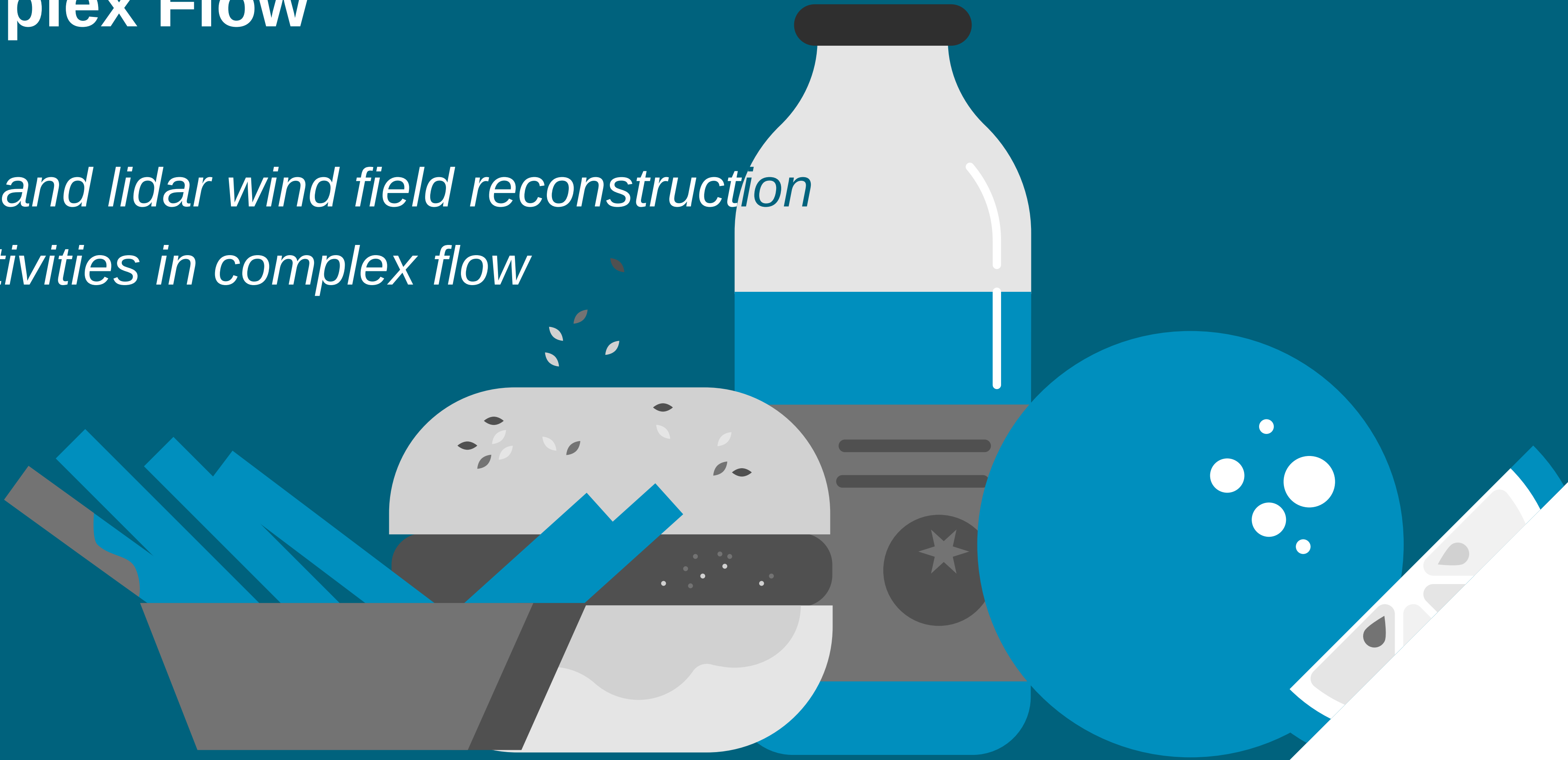
IEA Task 52 Lunch Webinar Series
Complex Flow

Point and lidar wind field reconstruction sensitivities in complex flow



Andrew Hastings Black, Frederic Delbos

14 February 2023

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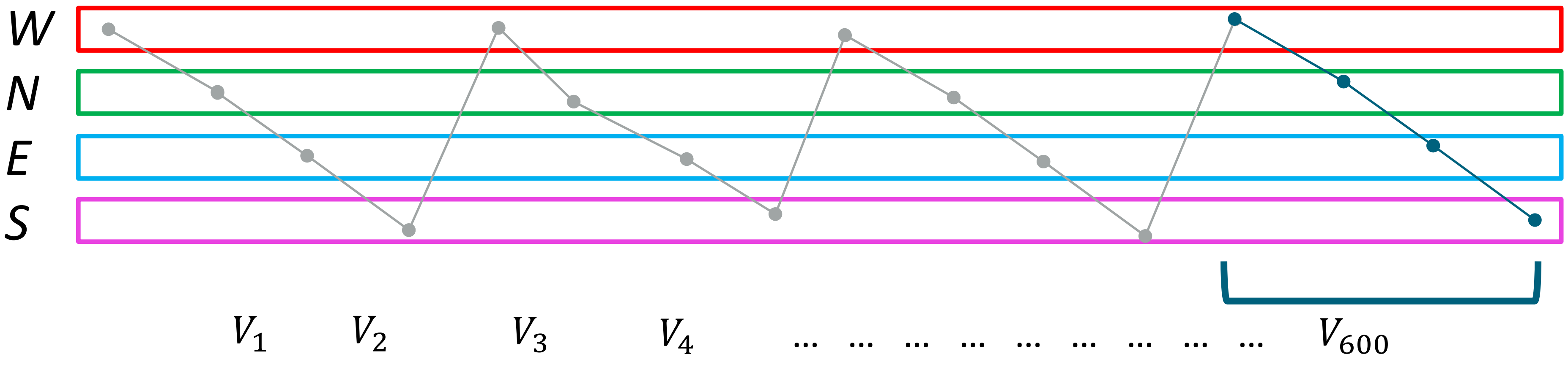
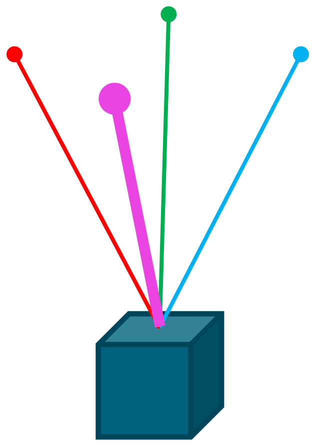
Background + Motivation

- IEA Task 32’s round robin yielded ambiguous results:
  - General lidar over-estimation of wind speed for many CFD and LES corrections
  - Some sites are quite extreme, slopes of ~20°
  - *Black et al* (2020) showed better performance for variety of sites and methods
- Recent literature shows turbulence can create subtle differences between anemometers and lidars depending on the wind field reconstruction (WFR)
  - Hybrid WFR eliminates turbulence sensitivities in flat terrain: *Rosenbusch et al* (2021)
  - *Lundquist and Robey* (2022) demonstrated variations in various stability conditions via LES
- In this presentation, we extend the theoretical basis for turbulent sensitivities, for point and lidar measurements to include complex flow
  - Bonus: from the Task 32 study, at Sites A, B, and C, the 1 Hz RTD data is available, enabling reprocessing with different WFR to sanity check this hypothesis

Wind Field Reconstruction Refresher

Scalar Averaging (Scalar WFR)





Scalar averaging generates a new wind speed every second, using the last four measurements

$$\frac{N-S}{2\sin\theta} = u_i \qquad \frac{E-W}{2\sin\theta} = v_i$$

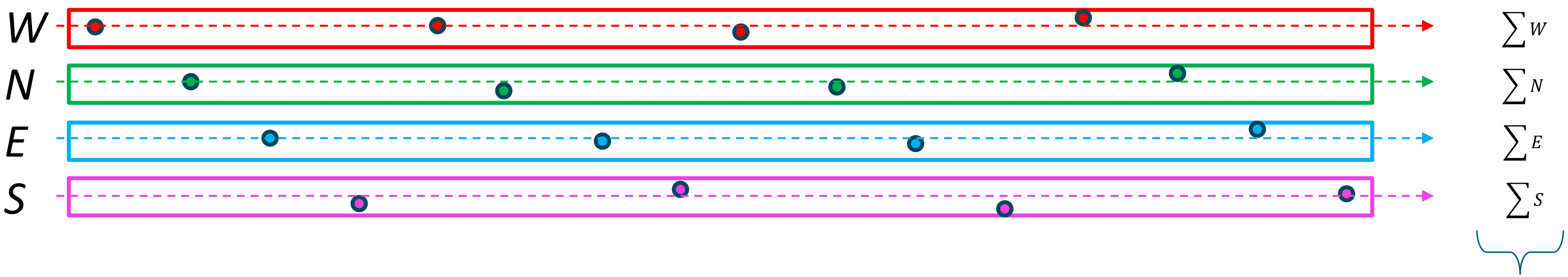
$$\sqrt{u_i^2 + v_i^2} = V_i$$

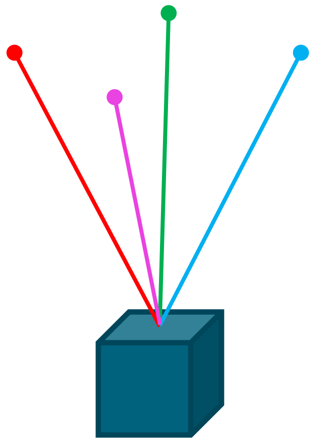
$$\overline{V_{scalar}} = \frac{1}{600}\sum_{i=1}^{600} V_i$$

These 1 Hz wind speeds are averaged at the end of the 10-minute period to generate the average wind speed

Wind Field Reconstruction Refresher

Vector Averaging (Vector WFR)





Vector averaging generates just one wind speed for each 10-minute period using the average radial beam measurements

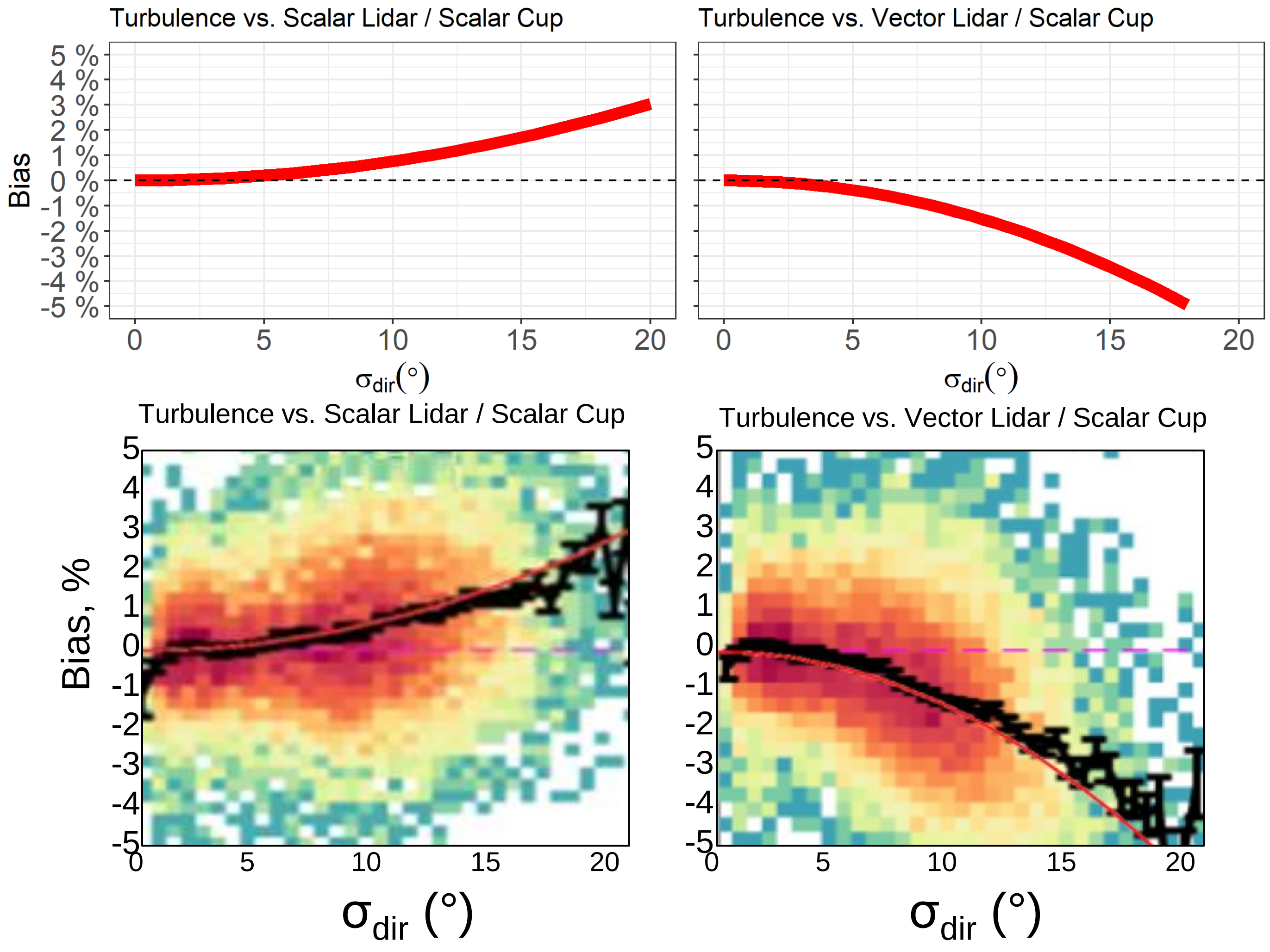
$$\frac{1}{2\sin\theta}\left[\sum N - \sum S\right] = \sum u$$

$$\frac{1}{2\sin\theta}\left[\sum W - \sum E\right] = \sum v$$

$$\overline{V_{vector}} = \frac{1}{150}\sqrt{\left(\sum u\right)^2 + \left(\sum v\right)^2}$$

Wind Field Reconstruction Refresher

Hybrid Averaging (Hybrid WFR)



- Comparisons of WindCube to high quality met masts agree very well with theory for scalar and vector averaging
- In WindCube v2.1, we combine the two methods and reduce the sensitivity to turbulence by an order of magnitude

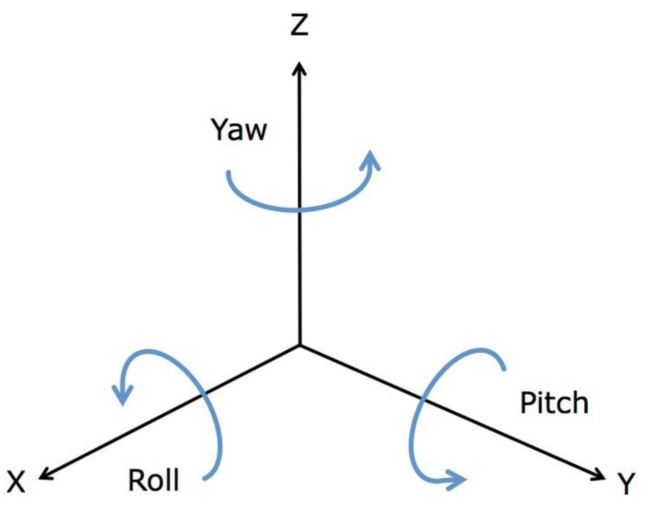
$$V_{Hybrid}^{lidar} = \frac{2}{3}V_{scalar}^{lidar} + \frac{1}{3}V_{vector}^{lidar}$$

$$= \frac{2}{3}V_{scalar}^{cup} + \frac{1}{3}V_{scalar}^{cup} + \frac{1}{6}\tilde{\sigma}^2 - \frac{1}{6}\tilde{\sigma}^2$$

$$= V_{scalar}^{cup}$$

Wind Measurement Theory
Reynolds Decomposition and SO(3) Rotation Group

$$R_x(\theta_x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & -\sin\theta_x \\ 0 & \sin\theta_x & \cos\theta_x \end{bmatrix}, \quad R_y(\theta_y) = \begin{bmatrix} \cos\theta_y & 0 & \sin\theta_y \\ 0 & 1 & 0 \\ -\sin\theta_y & 0 & \cos\theta_y \end{bmatrix}, \quad R_z(\theta_z) = \begin{bmatrix} \cos\theta_z & -\sin\theta_z & 0 \\ \sin\theta_z & \cos\theta_z & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$



Express measurement axes using SO(3) group

- **Example #1:** wind measurement along the North Beam of a WindCube lidar is derived:

$$R_y(-62°)R_z(0°)\boldsymbol{uU_N} = \mathcal{L}_N = \sin(62) * (\bar{u} + u') + \cos(62) * (\bar{w} + w')$$

- **Example #2:** wind measurement along ***v*** for ultrasonic anemometer: $R_z\,(90°)\boldsymbol{uU} = \boldsymbol{v}$

For boundary layer wind: $\boldsymbol{U} = \begin{bmatrix} \bar{u} + u' \\ \bar{v} + v' \\ w' \end{bmatrix}$ and measurement axis, ***u***

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Wind Measurement Theory

Simplest Configuration: USA in flat, turbulent flow

Measurements: $\boldsymbol{u} = \boldsymbol{u}$ and $R_z\,(90°)\boldsymbol{uU} = \boldsymbol{v}$

1. Expand all terms in the WFR $U_{scalar,1Hz} = \sqrt{\boldsymbol{u^2} + \boldsymbol{v}^2} = \sqrt{\bar{u}^2 + 2\bar{u}u' + u'^2 + \bar{v}^2 + 2\bar{v}v' + v'^2}$

2. Factor out the vector average $= \sqrt{\bar{u}^2 + \bar{v}^2}\sqrt{1 + \frac{1}{\bar{u}^2+\bar{v}^2}[2\bar{u}u' + u'^2 + 2\bar{v}v' + v'^2]}$

3. Apply 1st order binomial expansion $= U_{vector}\left[1 + \frac{1}{2U_{vector}^2}[2\bar{u}u' + u'^2 + 2\bar{v}v' + v'^2]\right]$

4. Apply 2nd order binomial expansion $= U_{vector}\left[1 + \frac{1}{2U_{vector}^2}[2\bar{u}u' + 2\bar{v}v' + (u'^2 + v'^2)\sin^2\varphi]\right]$

   not fully derived yet for all cases

where φ is the angle of the random fluctuations

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Wind Measurement Theory

Simplest Configuration: Anemometer in flat, turbulent flow

Measurements: $\boldsymbol{uU} = \boldsymbol{u}$ and $R_z\,(90°)\boldsymbol{uU} = \boldsymbol{v}$

3. Apply 1st order binomial expansion

$$= U_{vector}\left[1 + \frac{1}{2U_{vector}^2}\left[2\bar{u}u' + u'^2 + 2\bar{v}v' + v'^2\right]\right]$$

4. Summation over 10 minutes

$$= \overline{U_{vector} + \frac{1}{2U_{vector}^2}\left[2\bar{u}u' + u'^2 + 2\bar{v}v' + v'^2\right]}$$

Linear terms all drop out, only squared turbulent terms remain:

$$U_{scalar,10min} = U_{vector} + \frac{1}{2U_{vector}^2}\left[\overline{u'^2} + \overline{v'^2}\right]$$

5. Reformulate:

$$U_{scalar,10min} = U_{vector} + \frac{1}{2U_{vector}}\sum_{i,j=1}^{3}\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 0\end{bmatrix} \circ \begin{bmatrix}\overline{u'^2} & \overline{u'v'} & \overline{u'w'}\\ \overline{v'u'} & \overline{v'^2} & \overline{v'w'}\\ \overline{w'u'} & \overline{w'v'} & \overline{w'^2}\end{bmatrix} = U_{vector} + \frac{1}{2U_{vector}^2}\sum_{i,j=1}^{3}\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 0\end{bmatrix} \circ \boldsymbol{\tau_{ij}}$$

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Wind Measurement Theory
Anemometer in flat or complex turbulent flow

1st order Point measurement, flat, turbulent flow

$$U_{scalar,10min} = U_{vector} + \frac{1}{2U_{vector}^2} \sum_{i,j=1}^{3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \circ \begin{bmatrix} \overline{u'^2} & \overline{u'v'} & \overline{u'w'} \\ \overline{v'u'} & \overline{v'^2} & \overline{v'w'} \\ \overline{w'u'} & \overline{w'v'} & \overline{w'^2} \end{bmatrix} = U_{vector} + \frac{1}{2U_{vector}} \sum_{i,j=1}^{3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \circ \boldsymbol{\tau_{ij}}$$

What happens in complex flow?

Use SO(3) rotations again:

$R_x(\theta_x)R_y(\theta_y)\boldsymbol{U} =$ arbitrary tilt and roll applied to wind

1st order Point measurement, tilted, turbulent flow

$$\overline{U_{scalar}} = U_{vector} + \frac{1}{2U_{vector}^2} \sum_{i,j=1}^{3} \begin{bmatrix} \cos^2\theta_y + \sin^2\theta_y \sin^2\theta_x & \sin\theta_x \sin\theta_y \cos\theta_x & \sin\theta_y \cos\theta_y - \sin^2\theta_x \sin\theta_y \cos\theta_y \\ \sin\theta_x \sin\theta_y \cos\theta_x & \cos^2\theta_x & \cos\theta_x \sin^2\theta_x \cos^2\theta_y \\ \sin\theta_y \cos\theta_y - \sin^2\theta_x \sin\theta_y \cos\theta_y & \cos\theta_x \sin^2\theta_x \cos^2\theta_y & \sin^2\theta_x + \sin^2\theta_y \end{bmatrix} \circ \boldsymbol{\tau_{ij}}$$

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Wind Measurement Theory

General Forms

- After this derivation, a wide variety of point and lidar measurement geometries can all be expressed as a sum of the vector average and an “scalar inflation tensor”.

$$U_{scalar,10min} = U_{vector} + \frac{1}{2U_{vector}} \sum_{i,j=1}^{3} \begin{vmatrix} \not{f}_{ii} & \not{f}_{ij} & \not{f}_{ik} \\ \not{f}_{ji} & \not{f}_{jj} & \not{f}_{jk} \\ \not{f}_{ki} & \not{f}_{kj} & \not{f}_{kk} \end{vmatrix} \circ \begin{bmatrix} \overline{u'^2} & \overline{u'v'} & \overline{u'w'} \\ \overline{v'u'} & \overline{v'^2} & \overline{v'w'} \\ \overline{w'u'} & \overline{w'v'} & \overline{w'^2} \end{bmatrix} = U_{vector} + \frac{1}{2U_{vector}} \sum_{i,j=1}^{3} \begin{vmatrix} \not{f}_{ii} & \not{f}_{ij} & \not{f}_{ik} \\ \not{f}_{ji} & \not{f}_{jj} & \not{f}_{jk} \\ \not{f}_{ki} & \not{f}_{kj} & \not{f}_{kk} \end{vmatrix} \circ \boldsymbol{\tau_{ij}}$$

10-min scalar WFR average

10-min vector WFR average

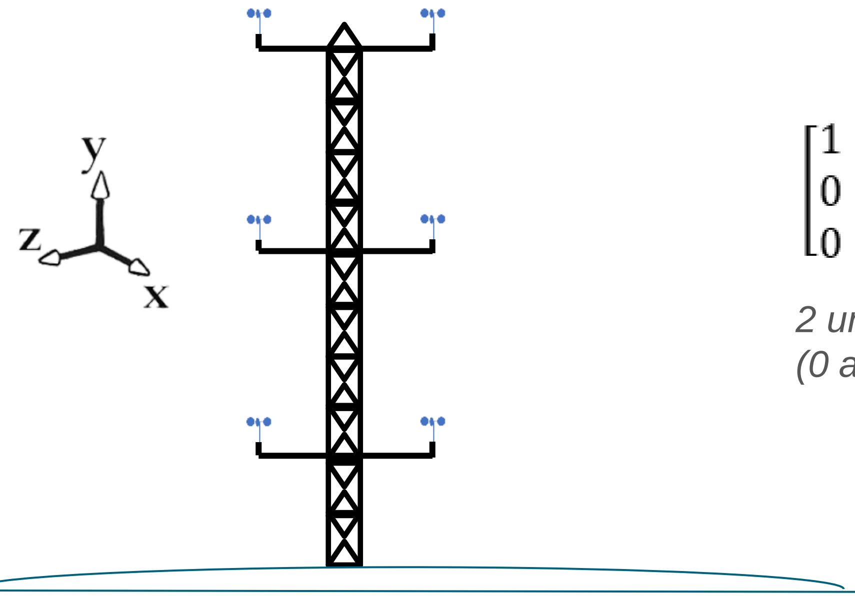
“Scalar Inflation Tensor”

- Up to 9 unknowns
- Measurement angles
- Flow tilt angles

Reynolds Stress Tensor

- Assuming isotropic turbulence, 6 unknowns
- $u'^2, v'^2, w'^2, u'w', v'w', u'v'$

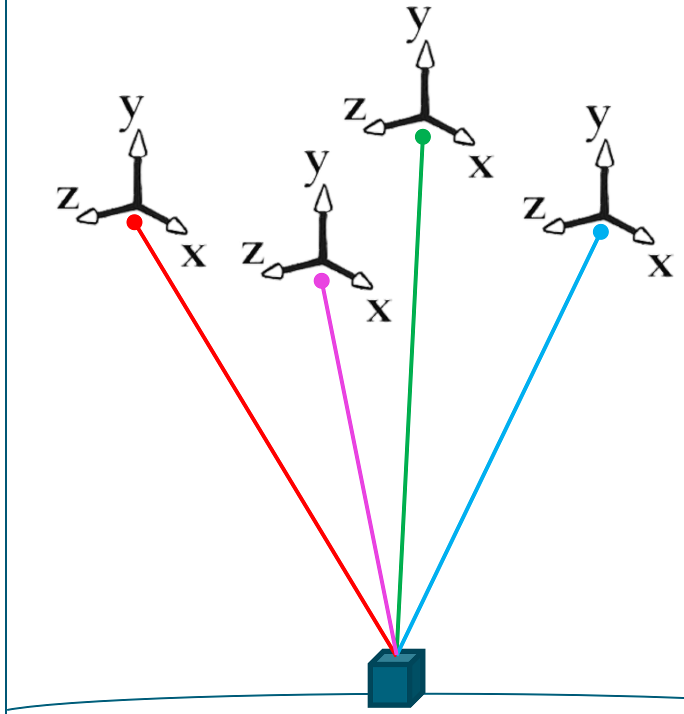
Point measurement, flat terrain



$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \boldsymbol{\tau_{ij}}$$

2 unknowns
(0 angles)

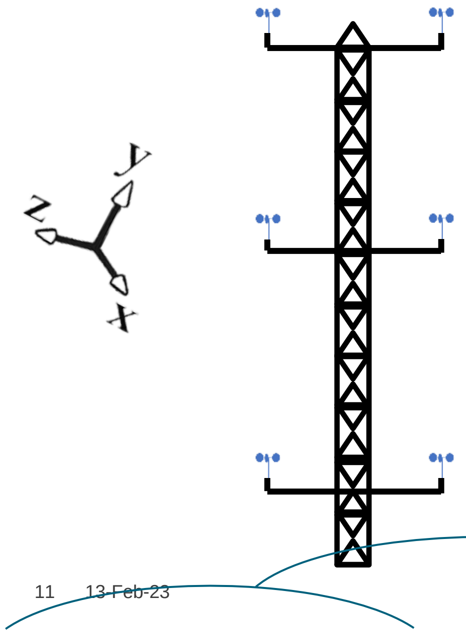
Lidar measurement, flat terrain



$$\begin{vmatrix} 1 & 0 & \tan\varphi \\ 0 & 1 & \tan\varphi \\ \tan\varphi & \tan\varphi & 2\tan^2\varphi \end{vmatrix} \cdot d\tau_{ij}$$

5 unknowns
(0 angles)

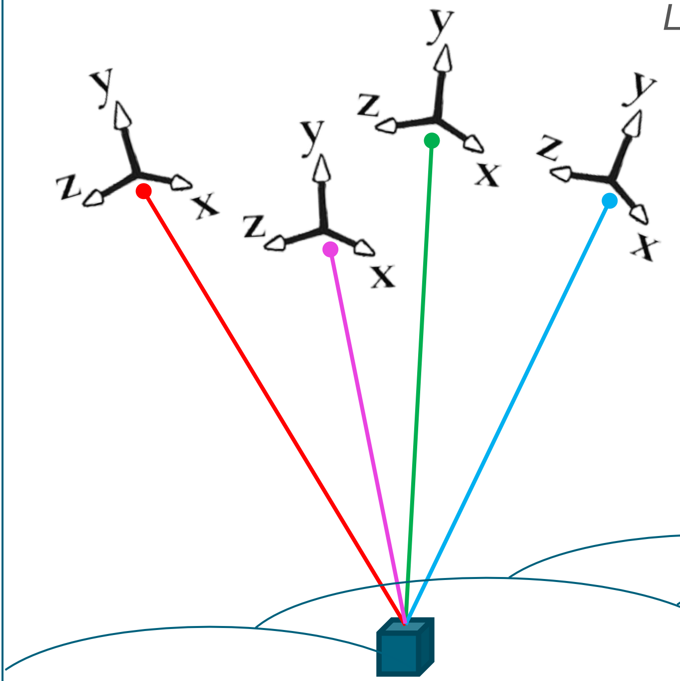
Point measurement, complex terrain



$$\begin{bmatrix} \cos^2\theta_y + \sin^2\theta_y \sin^2\theta_x & \sin\theta_x \sin\theta_y \cos\theta_x & \sin\theta_y \cos\theta_y - \sin^2\theta_x \sin\theta_y \cos\theta_y \\ \sin\theta_x \sin\theta_y \cos\theta_x & \cos^2\theta_x & \cos\theta_x \sin^2\theta_x \cos^2\theta_y \\ \sin\theta_y \cos\theta_y - \sin^2\theta_x \sin\theta_y \cos\theta_y & \cos\theta_x \sin^2\theta_x \cos^2\theta_y & \sin^2\theta_x + \sin^2\theta_y \end{bmatrix} \boldsymbol{\tau_{ij}}$$

8 unknowns
(2 angles)

Lidar measurement, complex terrain

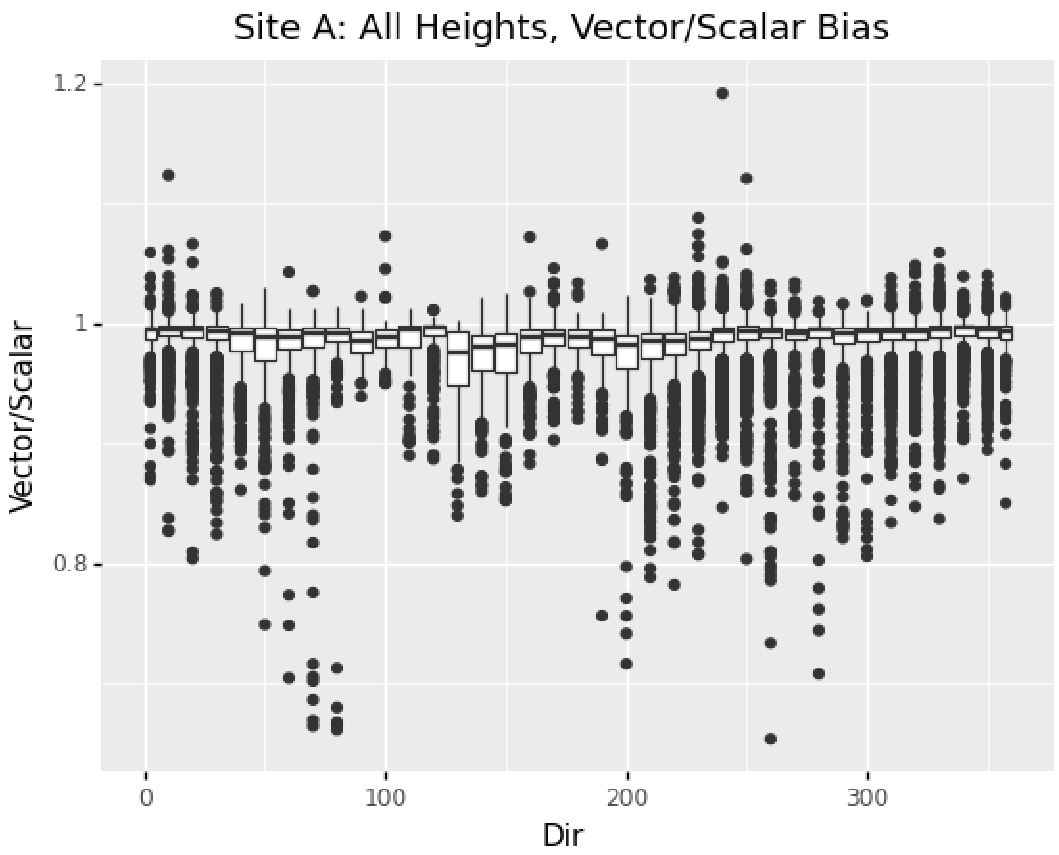


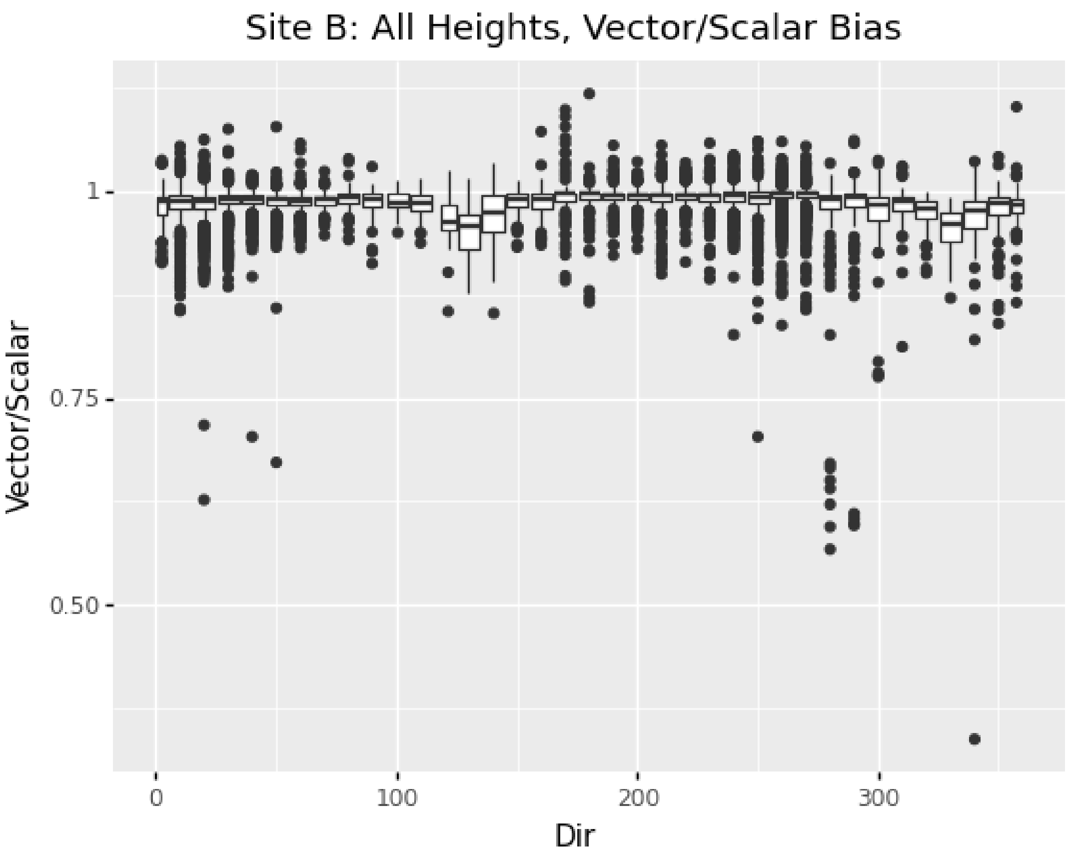
$$\begin{vmatrix} \tau_{ii} & \tau_{ij} & \tau_{ik} \\ \tau_{ji} & \tau_{jj} & \tau_{jk} \\ \tau_{ki} & \tau_{kj} & \tau_{kk} \end{vmatrix} \tau_{ij}$$

14 unknowns*
(8 angles)

IEA Task 32 Site Data **Preliminary Reanalysis**

Scalar vs. Vector WFR Comparison





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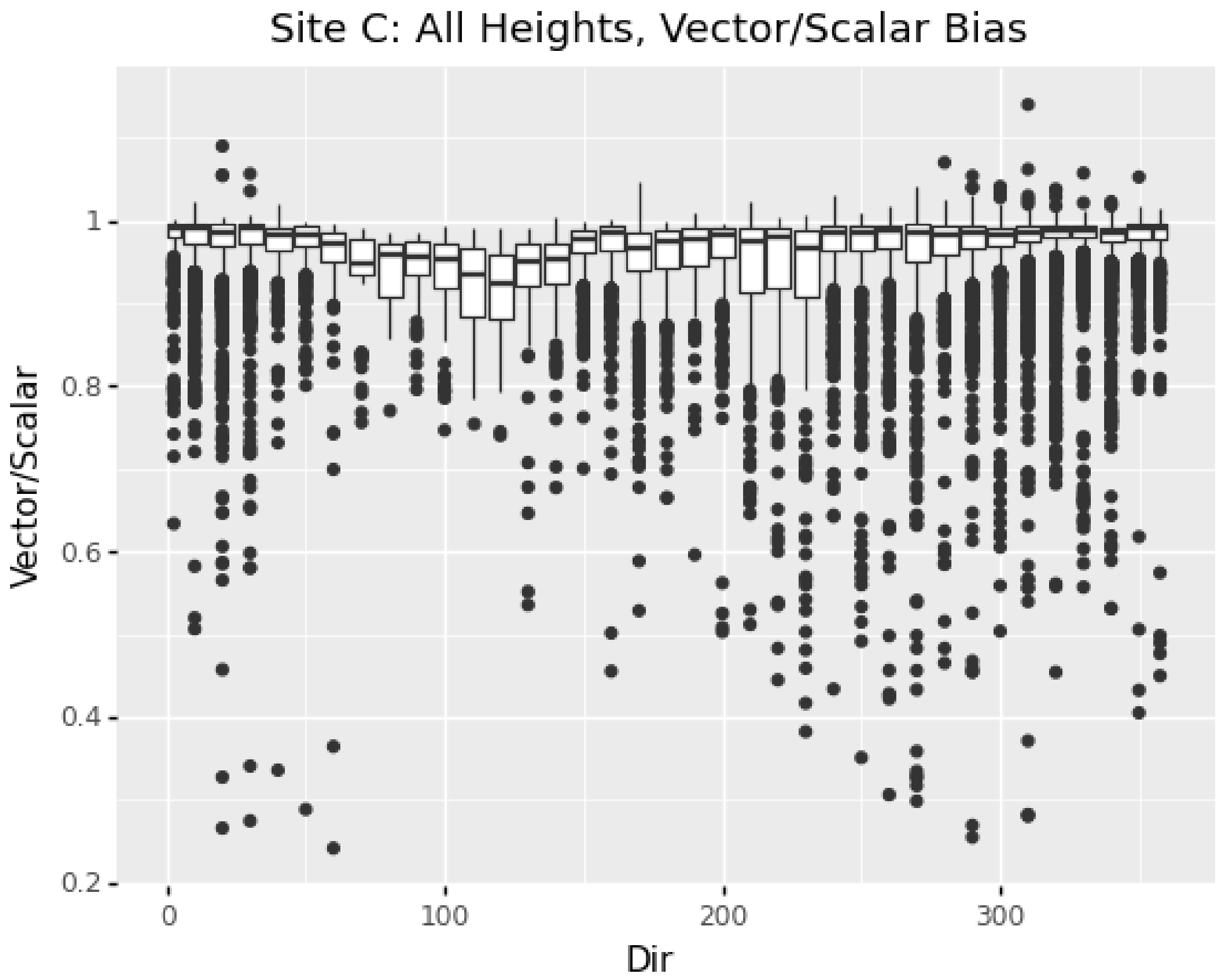
IEA Task 32 Site Data **Preliminary Reanalysis**
Scalar vs. Vector WFR Comparison



- Up to 7% bin average differences between Scalar and Vector WFR
- Overall differences on order of 3%
- Seemingly can explain some of the ambiguities in this dataset

- Next steps:
  - Expand to other sites
  - Reprocess USA data to estimate ReST terms

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Thoughts and Open Questions
Goal: Well-understood and Traceable Uncertainties

- In traditional IEC Classification (following 61400-50-2):
  - uncertainties estimated using linear functions of various atmospheric parameters and assumptions that suitable ranges of those parameters are captured in the Classification, Validation, and specific measurement campaigns
  - Hybrid WFR weightings validated, largely eliminates turbulence sensitivity
- For a Complex Terrain Classification :
  - Anemometers and lidars have sensitivities to Reynolds Stress Tensor
    - **Sensitivity “candidates” may need to be expanded**
  - No set methodology for estimating uncertainties of CFD correction itself
    - **Theory shows angles are even more critical than previously assumed**
  - **Need assumption of SMC conditions different than Classif / Valdation**

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Thoughts and Open Questions
Goal: Well-understood and Traceable Uncertainties

- Some proposals for traceable lidar complex terrain campaigns include met masts
  - WindEurope 2022: *Montavon et al* (2022); *Nixon et al* (2022)
    - **Could USAs be included to measure the Reynolds Stress Tensor + angles ?**
- LES and CFD are outstanding tools for lidar science
  - *Lundquist and Robey* (2022), *Mann and Kelberlau* (2020)
    - **Could we use LES and CFD for complex Classification instead of field campaigns?**
    - Classification → LES Classification → Flat Terrain Validation → Complex SMC
- What Hybrid WFR weights (α * scalar + β * vector) are ideal in complex terrain?
    - **Could they be site-dependent?**
    - **What are the key drivers? Full ReST, specific flow angles, stability, roughness length…**

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Thoughts and Open Questions

Goal: Well-understood and Traceable Uncertainties

- Today we showed only a few derivations…
  - **Need to methodically derive ReST sensitivities for more use cases →**
  - How to model a cup? Infinite circular measurement axes?
- Estimate errors for typical ReST values in flat and complex terrain (stable, unstable, neutral)
  - **Is there a 'canonical' ReST ?**
- Extend theory to 2nd order

Sensor	Tilt	Roll	Yaw	Simple	Complex
Ultrasonic				X	
Cup				X	
Lidar				X	
Ultrasonic	X	X			X
Ultrasonic	X	X	X		X
Lidar	X				X
Lidar		X			X
Lidar			X	X	
Lidar			X		X

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andrew.hastingsblack@vaisala.com

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