corresponds to the Hawking temperature and is inversely proportional to the quartic of the Schwarzschild radius. It implies that a lighter and smaller black hole exerts more pressure on its surrounding environment. It might shed light on the other thermodynamic aspects of the black hole.

Keywords: Hawking temperature; black hole thermodynamics; black holes;

1 Introductions

Black holes are the most enigmatic objects in physics, and physicists are working hard to fully comprehend their behavior. The classical black hole was an object that absorbed everything and from which nothing, not even a photon, could escape.

Bekenstien and other physicists have recently made advances in this area, demonstrating that black holes have entropy. Hawking's contributions to this field have demonstrated that its temperature is inversely correlated with the black hole's surface gravity. In this context, the concept of quantum effect or quantum-gravitational effect has emerged [1] [2] [3] [4]. This work laid the foundation for the theory of black hole thermodynamics [5] where for a classical black hole (Schwarzschild black hole without charge and spin) it's first law states,

$$\nabla E = T\nabla S \tag{1}$$

here the black hole mass M corresponds to the total energy of the system, or enthalpy. T is temperature, and S is the entropy. A remarkable aspect of this expression is that it does not incorporate the pressure terms $P\nabla V$, i.e. the corresponding change in volume V at pressure P, in contrast to the laws of thermodynamics of physical systems. It is important in the sense that it may help to understand the properties of a black hole by making an analogy with everyday things

This problem has been addressed, and it has been proposed that the variable cosmological constant might be the thermodynamic pressure in the first law of black hole thermodynamics [6] [7]. However, the question is, how does the cosmological constant incorporate itself into the particular individual black hole's pressure given that it is independent of the black hole's mass or other aspects? We thus consider the following fact: How do the fundamental properties of a black hole, such as mass, size, volume, and others, incorporated into the black hole's thermodynamical pres-

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sure? By deriving a mathematical expression for a black hole's pressure in terms of its attributes, we try to provide a solution to this intriguing question.

2 Derivation of black hole's pressure

Now, as we discussed earlier, the Hawking temperature is an integral and basic part of the first law of black hole thermodynamics. And this temperature is correlated with the properties of a black hole. Furthermore, this temperature corresponds to the cosmological constant or energy density of vacuum as written below [8],

$$\frac{R^2c^7}{hG^2} = \frac{hc^5}{G^2m^2} \tag{2}$$

where its first term is the mathematical expression of the cosmological constant or energy density of vacuum [9], and RHS is Hawking radiation power without a numerical factor [2]. We wrote it in the form of force because the desired expression of thermodynamic pressure is simply force per area, so this representation would be convenient for us.

Now that it has been discussed, the cosmological constant corresponds to the pressure in the first law of thermodynamics [6] [7]. Therefore, if we set the first terms of Eq.(2) as pressure P for the black hole thermodynamics, However, by taking $R \sim \frac{Gm}{c^2}$ as the Schwarzschild radius, the RHS will turn into the following,

$$P = \frac{hc^9}{G^4m^4} \tag{3}$$

it corresponds to,

$$P = \frac{hc}{R^4} \tag{4}$$

one can see that it denotes the pressure of the black hole; however, this expression contain the Planck constant thus we coined its name "quantum pressure of the black hole" for convenience, and hereafter we call it by this name in this paper. The origin of this quantum pressure might be the quantum effect of black holes or the quantum gravitational effect such as Hawking temperature.

3 Thermodynamical origin of black hole's pressure

This pressure can also be derived by using the law of thermodynamics where emergent force F is defined from Eq.(1) as:

$$F = \frac{T\nabla S}{\nabla R} \tag{5}$$

where R is space parameter and entropy is $S \sim k_B$ where k_B is Boltzmann constant. The pressure P can be denoted by $P = \frac{F}{A}$ where A is area, further by taking $E = T\nabla S = \frac{hc}{R}$, $\lambda \sim R$, volume of black hole $V \sim R^3$, radius of black hole which corresponds to $R \sim \frac{GM}{c^2}$, by substituting these all in Eq.(5), one can recover the Eq.(3); this is the thermodynamic origin of black hole's pressure. In this scenario one can say, it's thermodynamic pressure.

We can calculate the numerical value of this pressure by substituting the values of all the constants in Eq.(3); thus, we obtained the followings,

$$P \sim 10^{82}m^{-4} \tag{6}$$

this shows the numerical value of the thermodynamic pressure of a black hole. It suggests the pressure is at least 82 orders of magnitude, and the heavier black holes exert little pressure on their surroundings while the lighter black holes exert more pressure.

It hints that the black hole's pressure and energy density of vacuum is theoretically equivalent. Previously, this equivalence was discussed in refs [6] [7], and the possibility of similar kind of pressure in refs [10]. In previous work, we discussed it in a wider context [11]. However, its correspondence and congruence with the Hawking temperature and cosmological constants show it's consistency with existing theories.

4 Conclusions

Quantum pressure exists in the environment surrounding black holes. Theoretically, this quantum pressure corresponds to the Hawking temperature and vice versa. This quantum pressure is remarkable in that it is only a function of mass and is inversely proportional to the quartic of mass of the black hole; thus, a lighter black hole exerts more pressure on the environment. For the study of black hole thermodynamics, it might present a fresh perspective.

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