

Distribution of Prime Numbers, Twin Primes and the Goldbach Conjecture

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ABSTRACT:

The following paper deals with the distribution of prime numbers, the twin prime numbers and the Goldbach conjecture. Starting from the simple assertion that prime numbers are never even, a rule for the distribution of primes is arrived at. Following the same approach, the twin prime conjecture and the Goldbach conjecture are found to be true.

INTRODUCTION:

For finding a rule for distribution of prime numbers on the number-line, we begin with the simple assertion that prime numbers must be odd and not even. Making use of this intrinsic property of primes, we next deal with the twin prime conjecture, which states that, every even number greater than 2 is the sum of two primes. Further in a similar fashion, we find the truth of the Goldbach conjecture, which states that, there are infinitely many prime numbers whose difference is 2. As a matter of fact, 2 assumes the status of a special prime number, as we will see in this paper.

RULE FOR PRIME NUMBERS:

Prime numbers being always odd may be arbitrarily defined as:

$$\begin{array}{ll} \text{Or,} & P=2n-1 \qquad \dots(1) \\ & P=2n+1 \qquad \dots(2) \end{array}$$

Where, n belongs to natural numbers.

In equation (1) we may place,
 $2n=2n_1+1$;
which makes $n_1 = 0.5$ (read odd number decimal five)

Thus, we see that,

$$\begin{array}{ll} & 2(0.5) = P \\ \text{i.e.,} & 2\theta + 1 = P \qquad \dots (3) \end{array}$$

Using equation (2) in a similar fashion and placing,

$$2n = 2n_2 - 1,$$

we get,

$$n_2 = E.5 \text{ (read even number decimal five)}$$

Thus, we see that,

$$2(E.5) = P$$

i.e. ,

$$2E + 1 = P \quad \dots(4)$$

In arriving at equations (3) and (4) we have proceeded in an arbitrary manner, preferring one solution over all others, viz. preferring $P/2 = \theta.5$ and $P/2 = E.5$ over other possible solutions. This however does not interfere with the results that we have assumed for obvious reasons. Again, in equation (4), we can further say, for all prime numbers concerned,

$$E = 2\theta \quad \text{or} \quad E = 2(10^n);$$

where, θ does not belong to prime numbers except for 3,7 and 13 and n belongs to natural numbers.

(We have preferred one solution over others in all these treatments for the sake of simplicity, convenience and also from experience. The solutions which do not match with experience, we discard. Also even numbers such as 20, must be expressed by $E = 2(10^n)$ viz., $20 = 2(10^1)$, etc.)

Thus we find that we have primes that belong to two groups:

- 1) Odd Primes: defined by, $P = 2\theta + 1$
- 2) Even Primes: defined by, $P = 2E + 1$

[$E = 2\theta$, or $2(10^n)$, where θ does not belong to prime numbers, except for $P=3, 7$ and 13 , and n belongs to natural numbers].

However, these sets of prime numbers do not include 2 and we thereby treat it as a special case and place it separately. The periodic-table of prime numbers may thus be defined as shown in table 1.

<u>Odd Primes</u>	2	<u>Even Primes</u>
$P=2\theta+1$		$P = 2E + 1, E = 2\theta, \text{ or } 2(10^n),$ Where θ is not a prime number except for $P = 3, 7$ and 13 And n belongs to natural numbers

Table 1 : Periodic -table of prime numbers.

As we move towards infinity, the number of primes being infinite, the number of primes of each type must also tend to infinity.

TWIN PRIMES CONJECTURE:

The twin primes conjecture states that, there are infinitely many prime numbers whose difference is 2 (such as 3 and 5 or 10,006,427 and 10,006,429).

If, $\theta_1 + E_1 = P$, where θ_1 and E_1 are consecutive, then θ_1 and E_1 generates twin primes.

Let us define twin primes by,

$$P = P_1 + P_2;$$

where $P_1 \sim P_2 = 2$

Also, let,

$$P_1 = 2\theta_1 + 1$$

And

$$P_2 = 2E_1 + 1;$$

where, $E_1 = 2\theta$ or $2(10^n)$, θ is not a prime number except for $P = 3, 7$ and 13 .

Thus, if

$$P_1 \sim P_2 = 2$$

$$2\theta_1 + 1 \sim 2E_1 + 1 = 2$$

$$2(\theta_1 \sim E_1) = 2$$

Or, $\theta_1 \sim E_1 = 1$

i.e. θ_1 and E_1 must be consecutive as we have already asserted.

Now, as the number of θ_1 and $E_1 = P$ before a given number tends to infinity as we move towards larger and larger numbers, the number of twin primes they generate also tends to infinity i.e., there are infinitely many twin primes.

GOLDBACH CONJECTURE :

The Goldbach conjecture states that, every even number greater than 2 is the sum of two primes. Now, for any even number $E > 4$, we may have any of the following three different cases:

$$1) 2\theta_1 + 1 + 2\theta_2 + 1 = E \quad \dots(5)$$

$$2) 2\theta_1 + 1 + E_1 + 1 = E \quad \dots(6)$$

$$3) 2E_1 + 1 + 2E_2 + 1 = E \quad \dots(7)$$

Where $2\theta_1 + 1$, $2\theta_2 + 1$, $2E_1 + 1$ and $2E_2 + 1 < E$,

$E_1 = 2\theta$ or $2(10^n)$, θ is not a prime number, except for $P = 3, 7$ and 13 , and n belongs to natural numbers.

(4 being the sum of 2 and 2, where 2 is a special case of prime numbers leaves itself out of this rule).

From equations (5), (6) and (7), we get respectively, three cases :-

$$\begin{aligned} 1) E/2 - 1 &= \theta_1 + \theta_2 && \dots(8) \\ 2) E/2 - 1 &= \theta_1 + E_1 && \dots(9) \\ 3) E/2 - 1 &= E_1 + E_2 && \dots(10) \end{aligned}$$

Example for equation (8) :

$$98/2 - 1 = 48,$$

$$48 = 3 + 45 (\theta_1 + \theta_2);$$

Where, $2*3+1 = 7$ and $2*45+1 = 91$ are the required primes.

Example for equation (9) :

$$96/2 - 1 = 47,$$

$$47 = 3 + 44 (\theta_1 + E_1);$$

Where, $2*3+1 = 7$ and $2*44+1 = 89$ are the required primes.

Example for equation (10) :

$$46/2 - 1 = 22,$$

$$22 = 2 + 20 (E_1 + E_2);$$

Where $2*2+1 = 5$ and $2(10^1) + 1 = 41$ are the required primes.

Now as for any given even number $E > 4$, we can satisfy at least one of the equations (8), (9) and (10) and as there are infinitely many such odd numbers (θ_1, θ_2 , etc.) and even numbers (E_1, E_2 , etc.), we can safely verify the truth of the Goldbach conjecture and say that all even numbers may be expressed as the sum of two prime numbers. (Corresponding to $\theta_1, \theta_2, E_1, E_2$, etc.)

CONCLUSION:

In this paper, starting out with the intrinsic property of prime numbers, a rule for the distribution of prime numbers is found. Besides, the twin primes conjecture and the Goldbach conjecture is found to be true. 2 comes out as a special case in the rule of distribution of primes and therefore 4, which is the sum of 2 and 2 leaves itself out of our proof of the Goldbach conjecture.

References:

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