The *s*-Parameter on the Transform Integrals is a Constant

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ABSTRACT

The proposition of this paper is simple: the *s* parameter on any transform integral is a constant and not an independent variable. Every transform integral is a different independent sytem that has its own set of parameters. The integral is evaluated through all values of the independent variable such that it is constant.

Introduction

The transform integrals such as the Laplace transform, the Mellin transform, and the Fourier transform with real part of *s* equal to zero $\Re(s)=0,$ use *s* as one of the parameters in their integral equations. The logic is that *s* a variable that is independent of the independentt variable(e.g., *t*).

But because the transform integral is a single integral, it only need a single independent variable as one of its parameters; a double integral requires two independent variables, a triple integral needs three, and so on.

The integral is taken for all values of *t,* which means that the resulting integral wil be a constant quantity. Therefore, the only reason that *s* is independent of *t* is that it is a constant. If *s* is a variable, then it should also depend on *t.*

Because of this constant/variable dual role of *s* (or any parameter of same), some hypotheses or proofs are constructed through the use of tricks. Riemann in his 1859 paper [1] used several tricks to arrived at his desired outcomes: the faulty analytic continuation of $\zeta(s),$ using the Poisson summation to prove his invalid functional equation(which it does not), and the tricks he used to obtained his formula for the approximate number of primes less than a given quantity.

The Laplace Transform

The Laplace transform of *f*(*t*) is given by

(1) $$F(s) = \int_0^\infty f(t) e^{-st} dt,$$

where *t* is the independent variable, *s* is a complex constant, and *z* = *st* as the complex variable. The function *f*(*t*) and $\Re(s)$ are such that the integral in (1) is finite, *F*(*s*)<∞. Since the integral is obtained for all *t*,

$$F(s) = \text{constant}.$$

The derivative of *F*(*s*) with respect to *s* is meaningless and all the integral associated with *F*(*s*) will be zero

$$\int_s^s F(s) ds = 0,$$

and so

$$\frac{1}{2\pi i} \int_s^s F(s) e^{st} ds = 0.$$

For example, the Laplace transforms for

$$f(t) = \mu(t),\ e^{at}\mu(t),\ e^{i\omega_o t}\mu(t),\ \text{and}\ \cos(\omega_o t)\mu(t),$$

are

$$F(s) = \frac{1}{s},\ \frac{1}{s-a},\ \frac{1}{s-i\omega_o},\ \text{and}\ \frac{s}{s^2+\omega_o^2},$$

which are all constant, such that

$$\int_s^s F(s) ds = \int_s^s \frac{ds}{s} = \int_s^s \frac{ds}{s-a} = \int_s^s \frac{ds}{s-i\omega_o} = \int_s^s \frac{sds}{s^2+\omega_o^2} = 0.$$

Because *F*(*s*) is constant, all you can do on *F*(*s*) is to do a what-if analysis: what is *F*(*s*) if *s* = 2, or 4, or 5, etc. How can analytic continuation be applied to *F*(*s*) if it is constant? Or why apply it at all?

A complex function *F*(*z*) with a complex variable *z* which is similar in form to *F*(*s*) constitute an entirely different independent system. The function *F*(*z*) is a variable and is valid for every value of *z* in which *F*(*z*) ≠ ∞,

(2) $$F(z) = \frac{1}{z}\quad z\neq 0;\ \frac{1}{z-a}\quad z\neq a;\ \frac{1}{z-i\omega_o}\quad z\neq i\omega_o;\ \frac{z}{z^2+\omega_o^2}\quad z\neq \pm i\omega_o.$$

Consider another integral sytem, the contour integral

(3) $$f(\tau)=\frac{1}{2\pi i}\int_C F(z)e^{\tau z}dz$$

where *z* is a complex variable, *τ* is a real constant, and *f*(*τ*) is a constant. We can also consider (3) as the transform integral of *F*(*z*). The contour integrals on simple closed paths for (2) are

$$f(\tau)=1,\ e^{a\tau},\ e^{i\omega_o\tau},\ \cos(\omega_o\tau),$$

which are all constant. A real function *f*(*t*) with a real variable *t* which is similar in form to *f*(*τ*) is an entirely different independent system.

The Mellin Transform

The Mellin transform of *f*(*t*) is given by

(4) $$M(s)=\int_0^\infty t^{s-1}f(t)dt$$

where *t* is the independent variable and *s* is a complex constant. The function *f*(*t*) and $\Re(s)$ are such that the integral in (4) is finite, *M*(*s*)<∞. Since the integral is obtained for all *t*,

$$M(s)=\text{constant}.$$

The derivative of *M*(*s*) with respect to *s* is meaningless and all integral associated with *M*(*s*) will be zero

$$\int_s^s M(s)ds=0,$$

and so

$$\frac{1}{2\pi i}\int_s^s M(s)t^{-s}ds=0.$$

Consider the contour integral

$$f(\tau)=\frac{1}{2\pi i}\int_C M(z)\tau^{-z}dz \qquad \tau\neq 0,$$

where *z* is a complex variable, *τ* is a real constant, and *f*(*τ*) is the transform integral of *M*(*z*). A real function *f*(*t*) with real variable *t* that is similar in form to *f*(*τ*) is an entirely different independent system.

The following are constant quantities:

$$\zeta(s)=\sum_{n=1}^{\infty} n^{-s}, \qquad \Gamma(s)=\int_0^{\infty} e^{-t}t^{s-1}dt, \qquad \Gamma(s)\zeta(s)=\int_0^{\infty}\frac{t^{s-1}}{e^t-1}dt, \qquad \pi^{-\frac{s}{2}}\Gamma\left(\frac{s}{2}\right)\zeta(s), \text{ etc.,}$$

while these are complex functions

$$\zeta(z)=\sum_{n=1}^{\infty} n^{-z}, \qquad \Gamma(z)=\lim_{n\to\infty}\frac{n^z n!}{z(z+1)(z+2)(z+3)\cdots(z+n)}, \qquad \Gamma(z)\zeta(z), \qquad \pi^{-\frac{z}{2}}\Gamma\left(\frac{z}{2}\right)\zeta(z), \text{ etc.}$$

Riemann's Bag of Tricks

Consider now the contour integral

(5) $$\int_C \frac{(-z)^{s-1}}{e^z-1}dz = \int_{z_1}^{z_2}\frac{(-z)^{s-1}}{e^z-1}dz$$

where z_1 and z_2 are the endpoints of the path *C*. Since $\dfrac{1}{e^z-1} = \sum_{n=1}^{\infty} e^{-nz}$ and *s* is constant, we can express (5) as

$$(-1)^{s-1}\zeta(s)\int_{z_1}^{z_2} z^{s-1}e^{-z}dz,$$

and solve for the contour integral $\int_{z_1}^{z_2} z^{s-1}e^{-z}dz$. Since $e^{-z}=1-\frac{z}{1!}+\frac{z}{2!}-\frac{z}{3!}+\cdots,$ we have

$$\int_{z_1}^{z_2} z^{s-1}e^{-z}dz=\int_{z_1}^{z_2}\left(z^{s-1}-z^s+\frac{z^{s+1}}{2!}-\frac{z^{s+2}}{3!}+\cdots\right)dz,$$

$$=\left(\frac{z^s}{s}-\frac{z^{s+1}}{s+1}+\frac{z^{s+2}}{(s+2)2!}-\frac{z^{s+3}}{(s+3)3!}+\cdots\right)\Bigg|_{z_1}^{z_2},$$

and so

(6) $$(-1)^{s-1}\zeta(s)\int_{z_1}^{z_2} z^{s-1}e^{-z}dz=(-1)^{s-1}\zeta(s)\left(\frac{z^s}{s}-\frac{z^{s+1}}{s+1}+\frac{z^{s+2}}{(s+2)2!}-\frac{z^{s+3}}{(s+3)3!}+\cdots\right)\Bigg|_{z_1}^{z_2}.$$

The trick that Riemann used in his 1859 paper was to employ two different values for *s* in getting two contour integrals of (5) and then equating them. But it is clear from (6) that the contour integral is dependent on: the value of *s*, the endpoints of path *C*, and on the chosen path itself.

The contour integral of (5) below

$$I_1(s) = \int_{+\infty}^{+\infty} \frac{(-z)^{s-1}}{e^z - 1}\, dz \qquad \Re(s) > 1,$$

is

(7) $$I_1(s) = -2i\sin(\pi s)\,\zeta(s)\Gamma(s),$$

which is valid [2]. Obtaining the second contour integral of (5) using the Residue theorem is undefined but Riemann got around this by *excluding* the pole at $z = 0$,

$$I_2(s) = -2\pi i\left\{(-0)^{s-1} + \sin\left(\frac{\pi s}{2}\right) 2^s \pi^{s-1} \zeta(1-s)\right\} = \text{undefined} \qquad \Re(s) < 0.$$

The equation

$$I_2(s) = -2\pi i \sin\left(\frac{\pi s}{2}\right) 2^s \pi^{s-1} \zeta(1-s)$$

is, therefore, not a valid contour integral of (5). Because $I_1(s) \neq I_2(s)$, we have an *invalid* equation

(8) $$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\,\zeta(1-s).$$

But Riemann, instead of recognizing that the equation he obtained was faulty invoked a magic spell: ANALYTIC CONTINUATION. By invoking analytic continuation, Riemann made an invalid equation (8) to become valid.

It is now widely accepted by post-modernist mathematicians that

(9) $$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \qquad \Re(s) > 1,$$

and

(10) $$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\,\zeta(1-s) \qquad \Re(s) < 0,$$

Now (9) is valid while (10) is not. For example, if you substitute $s = 2$ in (9),

$$\zeta(2) = \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6},$$

while for (10)

$$\zeta(2) = 2^2 \pi \sin(\pi) \Gamma(-1)\,\zeta(-1) = 4\pi(0)(\infty)(\infty) = \text{undefined}.$$

If s = -2 in (9)

$$\zeta(-2) = \sum_{n=1}^{\infty} n^2 = \infty,$$

and s = -2 in (10)

$$\zeta(-2) \ = \ 2^{-2}\pi^{-3}\sin(-\pi)\Gamma(3)\zeta(3) = \frac{1}{4\pi^3}(0)(2)(1.202) = 0.$$

Notice that the same value of s has a different value of $\zeta(s)$ which is absurd.

Another trick that Riemann used on the same paper: using the Poisson summation formula to "prove" the *invalid equation*

(11) $$\pi^{-\frac{s}{2}}\Gamma\left(\frac{s}{2}\right)\zeta(s) \ = \ \pi^{-\left(\frac{1-s}{2}\right)}\Gamma\left(\frac{1-s}{2}\right)\zeta(1-s),$$

which was obtained from (8). I've shown in [3] that the Poisson summation formula invalidates Riemann's claim. Riemann used analytic continuation as "proof" to an invalid equation.

In the Poisson summation formula shown below

(12) $$\sum_{n=-\infty}^{\infty} e^{-\pi x(t+nT)^2} \ = \ \frac{1}{T}\sum_{n=-\infty}^{\infty}\frac{1}{\sqrt{x}}\, e^{-\frac{\pi}{x}(n/T)^2}\, e^{\frac{2\pi int}{T}},$$

t is the only independent variable while x (>0) is constant. The role of x is to make the summation on both sides of (12) to converge either slowly or rapidly. Thus, if we want to compute the sum on both sides of (12) at t = 0, T = 1, and with an appropriate value of x, we have

(13) $$\sum_{n=-\infty}^{\infty} e^{-\pi n^2 x} = \frac{1}{\sqrt{x}}\sum_{n=-\infty}^{\infty} e^{-\pi\frac{n^2}{x}}$$

and since x is constant, the integrals on both sides of (13) will be zero, that is

$$\int_x^x\left\{\sum_{n=-\infty}^{\infty} e^{-\pi n^2 x}\right\}dx = \int_x^x\left\{\frac{1}{\sqrt{x}}\sum_{n=-\infty}^{\infty} e^{-\pi\frac{n^2}{x}}\right\}dx$$

$$0 = 0$$

Hence the Poisson summation formula can neither prove nor disprove (11), and in the equation below

$$\pi^{-\frac{s}{2}}\Gamma\left(\frac{s}{2}\right)\zeta(s) = \int_0^{\infty} x^{\frac{s}{2}-1}\left\{\sum_{n=1}^{\infty} e^{-\pi n^2 x}\right\}dx,$$

x is a variable. The Poisson summation formula and the equation above are, therefore, two different independent systems.

The rest of Riemann's tricks were done on obtaining his formula for estimating the number of primes less than a given quantity.   I just want to point out that

$$f(t)=\frac{1}{2\pi i}\int_{a-\infty i}^{a+\infty i}\frac{\log\zeta(s)}{s}t^{s}ds$$

is *not valid* since *s* is constant and the integral with respect to *s* is zero, that is

$$\frac{1}{2\pi i}\int_{s}^{s}\frac{\log\zeta(s)}{s}t^{s}ds=0.$$

Even if we use the zeta function $\zeta(z)$ with complex variable *z* and real constant *τ* and get the contour integral of

$$f(\tau)=\frac{1}{2\pi i}\oint\frac{\log\zeta(z)}{z}\tau^{z}dz \qquad \tau\neq 0,$$

over a simple closed path using the Residue theorem, the contour integral is undefined

$$f(\tau)=\frac{2\pi i}{2\pi i}\left[\log\zeta(0)\right]\tau^{0}=\log\left(1+1+1+1+\cdots\right)=\log(\infty)=\text{undefined}.$$

Also

$$f(\tau)=-\frac{1}{2\pi i}\frac{1}{\log\tau}\oint\frac{d\frac{\log\zeta(z)}{z}}{dz}\tau^{z}dz$$

$$f(\tau) = -\frac{2\pi i}{2\pi i\,\log\tau}\left(\frac{\zeta'(0)}{\zeta(0)}\tau^{0} - \left\{\tau^{0}\frac{\zeta'(0)}{\zeta(0)} + \log\zeta(0)\,\tau^{0}\log\tau\right\}\right) = \text{undefined},$$

since

$$\frac{d\frac{\log\zeta(z)}{z}}{dz} = \frac{\zeta'(z)}{z\,\zeta(z)} - \frac{\log\zeta(z)}{z^{2}},$$

$$\zeta'(z)=-\sum_{n=1}^{\infty}\frac{\log n}{n^{z}},$$

and

$$\zeta'(0)=-\left(\log 2+\log 3+\log 4+\cdots\right)=-\lim_{n\to\infty}\log n!=-\infty=\text{undefined}.$$

Thus, his formula for obtaining the number of primes less than *x*, that is

$$f(x) = Li(x) - \sum^{\alpha}\left\{Li\left(x^{\frac{1}{2}+\alpha i}\right) + Li\left(x^{\frac{1}{2}-\alpha i}\right)\right\} + \int_x^{\infty}\frac{1}{x^2-1}\frac{dx}{x\log x} + \log\xi(0)$$

has got nothing to do with the trasform integral of $\frac{\log\zeta(z)}{z}$ because it is undefined. Riemann only obtained *f*(*x*) by using another constant parameter(e.g., *β*) as a variable.

The Fourier Transform

The Fourier transform of *f*(*t*) is given by

(14) $$F(\nu) = \int_{-\infty}^{\infty} f(t)e^{-2\pi i\nu t}dt$$

where *t* is the independent variable and the fundamental frequency *ν* is constant. The function *f*(*t*) is such that the integral in (14) is finite, $F(\nu)<\infty$. Since the integral is obtained for all *t* and *ν* is constant

$$F(\nu) = \text{constant},$$

and the formula for the inverse Fourier transform is zero, that is

(15) $$\int_{\nu}^{\nu} F(\nu)e^{2\pi i\nu t}d\nu = 0.$$

The derivative of *F*(*ν*) with respect to *ν* is meaningless and all the integral associated with *F*(*ν*) will be zero

$$\int_{\nu}^{\nu} F(\nu)d\nu = 0.$$

Aperiodic Functions Don't Have Fourier Series Representations

According to the prevailing interpretation, the Fourier transform can be used to obtain the continuous spectra of aperiodic functions. Aperiodic functions don't have any *frequency content* since they *don't* have Fourier series representations, although there are instances where they can produced periodic waves as in plucking a guitar string or throwing a rock on a pond.

Let *f*(*t*) be any aperiodic function with a period *T* approaching ∞, and whose integral for all *t* is finite,

$$\int_{-\infty}^{+\infty} f(t)dt = k < \infty.$$

All its Fourier coefficients $\{c_n\}$ will be zero

$$c_n = \lim_{T\to\infty} \frac{1}{T}\int_{-\infty}^{+\infty} f(t) e^{\frac{-2\pi i n t}{T}} dt = \lim_{T\to\infty} \frac{1}{T}\int_{-\infty}^{+\infty} f(t)\, dt = \lim_{T\to\infty} \frac{k}{T} = 0,$$

and its fundamental frequency v_o will be zero

$$v_o = \lim_{T\to\infty} \frac{1}{T} = 0,$$

and all its harmonics are zero, $nv_o = 0$.

The periodic function $e^{-2\pi i v t}$ in (14) has only *one* frequency v (*i.e.*, its fundamental frequency). If you say that v is a variable independent of t, then that is clearly false since the angular speed ω is the derivative of the angular displacement θ with respect to time t, that is

$$\omega = \frac{d\theta}{dt}.$$

If ω is constant, then we have a periodic function with fundamental frequency v and the angular displacement $\theta = \omega t = 2\pi v t$ is used for the periodic function $e^{-2\pi i v t}$. While if ω is a variable, then the angular displacement $\theta = ½\alpha t^2$ must be used with zero initial θ and zero initial ω. The angular acceleration α is now the constant parameter and the new transform integral is now

$$F(\alpha) = \int_{-\infty}^{\infty} f(t) e^{-i\frac{\alpha t^2}{2}} dt.$$

The term v simply means that the fundamental frequency v_o is a continuous quantity or that it can take any value on a continuous scale. In fact, the concept of frequency vanishes when ω is a variable and the function will be aperiodic.

Some Fourier Transforms

For the following examples, we use

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt.$$

1) $f(t) = 1 \qquad -\infty < t < +\infty$

$$F(\omega) = \int_{-\infty}^{\infty} e^{-i\omega t} dt = \int_{-\infty}^{\infty} \cos(\omega t) dt - i\int_{-\infty}^{\infty} \sin(\omega t) dt$$

$$2\int_0^{\infty} \cos(\omega t)\,dt = 2\left\{\int_0^{T} \cos\left(\frac{2\pi t}{T}\right)dt + \int_T^{2T} \cos\left(\frac{2\pi t}{T}\right)dt + \cdots\right\} = 0$$

$$F(\omega) = 0.$$

2) $f(t) = \mu(t)$

$$F(\omega) = \int_{-\infty}^{\infty} \mu(t) e^{-i\omega t}dt = \int_0^{\infty} \cos(\omega t)dt - i\int_0^{\infty} \sin(\omega t)dt$$

$$F(\omega) = \left\{\int_0^{T} \cos\left(\frac{2\pi t}{T}\right)dt + \int_T^{2T} \cos\left(\frac{2\pi t}{T}\right)dt + \cdots\right\} - i\left\{\int_0^{T} \sin\left(\frac{2\pi t}{T}\right)dt + \int_T^{2T} \sin\left(\frac{2\pi t}{T}\right)dt + \cdots\right\} = 0$$

$$F(\omega) = 0.$$

3) $f(t) = 1 \qquad -\tau/2 < t < \tau/2$

$$F(\omega) = \int_{-\tau/2}^{\tau/2} e^{-i\omega t}dt = \int_{-\tau/2}^{\tau/2} \cos(\omega t)dt - i\int_{-\tau/2}^{\tau/2} \sin(\omega t)dt$$

$$F(\omega) = 2\int_0^{\tau/2} \cos(\omega t)dt = \frac{2}{\omega}\sin\left(\frac{\omega\tau}{2}\right)$$

4) $f(t) = e^{i\omega_o t} \qquad -\infty < t < +\infty$

$$F(\omega) = \int_{-\infty}^{+\infty} e^{i\omega_o t} e^{-i\omega t}dt = \int_{-\infty}^{+\infty} e^{-i(\omega - \omega_o)t}dt$$

Letting $\Omega = \omega - \omega_o = 2\pi(1/T - 1/T_o) = 2\pi(T_o - T)/(TT_o) = 2\pi/\Phi$ where $\Phi = TT_o/(T_o - T)$.

For $\Omega \neq 0$,

$$F(\omega) = \int_{-\infty}^{+\infty} \cos(\Omega t)\, dt - i\int_{-\infty}^{+\infty} \sin(\Omega t)\, dt$$

$$2\int_0^{\infty} \cos(\Omega t)dt = 2\left\{\int_0^{\Phi} \cos\left(\frac{2\pi t}{\Phi}\right)dt + \int_{\Phi}^{2\Phi} \cos\left(\frac{2\pi t}{\Phi}\right)dt + \cdots\right\} = 0$$

$$F(\omega) = 0.$$

Conclusions

(a) The *s* parameter used in any transform integral is a constant.
(b) Because *s* is constant, the transform integral is also constant.
(c) Any function that is similar in form to the transform integral is a different independent system.
(d) The concept of analytic continuation is often used to justify an invalid result(s) and is being applied inappropriately.
(e) An aperiodic function doesn't have a Fourier series representation.
(f) The fundamental frequency v_o of a periodic function is a constant that can take any value on a continuous scale.

REFERENCES

[1] Riemann, Bernhard (1859). *On the Number of Prime Numbers less than a Given Quantity.*

[2] Evangelista, Armando M. (2018). *Riemann's Functional Equation is Not Valid and its Implication on the Riemann Hypothesis*. https://doi.org/10.5281/zenodo.1490765

[3] Evangelista, Armando M. (2018). *Disproof of Riemann's Functional Equation Using the Poisson Summation Formula*. https://zenodo.org/record/2652230#.XQEO-C17H9M