Trigonometric Equations and Series by Combinatorial Geometric Series

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Abstract: This paper presents the trigonometric equations and series with binomial coefficients defined in combinatorial geometric series. The trigonometric equations and series can be used in the fields of computing and cybersecurity.

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1. Introduction

When the author of this article was trying to compute the multiple summations of a geometric series [1-12], a new idea stimulated his mind to create a new type of geometric series. As a result, a combinatorial geometric series [12-20] was developed with new idea of binomial coefficients.

2. System of Binomial Coefficients

The combinatorial geometric series is derived from the multiple summations of geometric series. The coefficient of each term in the combinatorial geometric series [17-31] refers to the binomial coefficient [28-40].

$$\sum_{i_1=0}^{n}\sum_{i_2=i_1}^{n}\sum_{i_3=i_2}^{n}\cdots\sum_{i_r=i_{r-1}}^{n} x^{i_r} = \sum_{i=0}^{n} V_i^r x^i,$$

$\sum_{i=0}^{n} V_i^r x^i$ denotes the combinatorial geometric series and V_n^r the binomial coefficient.

$N = \{V_0^1, V_1^1, V_2^1, V_3^1, V_4^1, V_5^1, \cdots\} = \{1, 2, 3, 4,5, 6, \cdots\}$ is a set of natural numbers,

where $V_0^1 = 1;\ V_1^1 = 2;\ V_2^1 = 3;\ V_3^1 = 4;\ V_4^1 = 5;\ V_5^1 = 6;\ \cdots$

Let us show that V_n^r belongs to the set of natural numbers, $i.e.\ V_n^r \in N$,

where, $V_n^r = \dfrac{(n+1)(n+2)(n+3)\cdots(n+r)}{r!}$ and integers $n \geq 0, r \geq 1$.

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Let $V_n^r = V_{Q-1}^1$, where $Q = V_n^r = \dfrac{(n+1)(n+2)(n+3)\cdots(n+r)}{r!}$.

$$V_{Q-1}^1 = \frac{(Q-1+1)}{1!} = Q \quad (OR) \quad V_{V_n^r-1}^1 = \frac{(V_n^r - 1 + 1)}{1!} = V_n^r.$$

$\therefore$ V_n^r belongs to the set of natural numbers, $i.e. V_n^r \in N$.

Trigonometric Equations and Series

Let us construct a trigonometric equation with binomial coefficients defined in the combinatorial geometric series.

$$V_m^n \sin\theta_1 + V_p^q \cos\theta_2 = c, \text{where } c \text{ is a constant.}$$

If we know the values of V_m^n, V_p^q, θ_1, and θ_2, then we can find the value of c.
For example,

$$V_0^1 \sin 90^\circ + V_2^3 \cos 0^\circ = c \implies c = 11.$$

Note that we can use $\csc\theta$, $\sec\theta$, $\tan\theta$, and $\cot\theta$ in the place of $\sin\theta_1$ and $\cos\theta_2$.

Let us construct a general trigonometric series for the application of computing and cybersecurity.

$$\sum_{i=0}^{n}\left(V_i^{P_i} \sin\theta_1 + V_i^{q_i} \cos\theta_2\right) = d, \text{where } d \text{ is a constant.}$$

The following trigonometric series with binomial coefficients is equivalent to the Fourier series.

$$f(x) = \sum_{n=0}^{\infty}(V_n^a \sin n\pi x + V_n^b \cos n\pi x).$$

If the above trigonometric series (Fourier series) is finite and *x=0*, then

$$f(0) = \sum_{n=0}^{t}\left(V_i^a \sin n\pi 0 + V_i^b \cos n\pi 0\right) = V_t^{b+1} \Rightarrow \sum_{n=0}^{t} V_n^b = V_t^{b+1}.$$

3. Conclusion

In this article, we have introduced the trigonometric equations and series (Fourier series) with binomial coefficients defined in the combinatorial geometric series. These trigonometric equations and series can be used in the research areas of computing and cybersecurity.

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