SPACE ASTROMETRY

Relative mean errors of the five astrometric parameters for different scanning modes (Option A). By L. Lindegren 76-11-02

1. Formulae

When considering not only inclined scanning but also more complicated modes of scanning the sky (e.g. revolving scanning), it is no longer convenient to use the mean error at each epoch (σ_1 in Høg 76-03-12, p. 10) as a basis for error estimates, but the mean error per orbit (ϵ) must be used. With about 16 orbits per epoch for inclined scanning, the mean error σ_1 = 0."002 will correspond to ϵ = 0."008. The mean error of the astrometric parameter r_i is then

$$\epsilon_i = \sqrt{V_i}\ \epsilon. \tag{1}$$

Here, the variance V_i is the i:th diagonal element of the inverse of the 5x5 normal equation matrix $\overline{N}$ with the elements

$$N_{ij} = \sum_{k=1}^{n} p_i p_j , \tag{2}$$

obtained by summation over the n orbits during which the star was observed. p_i and p_j are elements of the vector $\overline{P}_k$ as given by (16) in Lindegren 76-10-19, i.e.

$$\begin{aligned} p_1 &= -\sin\eta = -\sin\beta_R/\cos\beta \\ p_2 &= -\cos\eta = -\cos\beta_R \sin(\lambda - \lambda_R) \\ p_3 &= -\tau\sin\eta \\ p_4 &= -\tau\cos\eta \\ p_5 &= R(\sin(\lambda-\lambda_\odot)\sin\eta + \sin\beta\cos(\lambda-\lambda_\odot)\cos\eta). \end{aligned} \tag{3}$$

The most general way to find the times τ of the observations is to go through with a computer the entire 3-year period of observation in small steps and test for each step if observation is possible. I have used the following procedure:

(a) With the assumption that $\lambda_\odot$ increases linearily with the time I have put

$$\tau = (\lambda_0 - 540^\circ)/360^\circ; \tag{4}$$

hence τ is measured in years and centred on the mid-point of the 3-year period $0 \leq \lambda_0 \leq 1080^\circ$. I also put $R = 1$.

(b) (λ_R, β_R) is given as a function of $\lambda_\odot$ or τ. For instance, inclined scanning with one year meridian scanning is described by

$$\begin{aligned} \lambda_R &= \lambda_\odot \\ \beta_R &= 0^\circ \text{ for } |\tau| < 0.5 \\ \beta_R &= 30^\circ \text{ for } |\tau| > 0.5. \end{aligned} \tag{5}$$

Revolving scanning is described by

$$\begin{aligned} \sin \beta_R &= \sin \zeta \, \sin(\varphi_o + K\lambda_\odot) \\ \sin(\lambda_R - \lambda_\odot) &= - \sin \zeta \, \cos(\varphi_o + K\lambda_\odot)/\cos \beta_R \end{aligned} \tag{6}$$

with three parameters: ζ = the constant angle between (λ_R, β_R) and the Sun, K = the speed of the revolving motion relative to the motion of the Sun, φ_o = zero-point of the revolving motion. For the example mentioned in the MDS, $\zeta = 60^\circ$, $K = 6$ (period 2 months), and $\varphi_o = 0^\circ$ (inferred from Fig. 4).

(c) If the full width of the field of view (in the z direction) is w (w = 50' has been used below), the condition for a star at (λ, β) being less than w/2 from a great circle with pole (λ_R, β_R) is

$$\begin{aligned} u(\tau) &= \sin \beta \, \sin \beta_R + \cos \beta \, \cos \beta_R \, \cos(\lambda - \lambda_R); \\ |u(\tau)| &< \sin(w/2). \end{aligned} \tag{7}$$

For each orbit (95^m period) for which this condition is met, the coefficients p_i in (3) and their contributions to the matrix $\overline{N}$ are computed. In practice, $u(\tau)$ is computed for steps in τ which are several times the orbital period T. If $u(\tau)$ and $u(\tau + \Delta\tau)$ have opposite signs, an iterpolated value of τ is used in (3) and the contributions $p_i p_j$ in (2) are factored by the number of orbits during which this star may be observed around epoch τ, viz.

$$n_\tau = \text{entier}\left\{ \frac{w.\Delta\tau}{|u(\tau + \Delta\tau) - u(\tau)|.T} + 0.5 \right\}. \tag{8}$$

2. Results

The Tables below give the mean errors ϵ_i based on ϵ = 0".008, to allow direct comparison with Høg 76-03-12. They also give n, the total number of observations (one per orbit), and m, the number of epochs over which the observations are distributed.

Table 1. Inclined scanning. Mean errors in milliarcsec.

β	$\Delta\lambda\cos\beta$	$\Delta\beta$	$\mu_\lambda\cos\beta$	μ_β	π	n	m
A. With $\beta_R = 30^\circ$ except for $\lvert\tau\rvert < 0.5$, when $\beta_R = 0$							
0°	2.1	1.6	2.1	1.2	3.4	86	6
30	1.7	1.2	1.6	1.1	2.9	104	6
50	0.92	0.91	0.91	1.1	2.1	182	6
58	0.50	1.1	0.50	1.4	2.2	418	6
B. With $\beta_R = 30^\circ$ except for $\lvert\tau\rvert < 0.5$, when $\beta_R = -30^\circ$							
0°	1.7	1.0	2.1	1.1	1.9	90	6
30	1.3	0.94	1.6	1.1	1.5	108	6
50	0.74	0.95	0.91	1.1	1.1	192	6
58	0.41	1.2	0.50	1.4	1.1	438	6

Table 2. Mean errors in milliarcsec for revolving scanning, $\varphi_0 = 0$ and $K = 6$.

λ	β	$\Delta\lambda \cos\beta$	$\Delta\beta$	$\mu_\lambda \cos\beta$	μ_β	π	n	m
A. With $\zeta = 60^\circ$								
90°	0°	1.2	0.68	1.32	0.79	1.4	191	30
90	30	0.74	1.0	0.86	1.2	1.3	176	36
90	60	1.1	1.2	1.2	1.4	1.1	99	36
90	80	1.9	1.3	1.5	1.5	1.8	82	35
B. With $\zeta = 45^\circ$								
90°	0°	1.6	1.3	1.9	1.4	1.8	66	18
90	30	1.2	1.0	1.4	1.2	1.6	126	24
90	60	0.95	1.0	1.1	1.2	1.2	130	36
90	80	1.2	1.1	1.2	1.3	1.2	108	36
C. With $\zeta = 30^\circ$								
90°	0°	2.1	0.70	2.4	0.74	2.4	178	18
90	30	2.3	0.74	2.6	0.83	2.4	178	18
90	60	0.55	0.81	0.64	0.94	1.6	313	36
90	80	0.97	0.91	1.1	1.1	1.4	153	36

Comments:

1. For inclined scanning, the solution will be considerably much stronger (especially for the parallax) if β_R alternates between $+30^\circ$ and -30° instead of between $+30^\circ$ and 0°. However, with no meridian scanning ($\beta_R = 0^\circ$), the polar regions will never be observed at all.

2. For revolving scanning, the strength of the solution generally increases with ζ, as was expected, but even with $\zeta = 30^\circ$ the solution is approximately as good as for inclined+meridian scanning (even somewhat better for the parallax).

3. The strength of the solution with revolving scanning varies greatly and in a complex way with the position (λ, β) of the star. In Table 2A – C, we had $\lambda = 90^\circ$ (arbitrary choice), but

the mean errors vary periodically with λ, as shown in Fig. 1 for $\zeta = 30^{\circ}$, $\varphi_{o} = 0$, $K = 6$, and $\beta = 30^{\circ}$.

4. Although it generally does not give much better accuracy than inclined scanning, revolving scanning still has two important advantages: first, it covers the entire sphere with reasonably homogeneous accuracy (for example with $\zeta = 45^{\circ}$). Second, the observations are distributed over many more epochs, making the solution much less sensitive to interruptions which will almost certainly occur several times during the three years of observation. The second point is by far the most important; in the process of optimising the scanning mode it will probably be necessary to sacrifice some accuracy in order to diminish the risk that the whole project comes out with a huge amount of data of very little astrometric value (for instance, if the satellite stops to work after two years of successful inclined scanning).

Fig. 2 shows the accuracies of the five astrometric parameters for revolving scanning with $\zeta = 45^{\circ}$, as functions of λ. The relative variations of the mean errors are smaller than for $\zeta = 30^{\circ}$ and only reflect the varying number of observations (see the top curve for n versus λ). This number varies abruptly; this is easily explained if one looks at a plot of $u(\tau)$ (Fig. 3).

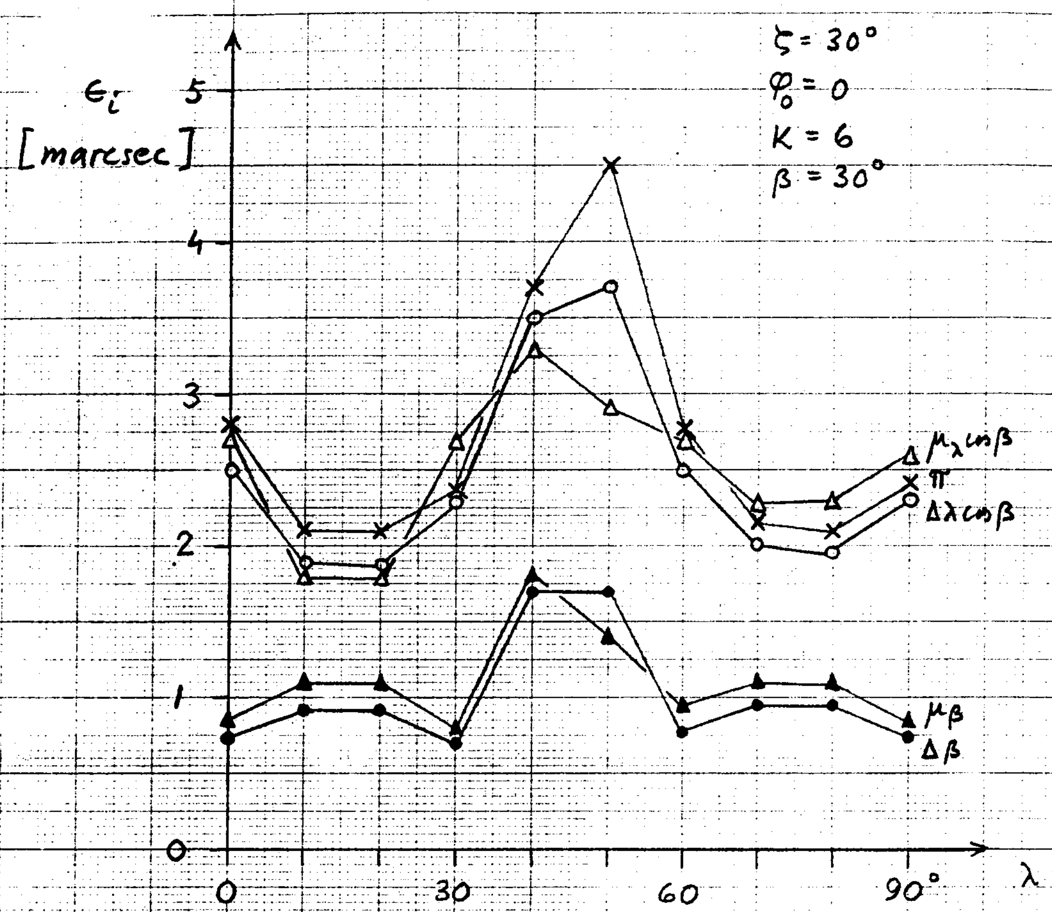

Fig. 1. Mean errors for revolving scanning with $\zeta = 30°$

Fig. 2
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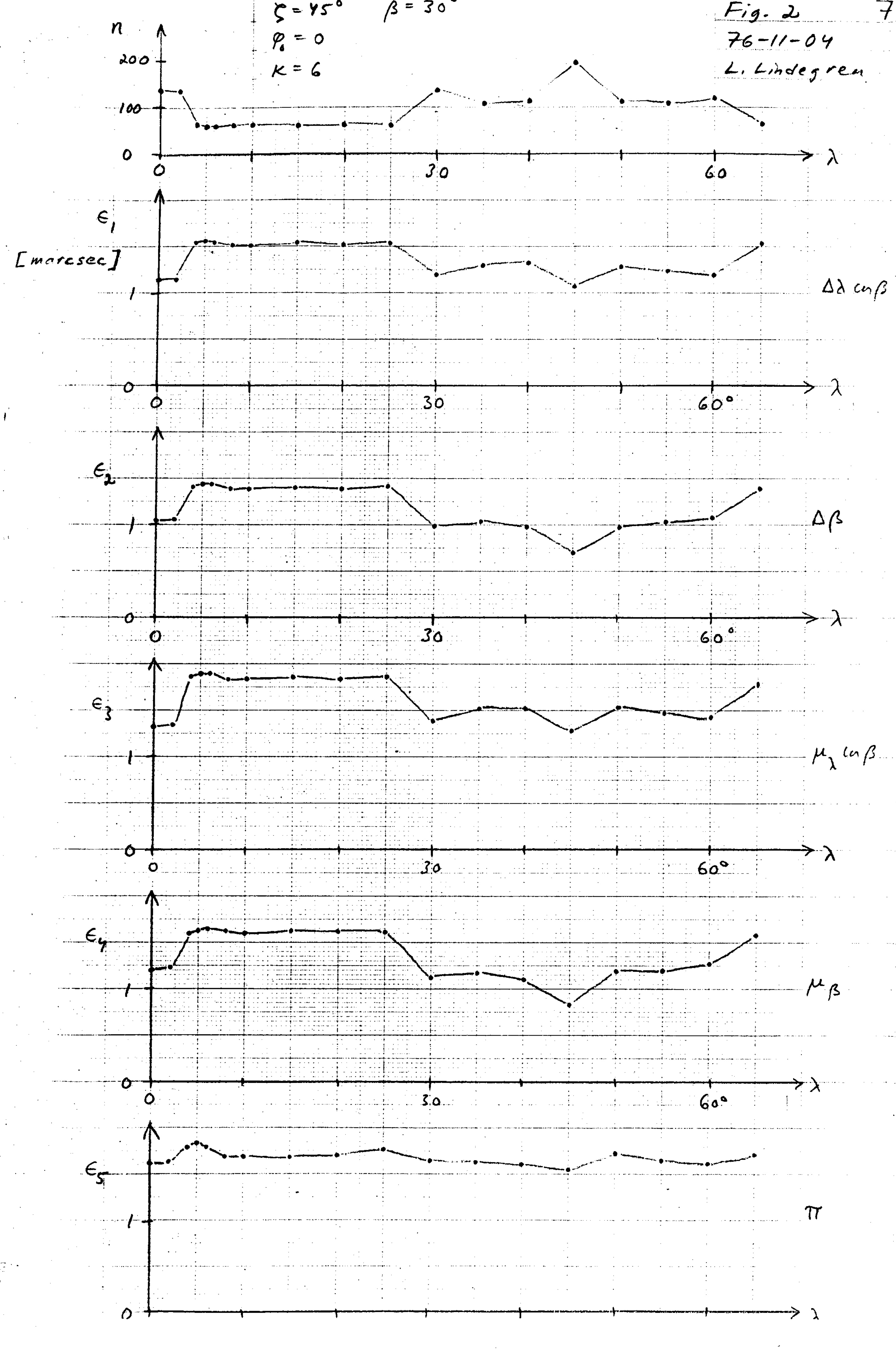

Fig. 3
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$u(\tau)$ versus $\lambda_\odot$ for $\zeta = 45°$, $\varphi_0 = 0$, $\kappa = 6$, $\beta = 30°$