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RELATIONS BETWEEN MATHEMATICS AND PHYSICS FROM A WITTGENSTEINIAN PERSPECTIVE

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Alyne Maria Rosa de Araujo Dias[2]

Professor at the Federal Institute of Education, Science and Technology of Pará (IFPA). Doctoral student of the Graduate Program in Science and Mathematics Education at the Federal University of Pará.

Paulo Vilhena da Silva[3]

Professor at the Faculty of Mathematics of the Federal University of Pará (UFPA), professor at the Graduate Program in Education in Science and Mathematics (PPGECM/UFPA) and professor at the Master's Degree in Mathematics in the National Network (PROFMAT/UFPA). He holds a PhD in Mathematics Education (UFPA).

Abstract: The present work is part of an ongoing doctoral research that aims to present a new way of seeing Mathematics in the teaching of Physics from the pragmatic perspective of the Austrian philosopher Ludwig Wittgenstein, who directs his second philosophy to language and its various uses. We consider, preliminarily, that most of the literature that brings Mathematics into the teaching of Physics present ways of seeing linked to traditional philosophical perspectives. However, our reflections, through Wittgenstein's philosophy, turn to Mathematics and Physics as language games that intertwine. Physics, with its empirical propositions, makes sense through the grammatical propositions of Mathematics. In this pragmatic perspective, it is not obvious for the student to perceive the meaning of Mathematics in the teaching of Physics, therefore, it is the Physics teacher's task to give meaning to it in this context, that is, the teacher must establish links between the mathematical language and the language of Physics, connect them. Thus, we bring the see-as concept, present in Wittgenstein's philosophy, as a metaphor that can enable the teacher to see Mathematics and establish connections with Physics, free of dogmatism and referentialism. We can say, based on the reflections, that this work can contribute to future reflections on the teaching of Physics when using mathematical language, as well as with research that deals with Wittgenstein's philosophy of language in Science and Mathematics Education.

Keywords: philosophy of language, Wittgenstein, Mathematics, Physics teaching, see-as

[2]Doctoral student of the Graduate Program in Science and Mathematics Education at the Federal University of Pará; email: alynerosaadias@gmail.com
[3]PhD from the Graduate Program in Science and Mathematics Education at the Federal University of Pará; email: pvilhena@ufpa.br

INTRODUCTION

This article presents reflections on the teaching of Physics with a focus on the use of Mathematics in this context. This proposal is justified by the fact that we perceive, through the speech of Physics teachers as well as in readings that bring the relationship between these two disciplines, that it is almost unanimous reports that the difficulties in learning Physics are due to the lack of understanding of Mathematics and some reference is attributed to it. The literature that deals with the insertion of Mathematics in the teaching of Physics shows that Mathematics plays the role sometimes as a tool, sometimes as a description of empirical facts, sometimes as a prerequisite for understanding Physics or even as a language of Physics. Most of these conceptions are supported by traditional and referential philosophical currents and these serve as a basis for the construction of documents that govern teaching, textbooks and, consequently, are taken to the classroom through the speeches and teaching actions, since the reproduce what they conceive about Mathematics in this context. In this perspective, we do not intend to exclude these ways of conceiving Mathematics, but to present a new way of seeing, free of dogmatism and referentialism. To do so, we rely on the philosophical perspective of the Austrian Ludwig Wittgenstein in his second philosophical moment through the concepts of language games and *seeing-as*. Therefore, we present a theoretical essay that will make it possible to reflect on how the teacher can see Mathematics in the context of teaching Physics. Thus, Mathematics and Physics are configured in this work as language games and *seeing-as* as a teacher's ability to establish Mathematics links in Physics teaching.

A LOOK AT WHAT THE LITERATURES SAY

Historically, in the Middle Ages, when logic had the role of guaranteeing the accuracy of theoretical thinking (VARGAS, 1996), Mathematics was inserted in the context of empirical sciences and this insertion came to be known as *the mathematization of nature.* , which would have the role of shaping the form of natural science. The universality attributed by men to Mathematics led to the belief that the essence of Mathematics would be in describing the facts of the world. It was, with the Analytical Geometry of Descartes (1637) and the differential and integral calculus of Newton and Leibniz that it was possible to analyze the physical phenomena mathematically, including Newton, shows through his work *Mathematical Principles of Natural Philosophy* that "any physical phenomenon observed empirically corresponds exactly to a mathematical model deduced from axioms pre-established as true" (VARGAS, 1996, p. 256).

It is believed, therefore, that the facts should be organized mathematically in order for them to be understood. Through D'Alembert, the process of *mathematization of nature was definitively accepted* : "The peak of the dominant belief since Galileo and Descartes was reached, that the world was a machine governed by mathematical rationality" (VARGAS, 1996, p. 257); and with Lagrange, Laplace and Legendre, Mathematics was consolidated as an instrument for the analysis of natural phenomena, as it is used today, and it was through the analytical mechanics equations of Lagrange and Laplace that Physics was completely mathematized.

Given this, there is no denying the importance of Mathematics in the constitution of physical concepts. However, the way of seeing Mathematics in the context of teaching Physics can be one of the factors that will contribute to the learning of Physics. And some literatures that bring the relationship between Mathematics and Physics teaching attribute to Mathematics the exclusively instrumental, tool role or believe that Mathematics is a prerequisite for the understanding of Physics.

Ataíde and Greca (2013) for example, bring Mathematics as the "skeleton that sustains the body of Physics". They consider that there is a lack of clarity in this relationship and this leads to non-learning in Physics. These authors say that in one of the literatures that they cataloged in their research, students cite that they know how to calculate, but do not know the meaning of Mathematics in Physics and these reports lead them to say that "students seem to believe that Mathematics implies only in specific operations with meaningless symbols, learning in a mechanical way" (ATAÍDE; GRECA, 2013, p. 211).

Another designation given to Mathematics in the teaching of Physics is in the work of Pietrocola (2002), who brings Mathematics as a structure of physical knowledge. Thus, Mathematics would be a mental structure that serves to express Science, as a criterion of scientificity in Physics, while emphasizing that many researchers consider Mathematics a language responsible for the failure in the teaching of Physics (PIETROCOLA, 2002).

> In Physics teaching, the mathematical language is often considered the main responsible for school failure. It is common for teachers to claim that their students do not understand Physics due to the fragility of their mathematical knowledge. For many, a good mathematical base in the years that precede the teaching of Physics is a guarantee of success in learning (PIETROCOLA, 2002, p. 90).

Another way of seeing Mathematics is in the work of Mannrich (2014), in which it is seen as a structuring language in the teaching of Physics. He begins the text by reflecting on the conception that one has of Mathematics in the difficulties encountered in the teaching of Physics:

> This culpability indicates a conception in which Mathematics would have a tool character, an "external" instrument that must be acquired elsewhere or at another time (mathematics classes) to be used by Physics (MANNRICH, 2014, p. 23).

This criticism that Mannrich (2014) makes, when considering Mathematics as a tool, is linked to a referentialist perspective, in which it would be external to language and that it should be learned in Mathematics classes and not in Physics classes, as if it did not compose the language of physics.

None of these conceptions historically constructed and presented in these works mentioned above bring language in the pragmatic perspective as a subsidy, but seek to be anchored in a traditional philosophical conception, such as empiricism, which conceives Mathematics as the descriptor of the facts of the world, disregarding the uses that it can have in each application context.

MATHEMATICS AND PHYSICS: THE INTERTWING OF LANGUAGE GAMES

The perspectives that place in Mathematics the responsibility for learning Physics, not considering the importance of the meaning attributed to it in each context, contribute to conceptual confusions in the context of teaching and learning. Thus, the pragmatic perspective of Wittgenstein's philosophy of language that we bring seeks to dissolve these confusions and suggests that we have a panoramic view of the uses that Mathematics can have in each context of application.

Wittgenstein, in his work *Philosophical Investigations* , through aphorisms, uses metaphors to better elucidate the relationship between games and language, leading us to reflect that just as games do not have an essence, but are related to each other, changing a rule, changes the game, as it works with language, changed the application context, changed the meaning of the word (WITTGENSTEIN, 1999).

In this kinship between game and language, Wittgenstein inserts the concept *of language games* and characterizes it as follows:

> The expression "language game" should emphasize here that speaking a language is part of an activity or a way of life.
> Put before you the multiplicity of language games by these and other examples:
> Giving orders and acting on orders –
> Describing an object by appearance or by measurement –

> Produce an object according to a description (design) –
> Report an event –
> Conjecture about an event
> Propose a hypothesis and prove it –
> Present the results of an experiment through tables and diagrams – (WITTGENSTEIN, 1999, § 23).

This multiplicity of tools that the language has is what leads to the understanding that his second philosophy can collaborate with the understanding of the insertion of Mathematics in the context of Physics and the teaching of Physics, bringing, in this context of research, Mathematics and Physics as a game of language that intertwine. Mathematics as a language game that has a set of rules, whose propositions do not describe facts of the world:

> Rules do not describe, they guide, they constitute meanings, they tell us what we can or cannot say. However, we cannot verify them as true or false, as they are necessary or grammatical propositions (TEIXEIRA Jr; SILVEIRA; SILVA, 2021, p. 4).

Physics, on the other hand, is an empirical science whose language game depends on other languages, including mathematical language. In this context of interweaving between grammatical propositions and empirical propositions, we perceive that there are conceptual confusions regarding the uses of Mathematics in the teaching of Physics, however “the need that they [grammatical propositions] express seems to *determine* the facts [empirical propositions]” (MORENO , 1993, p.92, emphasis added). As a way of clarifying part of these confusions that are constructed in relation to Mathematics, Gottschalk (2014) brings reflections on the uses of Mathematics, such as, for example, considering it a descriptor of the facts of the world (empiricist conception) as well as Mathematics as a constructed idea in the subject's mind (idealist/intuitionist conception), conceptions that seek ultimate foundations outside of language.

> What unites these two conceptions of mathematical activity (the idealist and the empiricist) is the belief that there would be an essential meaning of mathematical beings based on an intuition, or on an empirical action, both considered extralinguistic conditions for their acquisition. In other words, it is as if the mathematical activity referred to something existing *a priori* in the subject or in the external world, and which becomes linguistically expressed, where the mathematical language would have a merely descriptive or communicative function (GOTTSCHALK, 2014, p. 74).

Thus, in Mathematics as a language game, its words acquire meanings in each context and not an essential meaning. Paraphrasing Wittgenstein (1999, p. 138), it is the grammatical rules, in use, anchored to a praxis, that determine the meanings of words. As an application of this multiplicity of uses of the Mathematical language, we can think of Mathematics in the context of teaching itself in which one seeks to understand its rules and follow them; mathematical propositions in the context of teaching Physics, their propositions are conditions of meaning for empirical propositions (Gottschalk, 2006). " The danger lies in taking them for what they are not, intending to do more with them than they can allow" (MORENO, 1993, p.79).

SEE HOW IN THE LEARNING OF PHYSICS: MATHEMATICS *AS A* CONDITION OF MEANING

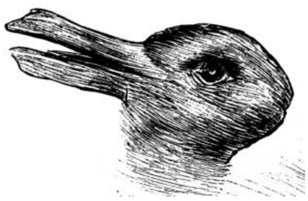
The concept of *seeing* in the traditional philosophical conception is related to a visual issue, seeing through the eyes, the mind, under a cognitive conception; already in the philosophical perspective of the second Wittgenstein, of pragmatism, *seeing* is related to the use of language with conceptual eyes, so that when you *see* something it is because you can interpret, understand the concept.

> "Is this thinking? Is this seeing?" - Doesn't that mean as much as "Is that interpreting? Is that seeing?" And to interpret is to think; and often causes a change in appearance. Can I say that the seeing of the aspect is akin to an interpreting? - My inclination was really to say: "It's like seeing an interpretation". Now, the expression of seeing is related to the expression of interpreting (WITTGENSTEIN, 1999, § 179).

The question of seeing, which was now taken as the essence of what is seen, in search of a reference, of an object that represents it, starts to understand the verb to *see in two different ways* , one expressing the perception of the thing as of the aspect, of the resemblance of objects, so the essence comes to be seen as a family of resemblance (GIANNOTTI, 1995).

It is through metaphors that Wittgenstein brings the concept of *seeing-as* , this being seeing through connections, perceiving the alternations of the meanings of words, perceiving the alternation of the figure. Among the various metaphors used by the Austrian to show the application of the concepts of *seeing* and *seeing-as* , let us turn to the figure of Jastrow's Duck-hare (1901) who, through him, proposes us to reflect on this new way of *seeing* .

Figure 1 : Joseph Jastrow's duck-hare figure (1901).



The application of *seeing* present in the *Philosophical Investigations* in the use of the duck-hare figure seeks to clarify that to immediately *see* the figure as a duck it is necessary to know the concepts that permeate this figure, to be able to describe its characteristics, to compare it with other ducks, etc., and to *see* the duck *as* a hare, there will be an astonishment (A hare!), making connections between the first concepts and the new concept, as well as getting to know the concepts of a hare. “We can only see the duck and rabbit aspects of the duck-rabbit figure if we are familiar with the appearance of these animals” BUDD (1989, p. 82). In *seeing-as* there is a new perception, a change of aspect, it would be to *see it* under another conceptual aspect, making comparisons, showing similarities.

However, noticing aspects requires mastery of techniques, it depends intrinsically on the visual experience of the beholder, because only those who master the technique of use can describe the perception of what was seen, are surprised by what they see, express their thoughts, because the thinking, seeing and interpreting require this experience. According to Hebeche (2002, p. 94), *seeing-as* is “a web of concepts: imagining, representing, thinking and wanting”, so that revelation takes place in this dynamism. Wittgenstein makes a comparison between noticing an aspect of a figure and noticing the aspect of a word, which require mastery of usage techniques (HEBECHE, 2002) that lead to similarities with other words in the language.

The Wittgensteinian *view-as* opens space for us to look at the teaching of Physics through mathematical language, to open our eyes to the ways in which teachers conceive Mathematics in the teaching of Physics, as well as which conceptions direct the practices of professors who train Physics in the use of mathematical language, because according to Wittgenstein (1999, p. 178, part II, section XI): “[...] we interpret it and *see* how we *interpret it* ”.

Budd (1989, p. 79, our translation) presents some types of noting aspects and among them we can apply the “seeing something as if it were destined to be a linguistic sign to see it as if it were going to be another linguistic sign” to the interweaving of the language games of Mathematics and Physics, so that we start to *see* a linguistic signal in Mathematics *as* another linguistic signal in Physics. However, noticing aspects requires skills, mastery of techniques and these must be presented by the teacher so that the student *sees* this *as* that, because the alternation of aspects is not obvious, since Mathematics is a human convention

> [...] seeing-as depends on mastering techniques, which in general are not obvious, but which need to be learned. A rule by itself does not support its applications, it does not tell us when to apply them. In general, new possibilities for applying a rule that we master in a given context are not obvious to us (SILVA; SILVEIRA, 2014, p. 29).

To exemplify the application of *seeing-as* , while mastering mathematical language techniques in Physics teaching, we present the following situation: when teaching polynomial functions of the first degree, we bring a polynomial function of the affine first degree, such as, for example, f (x) = 2 -3.x, which applied to the context of teaching Mathematics, the negative sign (-) allows us to classify it as a decreasing function, since the sign of its angular coefficient allows us to classify it like this, by mathematical convention; already in the context of teaching Physics, this same example can describe the movement of a body under observation, whose displacement can be described as s (t) = 2 - 3t, in this context the negative sign (-) agreed in Mathematics has another meaning in the context of Physics, it means that the body is traveling in the opposite direction to the direction stipulated as positive for movement.

Another application of the use of mathematical language in physics teaching can be seen when teaching derivatives, for example, $\frac{dy}{dx} = k$, where k is a constant. This concept, in the context of teaching Mathematics has a normative character, and from it some rules are possible, establishing a relationship between y and x variables of a function and in the context of teaching Physics it can be taught by the teacher so that the student *sees* the derivative *as* the rate of change between two physical quantities, the latter accompanied by a unit of measurement. An example of this application can be the rate of change of velocity with respect to time resulting in the acceleration of a body, $\frac{dv}{dt} = a$, being $a$the constant acceleration of a moving body.

An application that can also be shown in the context of Physics is the movement of a body falling vertically and the forces acting on that body, in which the teacher can connect mathematical knowledge in the following way: *see* an equation *as* equality of the forces acting on the body. About this example, if applied in higher education, we would say that the forces acting on the body will be the weight force that acts in the direction of the acceleration of gravity and the air resistance force that acts in the opposite direction to gravitation. In high school, the force of air resistance is negligible. Thus, the mathematical relationship that would give meaning to this phenomenon would be written by a mathematical equality of the type $F_r = P - F_{ar}$, and in it the teacher would show Mathematics establishing links between these languages until the student perceives the change of aspect, the meaning in the language game of Mathematics and in the language game of Physics. In this way, the resultant force equation would be shown as $m.a = m.g - \mu.v$and from it, see $a$how $\frac{dv}{dt}$, thus $m.\frac{dv}{dt} = m.g - \mu.v$. In this example, we realize that the same object can be seen from different aspects and these depend on the context in which they are found, they are games of different languages, but that intertwine.

FINAL CONSIDERATIONS

Our look at Mathematics in the teaching of Physics, in this philosophical perspective based on the concept of *seeing how* , leads us to discuss and try to understand what conception the professor who trains Physics has of Mathematics in the teaching of Physics and in which conception his teaching activity. From our point of view, it is up to the teacher to note the aspects and show the students that each context of use presents different aspects. In this way, we consider that Physics teachers who adopt a pragmatic perspective of language, through *seeing-as*, can contribute to their teaching not being dogmatic, while providing students with a panoramic view of the different uses of language.

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