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ILG‘OR fAJRIBA VA O[k]QITISH METODIKASI

OLIMPIADA VA MASALALAR YECHISH BO‘LIMI

TALAB, TAKLIF VA TAHLIL

GEOMETRIYANING ASOSIY TEOREMA VA FORMULALARINI BIRGALIKDA QO'LLASH YORDAMIDA YECHILADIGAN MASALALAR

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Ushbu maqolada turlicha qiyinlikdagi ba'zi geometrik masalalarni yechishda asosiy teoremalar va formulalarni birgalikda qo 'llashga doir masalalar o 'rganilgan.

Tayanch so'zlar: *teorema, formula, bissektrisa, mediana, balandlik, dioganal, urinma, perpendikulyar, burchak, qavariq.*

This article are stadied some geometric problems of different difficulties, which recuire multifunction using main theorems and formulas in geometry.

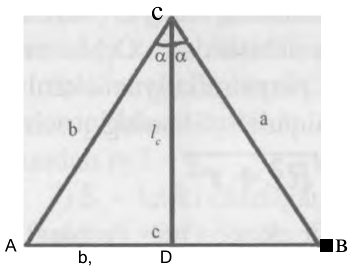
Keywords: *theorem, the formula, bicektor, median, perpendicular, plane angle, diagonal, tangent, angle, adjasrnt.*

В данной статье изучены некоторые различные по трудности геометрические задачи и их решение комбинированным применением основных теорем и формул геометрии.

Ключевые слова: *теорема, формула, биссектриса, медиана, высота, диагональ, касательная, перпендикуляр, угол, выпуклый.*

Turli qiyinlikdagi ba'zi geometrik masalalarni yechish geometriyaning (planimetriyaning) asosiy teorema va formulalarini birgalikda qo'llashni talab etadi. Shu tipdagi masalalarni o'rganish va yechish bilan shug'ullanish o'qituvchlar, talabalar va o'quvchilar uchun bilimlarini mustahkamlashda foydalidir. Quyida shu tipdagi bir nechta geometrik masalalarni o'rganamiz.

1-masala. Uchburchak burchagi bissektrisasining kvadrati unga yopishgan tomonlari ko'paytmasi bilan bissektrisa qarshi tomondan ajratgan kesmalar ko'paytmasi ayirmasiga tengligini, ya'ni $l^2 \sim ab - a_l b_l$ tenglikni isbotlang.



1 -rasm

Yechish: ACD va BCD uchburchaklarni qaraymiz (1-rasm).

1) Kosinuslar teoremasiga koʻra

$b^2 = b^2 + I^2$ - 2b l_c cosa ,

$a\backslash = a^2 + l^2$ - 2a l_c *cosa* bunda

a=Z C/2

Bundan 2 ^ *cosa* ^ b2+ I2 b2 ya'ni

2 a l_c cosa a 2 + i ^ - a ^ '

I^2 **(b - a)** = *ab* **(b — a) - (ab* - a^b) (A)**

2) Uchburchak bissektrisasining xossasiga koʻra a/b=a_l/$b_]$ ya'ni

ab^ajb. U holda **a** b^2 - a^2 **b - a_xb** b_x - **fltjbja- (b - a)**

va (A) tenglik l^2 **(b-a) = a b (b- a) - a_x** b_x **(b -a)** koʻrinishga keladi yoki b^a boʻIsa,

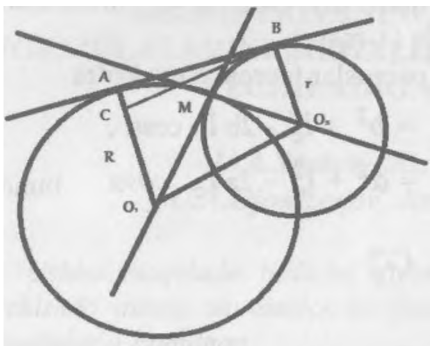
lc = a ^ ~ a***l^V*** Agar *a* = *b* boʻlsa, u holda *b=a=c/2* ,

$$a^2 - \frac{c^2}{4} \qquad l_c^2 = a^2 - \frac{c^2}{4}.$$

Demak, l^2=*ab*-$a_l b_j$ oʻrinli.

2-masala. Ikki aylana toʻgʻri burchak ostida kesishadi (ya'ni ular kesishish nuqtalaridan biri orqali oʻtkazilgan urinmalari oʻzaro perpendikulyar). Agar aylanalaming radiuslari *R* va *r* boʻlsa, bu aylanalar umumiy urinmasi kesmasi (uzunligini)ni toping.

Yechish: 0, va O_n berilgan aylanalar markazlari, M - ulaming kesishish nuqtalaridan biri, AB - umumiy urinma boʻlsin (2-rasm).



2-rasm

Aylanalaming to‘g‘ri burchak ostida kesishishidan OjM ning 0_2M ga perpendikulyar ekanligi kelib chiqadi. Shuning uchun

$$\mathbf{OiO}_{2}=\backslash/R^{2}+r^{2}.$$

Olaylik, R>r . 0,0;, ga parallel BC ni o‘tkazamiz va AABC ni qaraymiz (ZA=90°). Pifagor teoremasiga ko‘ra

$$AB=y/BC^{2}-AC^{2}=JR^{2}+r^{2}—(R—r)^{2}=y/2Rr.$$

Agar *R=r* bo‘lsa, u holda $AB=Op_2=R.\hat{}2$ Demak, *AB=^2Rr.*

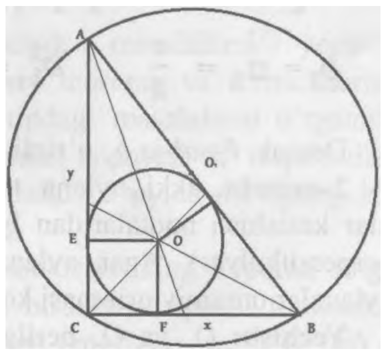
3-masaIa. Uchburchak tomonlari 25,24 va 7 ga teng. Unga ichki va tashqi chizilgan doiralar yuzlarini aniqlang.

Yechish: Masalani quyidagi tartibda yechish qulay: 1) Uchburchak tomonlarini *a, b, c* bilan belgilaylik. Faraz qilaylik *a-7, b=24, c=25.*

2) AABC ning turini aniqlaymiz: $a^2+b^2=625$ *va* $c^2=625$, ya’ni $a^2+b^2=c^2$ bo‘lganligidan Pifagor teoremasiga teskari teoremaga ko‘ra ДАВС - to‘g‘ri burchakli (3-rasm).

3). Uchburchakka tashqi chizilgan aylana markazi $0_($ - uchburchak tomonlariga o‘tkazilgan o‘rta рефеп-dikulyarlar kesishgan nuqtada bo‘ladi. To‘g‘ri burchakli uchburchakda tashqi chizilgan aylana markazi gipotenuzaning o‘rtasidir, ya’ni *R=c/2=12,5.*

4) Uchburchakka ichki chizilgan aylana markazi O bu uchburchak bissektrisalari kesishgan nuqtada bo‘ladi.



3-rasm

5) Aylanadan tashqaridagi nuqtadan o‘tkazilgan urinmalar kesmalari xossasiga ko‘ra *BD=BF=x, AE=AF=y.*

6) Shartga ko‘ra *BC=a=7* yoki *r+x=7. AC=b=24 yoki r+y=24* bu tengliklami qo‘shib *2r+(x+y)=31 yoki 2r+AB =31, 2r+25=31.* Bundan r=3.

7) S - Ichki chizilgan, S_2- tashqi chizilgan doiralar yuzlari bo‘lsin. U holda *S=nr=9n (kv. birlik.), S^itr² =12,5² n =624/4 n (kv. birlik.).*

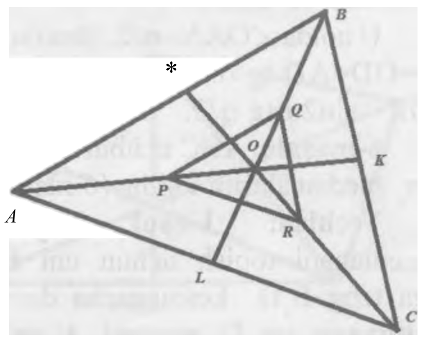
Eslatma: S_{MBC} *=abc/4R va* S_{AARC}*=pr* formulalardan foydalanib R va r radiuslami topish mumkin:

R =abc/2ab =c/2=12,5, r=S/p=ab/a+b+c=l 68/56=3

4-niasala. Yuzi 1 ga teng bo‘lgan ABC uchburchak berilgan. (4-rasm). Uchburchakning medianalari AK, BL va CN larda mos ravishda, Q va R nuqtalar olingan bo‘lib, *AP/PK=1, BQ/QL=l/2, CR/RN=5/4.* PQR uchburchakning yuzini toping.

Yechish: O nuqta ABC uchburchakning meridianalari kesishish nuqtasi bo‘lsin.

U holda S_{JAOB}-$S_A Bo_C$- S_{AAOC}. $=l/3S_{AABC}=l/3$. *APOQ-* uchburchakni qaraymiz. Bunda *OP=2/3AK-1/2AK=1/6AK* OQ *=2/3BL-l/3BL= 1/3BL. Va* S_{ApoQ}*=(l/2)OP OQsin< POQ,* S_{AAOB}*=(1/2)OA OBsin ZAOB* ekanligi e’tiborga olinsa, u holda



4-rasm

S_{APOQ} OP • OQ *j* AK ^ BL l

S_{AAOB} OA OB - AK* - BL 8

Bundan *S = 1/8* S_{AAOB} ga ega bo‘lamiz.

Shunga o‘xshash S_{AOpQ}*=(l/12)* S_{ABOC} S_{APO}*=(l/24)S* $_{dAOC}$ larga ega bo‘lamiz.

Shuning uchun S_{APQR}*=l/3(l/8+l/12+l/14) =1/12 (kv. birlik).*

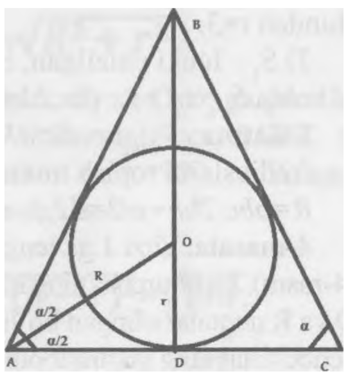
5-masala. Asosi a o‘tkir burchakli teng yonli uchburchakka ichki va tashqi chizilgan doiralar radiuslari nisbatini toping.

Yechish: ДABC teng yonli uchburchak. ZA=ZC=a bo‘lsin (5-rasm). AC=b belgilash olamiz. Sinuslar teoremasiga ko‘ra b/sinB=2R. Bunda R tashqi chizilgan doira radiusi.

Uchburchak asosidagi burchaklari a ga teng bo‘lganligidan Z B=180°-2a va sinB=sin(180°- a)=sin2a.

Demak, R=b/sin2a.

Endi O nuqta ichki chizilgan doira markazi bo‘lsin.



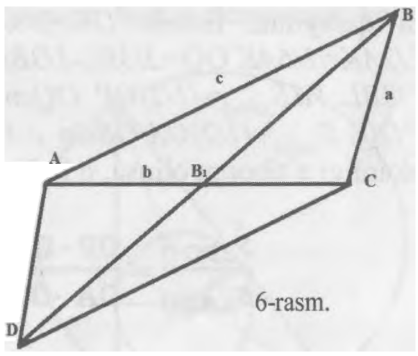
5-rasm

U holda ZOAA= a/2. Shuning uchun r=OD=AD tg a/2=(p/2) tg a/2. Bundan r/2 =sin2a tg a/2. Bundan r/R= sin2a tg a/2.

6-masala. ABC uchburchak tomonlari *a, b, c* ga teng. Uning m_a m_h m_c medianalarini toping (6-rasm).

Yechish: 1-usul. m=BB, b 1 medianani topish uchun uni $BB_($ ga teng B^ kesmagacha davom ettiramiz va D nuqtani A va C nuqtalar bilan birlashtiramiz.

Hosil bo‘lgan ABCD to‘rtburchak parallelogrammdir, chunki AC va BD dioganallar kesishish nuqtasida teng ikkiga bo‘linadi.



6-rasm.

Parallelogramm dioganallari kvadratlari yig‘indisi uning tomonlari kvadratlari yig‘indisiga teng, ya’ni $b^2+(2m_b)^2=2a^2+2c^2$.

Bundan m_b=-^ V2a^2 + 2c^2 — b^2. Xuddi shuningdek, m_u va

m_c medianalar topiladi.

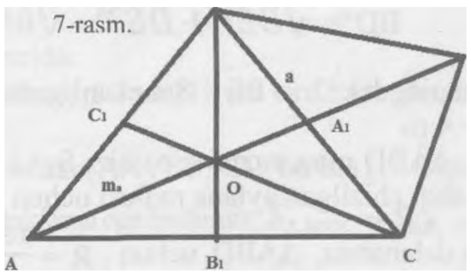
2-usul. ZAB|B=a bo‘lsin. U holda 68^=180° -a. Kosinuslar teoremasiga ko‘ra

C^2=— ***+ml - 2m$_b$ -*** cosa, a^2 = — + ***ml - 2m$_b$ -*** cos(180°-a;

Bu tengliklarni qo‘shib, cos(180°-a) = - cosa ekanligini e’tiborga olsak, $a^2+c^2=b^2/2+2m_b^2$- Bundan **ть** = ~ **V2a² + 2*C²* — *b²*.**

7-masala. ABC uchburchakda m_a, m_v *va* m_c medianalar berilgan. Uning *a, bva c* tomonlarini toping.

Yechish: a tomonni hisob-lab topish uchun AA_t kesma-ni *AjF* = *l/3m$_a$* masofagacha davom ettiramiz va *F* nuqtani *B* va C nuqtalar bilan tutashti-ramiz. O nuqta *IS.ABC* ning medianalari kesishgan nuqta bo‘lsin. *OBCF* to‘rtburchak -



Parallelogramm dioganallari xossasiga ko‘ra

a² + (frnj² = 2 (|m$_b$)² + 2 (fm$_c$)².

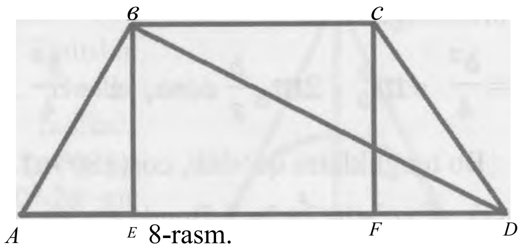
Bundan **^2m$_b$²** + $2m_c^2$ — m^2 topamiz. Xuddi shunga

o‘xshash *b* va *c* tomonlar topiladi.

8-masala. Trapetsiya asoslari 4 m va 16 m. Agar trapetsiyaga ichki va tashqi chizilgan aylanalar mavjudligi ma'lum bo'lsa, ulaming radiuslarini toping (8-rasm).

Yechish: Trapetsiya teng yonli bo'Isa, unga tashqi aylana chizish mumkin:

AB = CD va ABCD trapetsiyada AB+CD= BC + AD shart bajarilsa, unga ichki aylana chizish mumkin. BC = 4m, AD = 16m bo'lganligidan



8-rasm.

AB = **CD** =10, AE = FD = = 6 m.

BE = $\sqrt{AB^2 - AE^2}$ = V100— **36** = 8 m.

BD = $\sqrt{BE^2 + DE^2}$ = V64T100 =**2**ViI m.

Shuningdek, 2r = BE = 8m ekanligidan, ichki chizilgan aylana radiusi r = 4 m.

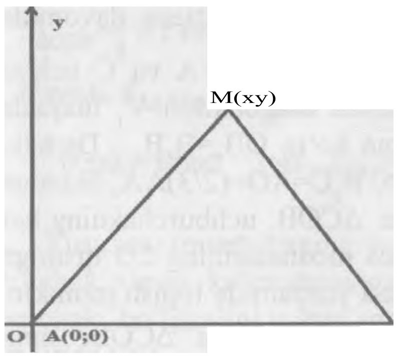
AABD ning yuzini topamiz: S = (1/2) AD BE = (1/2) 16-8=64m^2. Tashqi chizilgan aylana radiusi uchun R = abc/4S formuladan

foydalanamiz. AABD uchun R = 10. 2 = --^41. AABD ga
4- 64 **4**

tashqi chizilgan aylana radiusi trapetsiyaga tashqi chizilgan aylananing ham radiusidir.

9-masala. $|MA^2-MB^2|=kS_{AMAB}$ tenglik bajariladigan barcha M nuqtalar to'plami ikkita to'g'ri chiziqdan iborat ekanligini isbotlang (bunda A va B - berilgan nuqtalar, к >0-o^arтa8,8$_{ДMAB}$ -AMAB uchburchak yuzi).

Yechish. Koordinatalar metodini qo'llaymiz. AB=a bo'lsin. Koordinatalar sistemasini tanlaymiz (9-rasm).



9-rasm

UholdaA(0;0),B(a;0), M(x;y). Shuning uchun

$MA^2=x^2+y^2$, $MB^2=(x-a)^2+y^2$ u holda $MA^2-MB^2=x^2-y^2-y^2-(x-a)^2-y^2=2ax-a^2$

$|MA^2-MB^2|=|2ax-a^2| =a|2x-a|$.

MAB uchburchak balandligi moduli jihatdan M nuqtaning ordinatasiga teng, uchburchak asosi esa *a* ga teng. Shuning uchun $s_{amab}=1/2\,a|y|$- Shartga ko‘ra

$a|2x — a| <$ **-a|y| yoki |2x — a | = - |y|,bundan**

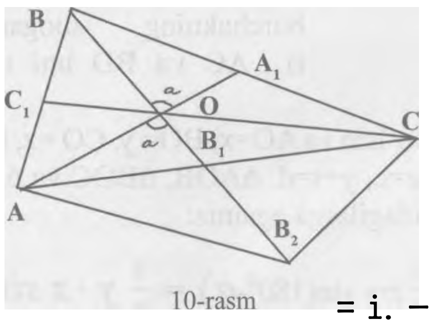
2x - a = -y, 2x - a = - - y yoki y = - (2x - a) va y = -~(2 x - a)

izlangan to‘g‘ri chiziq tenglamalaridir.

10-masala. Uchburchak medianalari 9 sm, 12 sm va 15 sm ga teng. Uning yuzini toping.

Yechish. ABC uchburchak medianalari AA,, BB_p CC_1 bo‘lsin (10-rasm).

Bunda oltita teng yuzli uchburchaklarga ega bo‘ lamiz: $S^\wedge{}_{OB}{}^\wedge = S\&_{BiDC}$,



10-rasm

$s_{AAOB_1} = - A0 - B_10 - sina =$

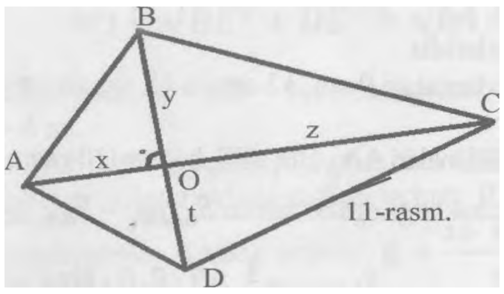
$= -'-'AA^- BB^ sina$ 1 2 1

$BO-A_1O-sina =$

$= i. - \cdot BBi \cdot - ЛA_r \cdot sina.$ 2 3 1 3 1

Natijada $S_{\&BOAi} = S^{\wedge}_{BDAi}$ va hokazo. 66, mediana davomida 06,ga teng 6,6,, kesmani qo‘yamiz va 6, nuqtani A va C uchlar bilan birlashtiramiz. AOC6, to‘rtburchakda diagonallari V, nuqtada teng ikkiga bo‘linadi: A B^BjC, shartga ko‘ra 06=6,6, . Demak, $AOC6_2$ parallelogramm. Shuning uchun, B,C=AO=(2/3)AA_r 8undan tashqari 06,=(2/3)66,, CO=(2/3)CC, va AC06, uchburchakning har bir tomoni AAБC uchburchakning mos medianasining 2/3 qismiga teng. $AC06_2$ ning yuzini Geron formulasi yordamida topish mumkin, so‘ngra 5ддвс =3 $Асов_2$-. Masala shartiga ko‘ra AC06, ning tomonlari 6 sm, 8 sm va lOsm. 6unda $6^2 + 8^2 = 10^2$ tenglik o‘rinli, demak Pifagor teoremasiga teskari teoremaga ko‘ra katetlari 6 sm va 8 sm bo‘lgan to‘g‘ri burchakli uchburchak. Shuning uchun

$$^3д\ сов, \quad \frac{-}{2}\cdot 6 \quad 8 = 24\ (sm^2)\ va\ S^{\wedge}_{BC} = 3 \cdot 24 = 72\ (sm^2)-$$



11-rasm.

11-masala. Har qanday qavariq to‘rtburchakning yuzi uning diagonallari bilan ular orasidagi burchak sinusi ko‘paytmasining yarmiga tengligini isbotlang.

Yechish: A6CD to‘rt-burchakning dioganalla-ri AC va 6D uni to‘rtta uchburchakka ajratadi (11-rasm).

Faraz qilaylik, AC=c, 6D=d bo‘lsin va AO=x, 60= y, CO =z, DO=t belgilashlar|| kiritsak, u holda x+z=c, y+t=d. AAOб, AБOC va ADOA uchburchaklar yuzlari uchun quyidagilarga egamiz:

$$1\ 11$$
$$s_{AAOB} = "x'y\ sinQf'\ s_{ABOC}\ 2\ 'y\ \sin(180°-a) = \sim y'z\ \sin a,$$

s_{acod} 4 z' t sina, $S_{\&D0A}$ x-t sin(180° -a) = J t - x sina.

Natijada $\$\&_{ABCD}$ = $\hat{}_{AAOB}$ S_{ABOC} $\hat{}_{ACOD}\hat{}_{ADOA}$ = j sina(y(x+z)+t(z+x)) =

= j (x+z)(y+z)sina yoki $S\hat{}_{ABCD}$= ~ c-d sina.

Xususan, trapetsiyaning yuzi uning dioganallari va ular orasidagi burchak sinusi ko‘paytmasiga teng va romb dioganallari orasidagi burchak bo‘lganligi uchun romb yuzi uning dioganallari ko‘paytmasi yarmiga teng.

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