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The 5 Equations that Generate All Composite Numbers

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This paper presents 5 equations that generate all existing composite numbers. To find these equations, we divided all natural numbers in 6 groups and found the equations that generate all composite numbers in each one of these groups by means of simple mathematical reasoning. Two different ways to obtain prime numbers by iteration are provided. An iteration method to find twin prime numbers is also described.

KEYWORDS: Prime numbers, composite numbers, twin prime numbers,

I. INTRODUCTION

For all natural numbers N, where N = {1, 2, 3, 4, 5…} each number is either a prime or a composite number, with the exception of number 1. A prime number is a natural number that is only divisible by itself and 1. A composite number is a natural number that is divisible by itself, 1 and at least a third number.

The first records of prime numbers date from the Rhind mathematical papyrus, around 1550 BC and already in ancient Greece prime numbers were studied in detail, with the finding of Mersenne prime numbers (prime numbers M that follow $M_n = 2^n-1$) and the Sieve of Eratosthenes (a method where the multiples of each prime are eliminated from the next natural numbers). Some of the learning from that time are still used today or are the foundation to develop similar methods (i.e. Sieve of Sieve of Pritchard, Sieve of Atkin, Sieve of Sundaram…)[1,2].

The most important use of prime numbers nowadays lies in computing and cybersecurity, in fields such as bank transactions, cryptocurrency or computer safety. [3] One of the most common ways to test if a number is or not a prime number is by using a primality test. Primality tests are algorithm to determine if an input number is prime or not and they can range from rather simple ones like the trial by division test or the 6k±1 method (other than 2 and 3, all prime numbers are known to be of the form 6k±1). The use of computers and algorithms helped to speed the process but this simpler methods become impractical for large numbers, and more sophisticated primality tests exist, such as the Fermat primality test, the Miller-Rabin and Solovay-Strassen primality test, the Frobenius primality test or the Baillie-PSW primality, to name a few. Some methods to find prime number, as the Miller Rabin primality test, are not 100% accurate as a price to their faster speed. Other methods pay a price on speed of calculation, for example the AKS primality test.

With the goal of finding new large prime numbers faster, the search of specific families of prime numbers is an used method. For example, the above mentioned Mersenne numbers. Other methods that are restricted to specific number forms include Pépin's test for Fermat numbers (1877), [4] Proth's theorem (c. 1878),[5] the Lucas–Lehmer primality test (originated 1856), and the generalized Lucas primality test.[2,6] Bernstein (2004) summarized 14 of methods to prove that an integer is prime, three additional methods to prove that an integer is prime if certain conjectures are true, and four methods to prove that an integer is composite. [7]

It is not the aim of this paper to go deeper into the algorithms to find and prove prime numbers, but to understand, by means of mathematical reasoning, the factors that generate all composite numbers and provide an explanation to the apparent randomness of their gaps. Prime gaps (the differences between consecutive primes) are still seen as to some degree arbitrarily occurrences and other questions remain unsolved. [8]

On the last years, more patterns on the apparent randomness of prime numbers have been found. [9] Dan Goldston, János Pintz, and Cem Yıldırım proved that there are infinitely many primes for which the gap to the next prime is as small as we want compared to the average gap between consecutive primes. [10], a phenomenon further studied by Soundararajan. [11] A phenomenon of interdependency between the structure of positive integers and the form of their prime factors was discovered by Karatsube in 2011. [12] In 2015 Granville developed further the findings of Zhang on twin prime numbers (2 prime numbers separated by only one composite number). [13, 14] Prime numbers near to each other tend to avoid repeating their last digits, and most importantly, as observed by Lemke and Soundararajan in 2016, all primes have a remainder of 1 or 5 when divided by 6 (otherwise, they would be divisible by 2 or 3) and the two remainders are on average equally represented among all primes. [15]. This occurrence in groups of 6, together with it happening in Euler's theorem (where 6n+1 are analysed) and the primality test, with almost all prime numbers are of the form 6n+1 or 6n-1 is from where the author starts his analysis.

II. ANALYSIS OF THE GENERATION OF COMPOSITE NUMBERS

The study presented in this paper starts from the following premise: If we could find a set of equations that generate all existing and only composite numbers, then we can prove that the occurrence of composite numbers is not random.

A way to be able to prove that a set of equations generates all and only composite numbers is to sort all natural numbers in groups and then find the equation/s that generate all composite numbers for each one of these groups. In 2016, Lemke and Soundararajan found that prime numbers are never divisible 6, an observation well aligned with the simpler primality tests. In this work all natural numbers N are sorted in 6 groups, A to F:

A contains all natural numbers generated by N = 1+6n where n = {0, 1, 2, 3, 4…}

B contains all natural numbers generated by N = 2+6n where n = {0, 1, 2, 3, 4…}

C contains all natural numbers generated by N = 3+6n where n = {0, 1, 2, 3, 4…}

D contains all natural numbers generated by N = 4+6n where n = {0, 1, 2, 3, 4…}

E contains all natural numbers generated by N = 5+6n where n = {0, 1, 2, 3, 4…}

F contains all natural numbers generated by N = 6+6n where n = {0, 1, 2, 3, 4…}

To help visualize the calculations and reasoning of this paper, the first 48 natural numbers are represented in Table 1, where each column is one of the groups described above. Number 1, not being considered a prime number, is already marked in red in Table 1:

Table 1. Natural numbers sorted in the groups A to F.

1+6n	2+6n	3+6n	4+6n	5+6n	6+6n
A	B	C	D	E	F
1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
…	…	…	…	…	…

A. The First Equation that Generates Only Composite Numbers, c1

The first prime number is 2, any multiple of 2 will be a composite number. Composite numbers that are multiples of 2 (c1) are generated by Equation 1:

Equation 1:

$$c1 = 2(n + 2)$$

where n = {0, 1, 2, 3, 4…}

if n=0 then c1=4, leaving this way the prime number 2 out of the composite generating equation.

Groups B , D and F contain only even numbers, while groups A, C and E contain only odd numbers. This is demonstrated by dividing the equation that generates each group by 2 and checking if the result is a natural number:

Group A: (1+6n)/2 = 0.5+3n will never be a natural number, none of these numbers are divisible by 2.

Group B: (2+6n)/2 = 1+3n will always be a natural number, all these numbers are divisible by 2.

Group C: (3+6n)/2 = 1.5+3n will never be a natural number, none of these numbers are divisible by 2.

Group D: (4+6n)/2 = 2+3n will always be a natural number, all these numbers are divisible by 2.

Group E: (5+6n)/2 = 2.5+3n will never be a natural number, none of these numbers are divisible by 2.

Group F: (6+6n)/2 = 3+3n will always be a natural number, all these numbers are divisible by 2.

From above, any number from groups B, D and F is divisible by 2 and therefore generated by c1, they are composite, with exception of the number 2. Table 2 shows all c1 composites marked in red for ease of visualization.

Table 2. All the composite numbers generated by c1 are marked in red.

1+6n	2+6n	3+6n	4+6n	5+6n	6+6n
A	B	C	D	E	F
1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
…	…	…	…	…	…

Equation 1, c1, is the first equation that generates only composite numbers but not all composite numbers. More equations are missing.

B. The Second Equation that Generates Only Composite Numbers, c2

The second prime number is 3, multiples of 3 are composite numbers c2 and are generated by equation 2:

Equation 2:

$$c2 = 3(n + 2)$$

where n = {0, 1, 2, 3, 4…}

if n=0 then c2=6, leaving the prime number 3 out of this composite generating equation.

Groups C and F contain only numbers which are 3 or multiples of 3, while groups A, B, C and E contain only numbers that are not multiple of 3. This is demonstrated by dividing by 3 the equation that generates each group and checking if the result is a natural number or not.

Group A: (1+6n)/3 = 1/3+2n will never be a natural number, none of these numbers are divisible by 3.

Group B: (2+6n)/3 = 2/3+2n will never be a natural number, none of these numbers are divisible by 3.

Group C:(3+6n)/3 = 1+2n will always be a natural number, all these numbers are divisible by 3.

Group D: (4+6n)/3 = 4/3+2n will never be a natural number, none of these numbers are divisible by 3.

Group E: (5+6n)/3 = 5/3+2n will never be a natural number, none of these numbers are divisible by 3.

Group F: (6+6n)/3 = 2+2n will always be a natural number, all these numbers are divisible by 3.

Table 3 includes now all composites generated by c3 also marked in red.

Table 3. All composite numbers multiple of 3 are now marked in red too.

1+6n	2+6n	3+6n	4+6n	5+6n	6+6n
A	B	C	D	E	F
1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
…	…	…	…	…	…

Equation 2, c2, is the second equation that generates only composite numbers but not all composite numbers. We have now covered all composite numbers in groups B, C, D and F but more equations are missing.

We have demonstrated that, besides numbers 2 and 3, all remaining prime numbers must belong to either group A (1+6n) or group E (5+6n). This is in good agreement with the primality test and previous findings where all prime numbers besides 2 and 3 are of the type 6n+1 and 6n-1, because 6n+1 equals a group A prime number and 6n-1 a group E prime number.

Not only prime numbers, but composite numbers will also be present in groups A and E. We must find the equations that define the composite numbers in groups A and E.

C. Defining how the Composite Numbers in Groups A and E Are Generated

By definition, any composite number can be obtained by multiplying 2 natural numbers or the same natural number twice, other than itself multiplied by 1.

Any multiplication of 2 natural numbers where at least one of them is even, will always result in a composite number which also is even. For example:

2n, where n = {1, 2, 3, 4…} contains all even numbers and m where m = {1, 2, 3, 4…} is any natural number. Then by equation 3, a multiplication of any m by any 2n, will always be divisible by 2, and therefore, a composite even number.

Equation 3:

$$\frac{2n \times m}{2} = n \times m$$

where n = {1, 2, 3, 4…} and m = {1, 2, 3, 4…}

We have already established when defining c1 composites, that all even numbers are only found in groups B, D and F. Therefore, any composite numbers in groups A and E cannot be even and cannot result from multiplying a number from B, D or F with any other number, because the result would be even, according to equation 3, but groups A and E contain only odd numbers. In short: composite numbers in groups A and B are not divisible by any number found in groups B, D or F.

In a similar way, any multiplication of 2 natural numbers where at least one of them is 3 or a multiple of 3, will always result in composite number multiple of 3. For example, 3n where n = {1, 2, 3, 4…} contains 3 and all numbers multiple of 3 and m where m = {1, 2, 3, 4…} is any natural number. Then by equation 4, the multiplication of any 3n by any m will be always divisible by 3, and therefore a multiple of 3 itself.

Equation 4

$$\frac{3n \times m}{3} = n \times m$$

We have already established when defining c2 composites, that all composite numbers multiple of 3 are only found in groups C and F. Therefore, any composite number in groups A and E cannot be a multiple of 3. Therefore any composite number in groups A and E cannot be a result of multiplying a number from C, or F with any other given number, because the result would be a multiple of 3 according to equation 4, but we have already established that numbers multiple of 3 cannot be found in groups A and E.

In short: composite numbers in groups A and E are not divisible by any number in groups C and F. Not by any number in groups B, D and F, as said earlier. If numbers in groups A and E are not divisible by any number found in B, C, D or F as already established, we have demonstrated that any composite number found in groups A and E is a result of multiplying 2 natural numbers also found in groups A and/or E.

D. The 3 Remaining Equations that Generate Only Composites, c3, c4 and c5

There are three ways to generate composite numbers using 2 numbers from groups A and/or E.

a) By multiplying 2 numbers from group A, generating a composite number c3:

Equation 5

$$c3 = (1 + 6n) \times (1 + 6m)$$

where n = {1, 2, 3, 4…} and m = {1, 2, 3, 4…}.

For all composite, there must be a solution other than itself multiplied by 1, so restricting the equation to not be able to generate 1 in any of the two numbers, all primers are ruled while still allowing to generate all composite. Neither n or m can be 0, otherwise the number resulting would be 1 for that one and c3 could then be a composite (e.g. n=0 and m=4), a primer (e.g. n=0 and m=1) or even 1 (if both n and m were 0).

b) By multiplying two numbers from group E generating a composite number c4:

Equation 6

$$c4 = (5 + 6n) \times (5 + 6m)$$

where n = {0, 1, 2, 3, 4…} and m = {0, 1, 2, 3, 4…}

c) By multiplying one number from group A and one number from group E, generating a composite number c5:

Equation 7

$$c5 = (1 + 6n) \times (5 + 6m)$$

where n = {1, 2, 3, 4…} and m = {0, 1, 2, 3, 4…}.

n cannot be 0, it would generate number 1 on that factor and a prime number could then be generated as well.

We have established the 3 equations that can generate all and only composite numbers in groups A and E. The next step is to check if all 3 equations can generate composite numbers in both groups A and E or not. We must also prove that that the prime numbers in groups A and E are not a valid solution of any of these 3 equations.

E. Finding what Composites Appear in Group A

If at least one composite number in A (1+6k) where k = {1, 2, 3, 4…} was a c3 composite, then it is resulting from multiplying two other numbers from group A (1+6n) and (1+6m), then equation 8 must have at least one valid solution, otherwise a composite number in group A would never be a c3 composite:

Equation 8

$$\frac{1 + 6k}{(1 + 6n) \times (1 + 6m)} = 1$$

n = {1, 2, 3, 4…}, m = {1, 2, 3, 4…} and k = {1, 2, 3, 4…}

$$1 + 6k = 1 + 6(n + m) + 36nm$$

$$6k = 6(n + m) + 36nm$$

$$k = n + m + 6nm$$

Because it is possible to obtain valid values of k for equation 8 using valid values for n and m, it will be possible to find c3 composite numbers in group A. For example, 91 is a group A number (1+6x15) and is a c3 composite originated by multiplying 7 and 13 (also group A numbers). In this example, k=15, n=1 and m=2.

If at least one composite number in A (1+6k) where k = {1, 2, 3, 4…} is a c4 composite it would result from multiplying two numbers from group E, (5+6n) and (5+6m), then equation 9 must have at least one valid solution.

Equation 9

$$\frac{1 + 6k}{(5 + 6n) \times (5 + 6m)} = 1$$

where n = {0, 1, 2, 3, 4…}, m = {0, 1, 2, 3, 4…} and k = {1, 2, 3, 4…}

$$1 + 6k = 25 + 30(n + m) + 36nm$$

$$6k = 24 + 30(n + m) + 36nm$$

$$k = 4 + 5(n + m) + 6nm$$

Because it is possible to obtain valid solutions of k for equation 9 using valid values for n and m, it will be possible to find c4 composite numbers in group A. For example, 55 is a group A number (1+6x9) and is a composite c4 originated by multiplying 5 and 11 (two numbers from group E) with k=9 and where n=0 and m=1.

If at least one composite number in A (1+6k) where k = {1, 2, 3, 4…} is a c5 composite, then it is the result of multiplying one number from group A (1+6n) by one number from group E (5+6m), then equation 10 must have at least one valid solution, otherwise a composite number in group A would never be a c5 composite:

Equation 10

$$\frac{1+6k}{(1+6n)\times(5+6m)}=1$$

where n = {1, 2, 3, 4…}, m = {0, 1, 2, 3, 4…} and k = {1, 2, 3, 4…}

$$1+6k=5+6m+30n+36nm$$
$$6k=4+6m+30n+36nm$$
$$k=\frac{4}{6}+m+5n+6nm$$

Equation 10 cannot result in a valid solution, because when using valid values for n and m, a number which is not natural is always obtained. Since k must be a natural number too, c5 composites can never found in group A.

In short: Group A is formed by number 1, prime numbers, composite numbers obtained by equation 8 (c3) and composite number obtained by equation 9 (c4). This can be easier visualized in Table 4 (now extended), where all numbers in group A are either prime, result of multiplying A_nxA_m or result of multiplying E_nxE_m, and of course, number 1.

Table 4. All composite except those in group E are marked in red, also 1.

1+6n	2+6n	3+6n	4+6n	5+6n	6+6n
A	B	C	D	E	F
1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60
61	62	63	64	65	66
67	68	69	70	71	72
73	74	75	76	77	78
79	80	81	82	83	84
85	86	87	88	89	90
91	92	93	94	95	96
…	…	…	…	…	…

F. Finding what Composites Appear in Group E

If at least a composite number in E (5+6k) where k = {0, 1, 2, 3, 4…} is a c3 composite, it is resulting from multiplying two numbers from group A, (1+6n) and (1+6m), then equation 11 must have at least one valid solution:

Equation 11

$$\frac{5+6k}{(1+6n)\times(1+6m)}=1$$

where n = {1, 2, 3, 4…}, m = {1, 2, 3, 4…} and k = {0, 1, 2, 3, 4…}

$$5+6k=1+6(n+m)+36nm$$
$$6k=-4+6(n+m)+36nm$$
$$k=-\frac{4}{6}+(n+m)+6nm$$

Equation 11 cannot result in a valid solution, because when using natural numbers for n and m, there will be always decimals in the solution, hence an invalid k. Therefore c3 composites can never found in group E.

If at least a composite number in E (5+6k) where k = {0, 1, 2, 3, 4…} is a c4 composite, it is resulting from multiplying two other numbers from group E (5+6n) and (5+6m), then equation 12 must have at least one valid solution.

Equation 12

$$\frac{5+6k}{(5+6n)\times(5+6m)}=1$$

where n = {0, 1, 2, 3, 4…}, m = {0, 1, 2, 3, 4…} and k = {0, 1, 2, 3, 4…}

$$5+6k=25+30(n+m)+36nm$$
$$6k=20+30(n+m)+36nm$$
$$k=\frac{10}{3}+5(n+m)+6nm$$

Equation 12 cannot result in a valid solution, because when using natural numbers or 0 for n and m, a number which is not natural is always obtained for k. Since k must be a natural number too, c4 composites can never found in E.

If at least one composite number in group E (5+6k) where k = {0, 1, 2, 3, 4…} is a c5 composite, then it is the result of multiplying one number from group A (1+6n) by one number from group E (5+6m) and equation 13 must have at least one valid solution.

Equation 13

$$\frac{5+6k}{(1+6n)\times(5+6m)}=1$$

where n = {1, 2, 3, 4…}, m = {0, 1, 2, 3, 4…} and k = {0, 1, 2, 3, 4…}

$$5+6k=5+6m+30n+36nm$$
$$6k=6m+30n+36nm$$
$$k=m+5n+6nm$$

Equation 13 can obtain valid values of k using valid values for n and m, therefore it will be possible to find c5 composite numbers in group E. For example, 65 is a group E number (5+6x10) and is a composite c5 originated by multiplying 13 (group A) and 5 (group E) with k=10, n=2 and m=0.

In short: Group E is formed by prime numbers and by c5 composite numbers. Table 5 shows all c5 composite numbers in group E marked in red.

Table 5. All composite numbers and 1 are now marked in red.

1+6n	2+6n	3+6n	4+6n	5+6n	6+6n
A	B	C	D	E	F
1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60
61	62	63	64	65	66
67	68	69	70	71	72
73	74	75	76	77	78
79	80	81	82	83	84
85	86	87	88	89	90
91	92	93	94	95	96
…	…	…	…	…	…

G. The 5 Equations that Generate All Composites Numbers

We have now a set of 5 equations that generate all existing composite numbers:

Equation 1 for all composites in groups B, D and F:

$$c1 = 2(n + 2)$$

where n = {0, 1, 2, 3, 4…}

Equation 2 for all composites in groups C and F:

$$c2 = 3(n + 2)$$

where n = {0, 1, 2, 3, 4…}

Equation 5 for part of the composites in group A:

$$c3 = (1 + 6n) \times (1 + 6m)$$

where n = {1, 2, 3, 4…} and m = {1, 2, 3, 4…}

Equation 6 for part of the composites in group A:

$$c4 = (5 + 6n) \times (5 + 6m)$$

where n = {0, 1, 2, 3, 4…} and m = {0, 1, 2, 3, 4…}

Equation 7 for all composites in group E:

$$c5 = (1 + 6n) \times (5 + 6m)$$

where n = {1, 2, 3, 4…} and m = {1, 2, 3, 4…}

These 5 equations were programmed and tested up to the first 10.000 natural numbers, and the results obtained compared with available recorded results on primes and composites. We can, for means of demonstration, check the first 100 natural numbers using this method, we have displayed it in Table 6.

Table 6. The first 100 natural numbers classified

Number	Prime or Composite Family
1	not a prime number by definition
2	prime number
3	prime number
4	c1 composite number
5	prime number
6	c1 and c2 composite number
7	prime number
8	c1 composite number
9	c2 composite number
10	c1 composite number
11	prime number
12	c1 and c2 composite number
13	prime number
14	c1 composite number
15	c2 composite number
16	c1 composite number
17	prime number
18	c1 and c2 composite number
19	prime number
20	c1 composite number
21	c2 composite number
22	c1 composite number
23	prime number
24	c1 and c2 composite number
25	c3 composite number
26	c1 composite number
27	c2 composite number
28	c1 composite number
29	prime number
30	c1 and c2 composite number
31	prime number
32	c1 composite number
33	c2 composite number
34	c1 composite number

35	c5 composite number
36	c1 and c2 composite number
37	prime number
38	c1 composite number
39	c2 composite number
40	c1 composite number
41	prime number
42	c1 and c2 composite number
43	prime number
44	c1 composite number
45	c2 composite number
46	c1 composite number
47	prime number
48	c1 and c2 composite number
49	prime number
50	c1 composite number
51	c2 composite number
52	c1 composite number
53	prime number
54	c1 and c2 composite number
55	c4 composite number
56	c1 composite number
57	c2 composite number
58	c1 composite number
59	prime number
60	c1 and c2 composite number
61	prime number
62	c1 composite number
63	c2 composite number
64	c1 composite number
65	c5 composite number
66	c1 and c2 composite number
67	prime number
68	c1 composite number
69	c2 composite number
70	c1 composite number
71	prime number
72	c1 and c2 composite number
73	prime number
74	c1 composite number
75	c2 composite number
76	c1 composite number
77	c5 composite number
78	c1 and c2 composite number
79	prime number
80	c1 composite number
81	c2 composite number
82	c1 composite number
83	prime number
84	c1 and c2 composite number
85	c4 composite number
86	c1 composite number
87	c2 composite number
88	c1 composite number
89	prime number
90	c1 and c2 composite number
91	c3 composite number
92	c1 composite number
93	c2 composite number
94	c_1 composite number
95	c_5 composite number
96	c_1 and c_2 composite number
97	prime number
98	c_1 composite number
99	c_2 composite number
100	c_1 composite number

Additionally this shows that for any number x, the next composite number c>N will be the smallest possible solution, cy, from any of the 5 equations of the set being cy>x. Any numbers between x and cy are prime numbers, as long as cy is the next possible valid solution of the set larger than x.

H. Obtaining Prime Numbers by Iteration

For any composite number of group A (1+6k) where k = {1, 2, 3, 4…}, using equations 5 and 6 we have seen that k can either be:

k=n+m+6nm (from equation 8)

where n = {1, 2, 3, 4…} and m = {1, 2, 3, 4…}

or

k=4+5(n+m)+6nm (from equation 9)

where n = {0, 1, 2, 3, 4…} and m = {0, 1, 2, 3, 4…}

Therefore, for any N value of k that cannot be obtained using valid solutions of n and m in any of these two equations, then 1+6k will be a prime number. For example, 7 is a prime number from group A with k=1 that cannot be obtained by valid values of n and m in equations 5 and 6.

On a similar way, for group E, any number (5+6k) where k = {0, 1, 2, 3, 4…} that cannot be generated using equation 13 will be a prime number.:

k=m+5n+6nm

where n = {1, 2, 3, 4…} and m = {0, 1, 2, 3, 4…}

For example, 29 is a prime number from group E with k=4 that cannot be obtained using valid soluitons for m and n.

Therefore, for a given number x,), if it belongs to A or to E (by means of checking if is of the form 1+6k or 5+6k) we can iterate equation 7 (if is from E) or equations 5 and 6 (if it is from A) and if no valid solution is found, then x is a prime number.

The authors of this paper tested this iteration method up to the first 10.000 natural numbers and compared with one of the existing simpler primality test. The results were found correct but the speed of iteration slower than the already established primality test.

I. Twin Prime Numbers

A twin prime is a prime number separated just by one composite from another prime number—for example, either member of the twin prime pair (41, 43).

If we divide all natural numbers N in the six groups A to F described in this paper, the only way to obtain prime numbers would be that one of them belongs to group E and the other to group A, being the prime number from group E pE 5+6k the smaller one of the pair and the primer number form column A pA = 1+6(k+1) the larger one of the pair. Following the reasoning from section H, to find 2 twin prime numbers, one would have to iterate:

k=m+5n+6nm (from equation 13)

where k = {0, 1, 2, 3, 4…} until a k is found where there is not a single combination of n and m where n = {1, 2, 3, 4…} and where m = {0, 1, 2, 3, 4…} that originates a prime number. Followed by iterating equations 5 and 6, for k+1:

(k+1)=n+m+6nm (from equation 8)

where n = {1, 2, 3, 4…} and where m = {1, 2, 3, 4…}

and

(k+1)=4+5(n+m)+6nm (from equation 9)

where n = {0, 1, 2, 3, 4…} and where m = {0, 1, 2, 3, 4…}

If none of the two can obtain a valid value of k using valid values for n and for m, then the conditions are met, and 5+6k and 1+6(k+1) are twin prime numbers. As an example, we can look at 41 (group E) and 43 (group A), two twin prime numbers.

41 = 5+(6x6)

43 = 1+(6x7)

In this case, k=6

6=m+5n+6nm

where n = {1, 2, 3, 4…} and here m = {0, 1, 2, 3, 4…}

A valid solution cannot be obtained. n=1 m=0 gives a value of 5 for k. n=m=1 gives a value for k of 12, and of course, increasing n or m would only increase the value of k. 41 is our lowest prime number of the twin set and has a k of 6.

Now we continue by replacing k by 6 in the two remaining equations:

(6+1)=n+m+6nm (from equation 8)

where n = {1, 2, 3, 4…} and where m = {1, 2, 3, 4…}

A valid solution cannot be found. n=m=1 gives a value for k of 7, and of course, increasing n or m would only increase it.

and

(6+1)=4+5(n+m)+6nm (from equation 9)

where n = {0, 1, 2, 3, 4…} and where m = {0, 1, 2, 3, 4…}

A valid solution cannot be found. n=m=0 leads to k=3. n=1 and m=0 leads to k=8. n=0 and m=1 leads to k=8, and of course increasing them more would only increase k.

k=6 does not find a valid set of values for m and n for any of the 2 equations. Therefore is a prime of the form 1+6(k+1), 43. 41 and 43 are twin prime numbers.

III. RESULTS

By dividing all natural numbers in 6 groups, A to F, and following the reasoning behind the generation of all composite numbers in each group, we have a set a 5 equations. These 5 equations define that a composite number c, must fulfill at least one of the following conditions:

$c1=2(n+2)$ where n = {0, 1, 2, 3, 4…}

$c2=3(n+2)$ where n = {0, 1, 2, 3, 4…}

$c3=(1+6n)\times(1+6m)$ where n = {1, 2, 3, 4…} and m = {1, 2, 3, 4…}

$c4=(5+6n)\times(5+6m)$ where n = {0, 1, 2, 3, 4…} and m = {0, 1, 2, 3, 4…}

$c5=(1+6n)\times(5+6m)$ where n = {1, 2, 3, 4…} and m = {0, 1, 2, 3, 4…}

Any number not fulfilling at least one of the equations is a prime number.

It is also possible to look for prime numbers by iteration, looking for values of k in group A or in group E that cannot be obtained valid solutions of n and m.

For group A, from equations 8 and 9 we have seen that k can either be:

k=n+m+6nm (for equation 8)

where n = {1, 2, 3, 4…}, m = {1, 2, 3, 4…} and k = {1, 2, 3, 4…}

or

k=4+5(n+m)+6nm (for equation 9)

where n = {0, 1, 2, 3, 4…}, m = {0, 1, 2, 3, 4…} and k = {1, 2, 3, 4…}

Therefore, for a valid value of k that cannot be obtained in any of the two equations using valid solutions for n and m, 1+6k is a prime number.

A prime number from group E is obtained by using equation 13:

k=m+5n+6nm

where n = {1, 2, 3, 4…}, m = {0, 1, 2, 3, 4…} and k = {0, 1, 2, 3, 4…}

For a valid of k that cannot be obtained using valid solutions for n and m, then 5+6k is a prime number.

Finally, the method to obtain prime numbers by iteration presented in this paper can be used to obtain twin prime numbers. First by finding a value of k that generates a prime number in the group E (5+6k). Then by iteration in equations 5 and 6. If 1+6(k+1) is also a prime number, then 5+6K and 1+6(k+1) are twin prime numbers.

IV. CONTRIBUTIONS

Segura J. J.: Mathematical reasoning and writing.
Barchino R.: Programming, and iterations.

The authors do not incur into any conflict of interests by publishing these results.

V. ANNEX

The equations provided in this paper were tested by programming the following algorithm in Java

```java
public class CompositeNumbers {
   public boolean isComposite(long testNumber, long n, long m) {
      return isC1(testNumber, n) ||
            isC2(testNumber, n) ||
            isC3(testNumber, n, m) ||
            isC4(testNumber, n, m) ||
            isC5(testNumber, n, m);
   }

   private boolean isC1(long testNumber, long n) {
      return testNumber == c1(n);
   }
   public long c1(long n) {
      return 2 * (n + 2);
   }
   private boolean isC2(long testNumber, long n) {
      return testNumber == c2(n);
   }
   public long c2(long n) {
      return 3 * (n + 2);
   }
   private boolean isC3(long testNumber, long n, long m) {
      if (n == 0 || m == 0) return false;
      return testNumber == c3(n, m);
   }
   public long c3(long n, long m) {
      return (1 + 6 * n) * (1 + 6 * m);
   }
   private boolean isC4(long testNumber, long n, long m) {
      return testNumber == c4(n, m);
   }
   public long c4(long n, long m) {
      return (5 + 6 * n) * (5 + 6 * m);
   }
   private boolean isC5(long testNumber, long n, long m) {
      if (n == 0) return false;
      return testNumber == c5(n, m);
   }
   public long c5(long n, long m) {
      return (1 + 6 * n) * (5 + 6 * m);
   }
}
```

The values obtained were compared to those obtained by well-established primality test:

```java
public class PrimeNumbers {
   private CompositeNumbers compositeNumbers = new CompositeNumbers();
   public boolean isPrime(long testNumber) {
      if (testNumber == 1) return false;
      if (testNumber < 3) return true;
      long top = testNumber / 2;
      for (long n = 0; n <= top; n++) {
         if (compositeNumbers.c1(n) > testNumber) {
            return true;
         }
         for (int m = 0; m <= top; m++) {
            if (compositeNumbers.isComposite(testNumber, n, m)) {
               return false;
            }
            if (compositeNumbers.c3(n, m) > testNumber) {
               break;
            }
         }
      }
      return true;
   }
}
```

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