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Application of Fuzzy Linear Programming and Branch and Bound Methods in Optimizing Production Raw Material Supply (Case Study Berkah Coffee Shop)

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Abstract:- Berkah Coffee is a coffee shop that produces various types of drinks, namely red velvet, matcha, chocolate, taro, and caramel chocolate. Berkah Coffee often experiences unstable purchases of uncertain beverage production, which creates obstacles for Berkah Coffee in formulating the amount of each type of drink to be produced. There is a need for production planning so that all available resources can be used optimally using the Fuzzy Linear Programming and Branch methods. and Bound with the help of LINDO Software. Fuzzy Linear Programming aims to find acceptable solutions based on the objective function and constraint function. Based on the results obtained using the Fuzzy Linear Programming method which is still in the form of decimal numbers can be converted into integers, based on the results of the Fuzzy Linear Programming and Branch and Bound methods can increase Berkah Coffee's profits by 9.7% or Rp. 346,000 from the company's profits by the value of λ is 0.6. And for the addition of each raw material by 0.4.

Keywords:- *Linear Programming, Fuzzy Linear Programming, Branch and Bound, LINDO Software.*

I. INTRODUCTION

Linear programming is a general model for optimally allocating limited resources, namely maximizing profits or minimizing costs. However, in practice, these assumptions are difficult to fulfill because a lot of information is not terministic data. To overcome the uncertainty assumption, fuzzy set theory is applied to linear programming, which is called fuzzy linear programming, which is one of the profit optimization methods. The fuzzy logic approach is used for optimal resource allocation and determining the value of production constraint intervals, and the most optimal value is obtained through the interval [4]. Fuzzy linear programming has been widely used to solve production optimization cases. In Sitanggang and Mustika's research, fuzzy linear programming was applied to optimizing the amount of production and profits at K-Bakery by using five variables with a tolerance of 10% [10].

Results that have been resolved using fuzzy linear programming often produce values in the form of fractions. But in the case of production optimization, the expected solution must be an integer. Therefore, a method is needed to produce a round value from the results obtained by fuzzy linear programming. One approach that is often used in solving this problem is the branch and bound method. Research on Branch and Bound was conducted by Supatimah, et al., optimizing profits with the Branch and Bound method at Me Laundry, with the aim of this research being to find optimal profits obtained by the Me Laundry business [11].

The Branch and Bound method are a method for producing optimal solutions to linear programs that produce integer decision variables. As the name implies, this method limits the optimal solution that will produce fractions by making upper and lower branches for each decision variable that has a fractional value so that it has an integer value, so that each restriction will produce a new branch [3].

Production optimization can be done using the simplex linear programming method to determine the optimal number of products and maximize the seller's profit. Solving the problem through the simplex linear programming method requires appropriate data as the objective function and the limit function. The amount of profit earned is defined as the objective function, while the amount of raw materials for each type of beverage product, the amount of material available, the production capacity, and the number of beverage orders are defined as the boundary functions.

Berkah Coffee is a coffee shop located on Jl. Trans Sulawesi No. 393. This shop produces various types of drinks. Berkah Coffee often experiences purchasing volatility due to erratic beverage production. This is a problem for the availability of raw materials for making beverage products. The problem in this research is the problem of linear programming, namely production optimization. However, based on the data obtained, there are fuzzy parameters, namely in the supply of raw materials. Therefore, this study uses the fuzzy linear programming and branch and bound methods.

II. LITERATURE REVIEW

A. *Linear Program*

Linear programming is a method for making decisions among various alternative activities when these activities are limited by certain activities. The decisions to be taken are expressed as objective functions, while the constraints encountered in making these decisions are expressed in the form of constraints. The general form of a linear program can be formulated in a mathematical form as follows [6].

➢ *Max:*

$$Z = \sum_{j=1}^{n} c_j x_j, \quad (1)$$

- *With Constraints:*

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i, \quad (2)$$

$$x_j \geq 0, \quad (3)$$

- With $i = 1, 2, 3, \ldots, m$ Dan $j = 1, 2, 3, \ldots, n$.

B. *Fuzzy Set*

A fuzzy set is a generalized set of strict and accurate sets. Each fuzzy set is characterized by a membership function, so that the membership of each element of this set is determined by the degree of membership in the set [7]. In a crisp set. the membership value of an item x in a set A, which is often written as $\mu_A(x)$, has two possibilities, namely one and zero [5].

➢ *Fuzzy Linear Programming*

A fuzzy linear program is a linear program that is expressed with an objective function and a constraint function that has fuzzy parameters and fuzzy inequalities. In a fuzzy linear program, we will look for something that is a Z value, which is an objective function that will be optimized so that it is subject to the constraints modeled using fuzzy sets (Kusumadewi & Purnomo, 2010). One example of a classic linear programming model [15]

➢ *Max:*

$$f(x) = c^T x, \quad (4)$$

- *With Constraints*:

$$Ax \leq b \quad (5)$$

$$x \geq 0 \quad (6)$$

- With $\boldsymbol{c}, \boldsymbol{x} \in \boldsymbol{R}^n, \boldsymbol{b} \in \boldsymbol{R}^m, \boldsymbol{A} \in \boldsymbol{R}^{mxn}$.

The membership function for the fuzzy set "decision" model can be expressed as:

$$\min\{\mu_i[B_i x]\}. \quad (7)$$

- From here, it is seen that $\mu_i[B_i x] = 0$ if the i-th constraints are actually violated $\mu_i[B_i x] = 1$ if the i-th constraints are strictly adhered to (similar to strict value constraints). Thevalue of $\mu_i[B_i x]$ will monotonically decrease over the interval $[d_i, d_i + p_i]$, i.e.

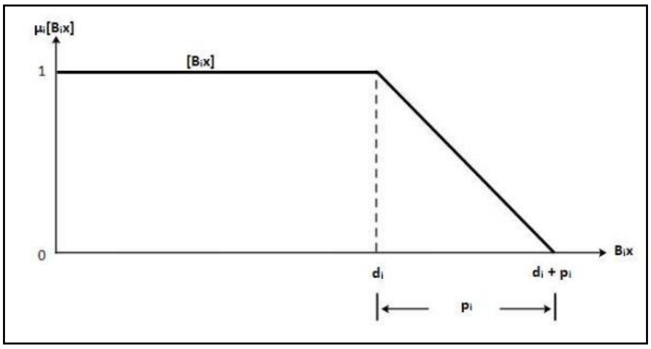

Fig 1 Membership Function

$$\mu_i[B_i x] = \begin{cases} 1; & B_i x \leq d_i \\ 1 - \dfrac{B_i x - d_i}{p_i}; & d_i < B_i x \leq d_i + p_i \\ 0; & B_i x > d_i + p_i \end{cases} \quad (8)$$

- From Figure 1, it can be seen that the greater the value of X in the domain, the smaller the membership value will tend to be. So that at the defuzzification stage to find the $\lambda -$ cut value can be calculated as $\lambda = 1 - t$, Thus a new form of linear programming will be obtained as follows [5]:

Max:	$\lambda,$	(9)
with constraints:	$\lambda p_i + B_i x \leq d_i + p_i,$	(10)
	$i = 1, 2, \ldots, m,$	(11)
	$x \geq 0.$	(12)

➢ *Fuzzification*

Fuzzification process is a process to change non-fuzzy variables (numerical variables) into fuzzy variables (linguistic variables). This fuzzification process is carried out to obtain values from the lower model $(t = 0)$ and the upper model$(t = 1)$ which are formed from the initialization of the decision variables and constraints.

- The lower limit of the optimal value is denoted by Z_L which is obtained from solving the following linear program:

✓ *Max:*

$Z_L = \sum_{j=1}^{n} c_j x_j,$	(13)

✓ *With Constraints*:

$\sum_{j=1}^{n} a_{ij} x_j \leq b_i, i = 1,2,3,\dots,m.$	(14)
$x_j \geq 0, j = 1,2,3,\dots,n.$	(15)

- The upper limit of the optimal value is denoted by Z_U, which is obtained from solving the following linear programming problem:

✓ *Max:*

$Z_U = \sum_{j=1}^{n} c_j x_j,$	(16)

✓ *With Constraints:*

$\sum_{j=1}^{n} a_{ij} x_j \leq b_i + p_l, i = 1,2,3,\dots,m,$	(17)
$x_j \geq 0, \quad j = 1,2,3,\dots,n.$	(18)

✓ *Where p_l is the tolerance given to the th–i constraint ($i = 1,2,3,\dots,m$) [14].*

➢ *Defuzzification*

Decisions resulting from each fuzzification process are in fuzzy form. This result must be converted back into a non-fuzzy numeric variable through a defuzzification process. After doing the calculations to get the values of the lower model ($t = 0$) end the upper model ($t = 1$), new constraints will be formed to determine the fuzzy value [14].

Max:	$\lambda,$	(19)
with constraints:	$-\lambda(Z_U - Z_L) + cx \geq Z_L,$	(20)
	$\lambda p_i + B_i x \leq d_i + p_i; i = 0,1,2,\dots,m,$	(21)
	$x \geq 0, dengan\ \lambda \in [0,1], x \geq 0.$	(22)

C. Integer Linear Programming

Integer linear programming, or integer linear programming, is a form of linear programming with the decision variables being integers, so that in the general form of linear programming there is an additional requirement that the decision variables be integers. Completion with the Integer linear programming method consists of 2 methods, namely the branch and bound method and the Gomory cutting plane method (Cutting Plane). The general form of integer linear programming with the maximizing case is as follows [3]:

Max:	$Z = \sum_{j=1}^{n} c_j x_j.$	(23)

with constraints:	$\sum_{j=1}^{n} a_{ij} x_j (\leq, =, \geq) b_i, \quad i = 1,2,3,\dots,m,$	(24)
	$x_j \geq 0, x_j \in (0,1,2,\dots), \quad j = 1,2,3,\dots,n.$	(25)

➢ *Branch and Bound*

The Branch and Bound method is one of the methods to produce optimal completion of linear programs that produce integer decision variables. The basic principle of this method is to break down the feasible region of a linear programming problem by creating subproblems. There are two basic concepts in the branch and bound method [8]:

- Branching is the process of dividing a problem into subproblems that may lead to a solution.
- Bounding is a process to find or calculate upper and lower bounds for optimal solutions to subproblems that lead to solutions.

➢ *Maximization with the Branch and Bound Method [8]:*

- Solve linear programming problems with the simplex method solve problems without limiting integer numbers.
- Examine the optimal solution; if the expected decision variable is an integer number, the optimal integer solution has been reached. If one or more of the expected decision variables are not integers, proceed to step 3.
- Make the solution at the completion of step 1 the upper limit and the lower limit a solution whose decision variables have been rounded down.
- Choose the variable that has the largest fractional value (meaning the largest decimal number of each variable) to be branched into sub-problems. The goal is to eliminate solutions that do not meet the integer requirements of that problem. The branching is done mutually exclusively to meet the requirements, with the guarantee that no feasible solutions are included. The result is a sub-problem with constraint $\leq$ or constraint $\geq$.
- For each sub-problem, the optimum value of the objective function is set as the upper limit. The optimal solution that is integrated becomes the lower bound (solutions that were not previously integers are then integrated). Sub-problems that have an upper limit lower than the existing lower limit are not included in the next analysis. A feasible (feasible) integer solution is as good or better than the upper limit for each sub-problem being searched for. If such a solution occurs, a sub-problem with the best upper bound is selected for branching. Go back to step 4.

D. Software LINDO

Lindo software is designed to find solutions to linear problems quickly by entering data in the form of formulas in linear form. Lindo software provides many benefits and conveniences in solving optimization and minimization problems. The Lindo Software Model has at least three requirements: objective functions, variables, and constraints (constraint functions) [2].

III. RESULTS

A. Data Processing

➢ *Max*

$$Z = 10000x_1 + 11000x_2 + 10000x_3 + 11000x_4 + 12000x_5$$

➢ *With Constraints:*

$25x_1 + 35x_2 + 40x_3 + 35x_4 + 25x_5 \leq 17760 + 17760t,$	$x_5 \geq 8.$
$10x_1 + 10x_2 + 10x_3 + 10x_4 + 10x_5 \leq 10000 + 500t,$ $100x_1 + 100x_2 + 100x_3 + 100x_4 + 100x_5 \leq 120000 + 12000t$ $x_1 + x_2 + x_3 + x_4 + x_5 \leq 353,$ $8x_3 + 6x_5 \leq 2500 + 500t,$ $15x_1 \leq 1500 + 500t,$ $4x_2 \leq 1000 + 500t,$ $4x_4 \leq 1000 + 500t,$ $8x_5 \leq 750 + 750t,$ $x_1 \geq 41,$ $x_2 \geq 33,$ $x_3 \geq 70,$ $x_4 \geq 50,$ $x_5 \geq 8.$	

To calculate the results of the constraints that have been determined using the fuzzy linear programming method, it is necessary to do calculations with $(t = 0)$ and $(t = 1)$.

Table 1 Fuzzification

Fuzzyfikasi	Fuzzyfikasi Results
$t = 0$	$x_1 = 41, x_2 = 33, x_3 = 70, x_4 = 115{,}3$ and $x_5 = 93{,}8$. With the objective function value $Z_L = 3.865.750$
$t = 1$	$x_1 = 41, x_2 = 33, x_3 = 70, x_4 = 50$ *and* $x_5 = 159$. With the objective function value $Z_U = 3.931.000$.

After doing the calculations for $(t = 0)$ and $(t = 1)$ then the p_0 value can be determined, which is the result of subtracting the Z value at the time $(t = 0)$ and $(t = 1)$ i.e.

$p_0 = Z_U - Z_L$
$p_0 = 3.931.000 - 3.865.750 = 65.250$

Table 2 Finally, a Fuzzy Linear Programming Model can be Formed as Follows:

Max $:\lambda,$
with constraints: $-65250\lambda + 10000x_1 + 11000x_2 + 10000x_3 + 11000x_4 + 12000x_5 \geq 3865750,$
$17760\lambda + 25x_1 + 35x_2 + 40x_3 + 35x_4 + 25x_5 \leq 35520,$
$500\lambda + 10x_1 + 10x_2 + 10x_3 + 10x_4 + 10x_5 \leq 10500,$
$12000\lambda + 100x_1 + 100x_2 + 100x_3 + 100x_4 + 100x_5 \leq 132000,$
$x_1 + x_2 + x_3 + x_4 + x_5 \leq 353,$

$500\lambda + 8x_3 + 6x_5 \leq 3000,$
$500\lambda + 15x_1 \leq 2000,$
$500\lambda + 4x_2 \leq 1500,$
$500\lambda + 4x_4 \leq 1500,$
$750\lambda + 8x_5 \leq 1500,$
$x_1 \geq 41,$
$x_2 \geq 33,$
$x_3 \geq 70,$
$x_4 \geq 50,$
$x_5 \geq 8.$

Using the LINDO software we get $\lambda = 0{,}6,\ x_1 = 41, x_2 = 33,\ x_3 = 70,\ x_4 = 76{,}8$ and $x_5 = 132{,}2$. With the value of the objective function $Z = 3.904.200$. After the results are obtained in fuzzy linear programming, Then use the branch and bound method and get the following results:

Table 3 The Branch and Bound Method and Get the Following Results

Solution Non Integer	**Solution Integer**
$x_1 = 41$	$x_1 = 41$
$x_2 = 33$	$x_2 = 33$
$x_3 = 70$	$x_3 = 70$
$x_4 = 76{,}8$	$x_4 = 78$
$x_5 = 132{,}2$	$x_5 = 131$
$Z = 3.904.200$	$Z = 3.903.000$

The profit earned by the company is IDR 3.557.000 while using fuzzy linear programming and branch and bound, the profit is IDR 3.903.000. In other words, the company's profits increased by 9,7% or IDR 346.000 by producing 41 Red Velvet drinks, 61 Matcha drinks, 70 chocolates, 50 taros, and 131 caramel chocolates.

IV. CONCLUSION

Based on the results of the data processing and analysis that has been carried out, it can be concluded that the application of fuzzy linear programming and branch and bound methods can increase Berkah Coffee's profits by 9,7% or IDR 346.000, with the addition of 0.4 raw materials for each raw material. Then the addition of sweetened condensed milk is 7104 gr, creamer is 200 gr, UHT milk is 4800 gr, chocolate powder is 200 gr, red velvet powder is 200 gr, pure matcha powder is 200 gr, pure taro powder is 200 gr, and caramel syrup is 300

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